Adaptive significance level

Adriano Polpo[1]

[1]Department of Mathematics and Statistics, University of Western Australia, Australia

*contact: adriano.polpo@uwa.edu.au

September 23, 2021

Abstract

We present the solution to all problems in a hypothesis test (ok, maybe not all, but most of them). This first sentence aims to attract the public, and if you want to know if we solved all problems, come to the seminar! Will you agree or disagree?

The traditional **abstract**:
A testing procedure, based in Pereira et al. (2017), is presented using frequentist and Bayesian tools, with a significance level that is a function of sample size, obtained from a generalized form of the Neyman-Pearson Lemma that minimizes a linear combination of α, the significance level, and β, the probability of rejecting the null hypothesis. Considering $\mathcal{X}$ the sample space, $\boldsymbol{x} \in \mathcal{X}$ a possible sample, Θ the parametric space, $\boldsymbol{H}$ as the null hypothesis, and $\boldsymbol{A} = \Theta \backslash \boldsymbol{H}$ as the alternative hypothesis. We compute the prior predictive distribution of $\boldsymbol{x}$ given $\boldsymbol{H}$, $f_{\boldsymbol{H}}(\boldsymbol{x})$, and given $\boldsymbol{A}$, $f_{\boldsymbol{A}}(\boldsymbol{x})$. Then, we order the sample space by the ratio $f_{\boldsymbol{H}}(\boldsymbol{x})/f_{\boldsymbol{A}}(\boldsymbol{x})$, which is the Bayes factor, $\mathrm{BF}(\boldsymbol{x})$. Applying the generalized Neyman-Pearson Lemma, the test rejects $\boldsymbol{H}$ in favour of $\boldsymbol{A}$ if $\mathrm{BF}(\boldsymbol{x}) < k$, does not reject $\boldsymbol{H}$ if $\mathrm{BF}(\boldsymbol{x}) > k$, and is indifferent if $\mathrm{BF}(\boldsymbol{x}) = k$. k is a constant that define which hypothesis is more important to the research. Typically, is assumed $k = 1$, the hypothesis $\boldsymbol{H}$ and $\boldsymbol{A}$ are equally important. If the experiment yields a result $\boldsymbol{x}_0$, the p-value is calculated as the probability, given probability (density) function $f_{\boldsymbol{H}}(\boldsymbol{x})$, of a point in the sample space favouring $\boldsymbol{A}$ as much as or more than $\boldsymbol{x}_0$ does. The resulting hypothesis tests do not violate the Likelihood Principle and do not require any constraints on the dimensionality of the parameter space. It is worth noting that the approach described here is compatible with guidelines in the ASA's statement on p-values (Wasserstein and Lazar, 2016).

This is joint work with many authors, and I talk about this too!

References

C.A.B. Pereira, E.Y. Nakano, V. Fossaluza, L.G. Esteves, M.A. Gannon, and A. Polpo (2017). "Hypothesis tests for Bernoulli experiments: Ordering the sample space by Bayes factors and using adaptive significance levels for decisions," *Entropy*, 19(12), 696.

R.L. Wasserstein, and N.A. Lazar (2016). "The ASA's statement on p-values: Context, process, and purpose," *The American Statistician*, 70(2), 129–133.