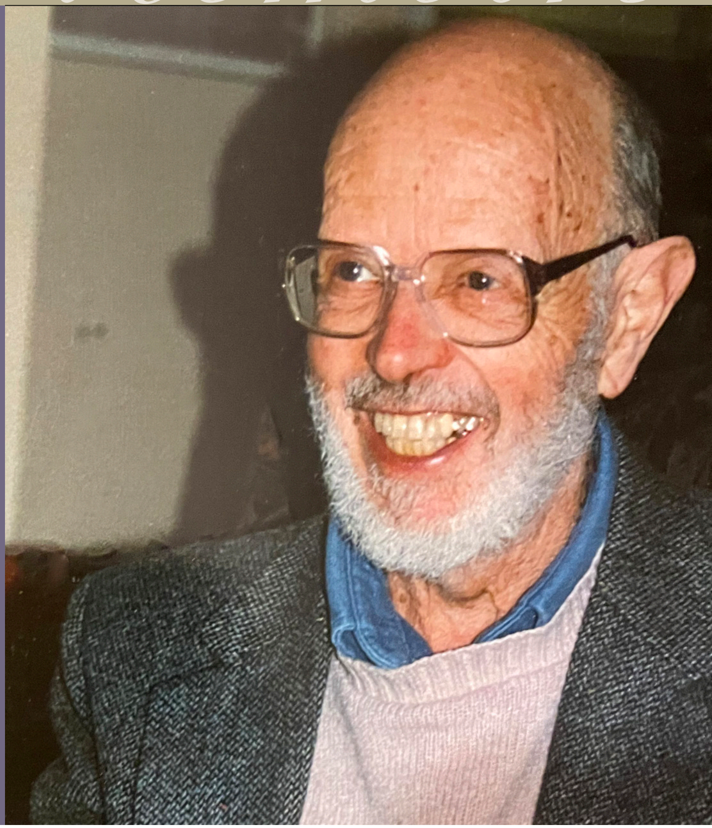
Erich L. Lehmann

1917–2009

BIOGRAPHICAL *Memoirs*



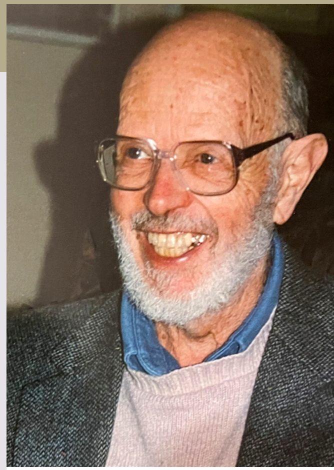
A Biographical Memoir by
P. J. Bickel
and K. A. Doksum

NATIONAL ACADEMY OF SCIENCES

ERICH LEO **LEHMANN**

November 20, 1917–September 12, 2009

Elected to the NAS, 1978



Photography by the Lehmann family.

By P. J. Bickel and K. A. Doksum

Statistics is a confluence of many streams originating in all the sciences: physical, biological, and social. Loosely it can be thought of as the theory and practice of analysis of data using probability models and measures of random variation. Notable names that appear include Bernoulli, Laplace, Gauss, Boltzman, Mendel, Quetelet, and many others. The foundations of the modern mathematical theory of statistics were laid primarily in the twentieth century by R. A. Fisher, J. Neyman, and A. Wald. The ideas of these founders and other foundational ones, such as the Bayesian approaches revived in various ways by H. Jeffreys and L. J. Savage, resulted in an exciting but somewhat disorganized picture. It fell to the next generation to bring these ideas into a system, which new researchers could fit into. Such systematization was Erich Lehmann's great achievement. His two books, *Testing Statistical Hypotheses* (TSH), first published in 1959, and *Theory of Point Estimation* (TPE), first published in 1983, were the basic graduate texts of the field in most parts of the world for several generations of students. They were enriched by Lehmann's own research as well as that of many others as statistics developed and new ideas and areas such as robustness and semiparametric statistics developed. Lehmann contributed to these areas directly as well through his many students, as we will discuss later. He drew in younger collaborators for later editions of his books, such as G. Casella to TPE[1] and J. Romano to TSH.[2] The books were brought up to date though the basic structure and style remained uniquely his.

We now turn to his life. He presents a charming record, highlighting also the development of statistics during his lifetime through his interactions with the founders, seminal figures in the field and various colleagues in his memoir, *Reminiscences of a Statistician: The Company I Kept*, published in 2008, just a year before his death.[3]

He was born in Strasbourg, Alsace-Lorraine, on November 20, 1917, while his father was an officer in the occupying German army during World War I. His family was of Jewish descent and had lived in Frankfurt since medieval times. They fled to Zürich, Switzerland, from Frankfurt in 1933 when Hitler came to power in Germany. He completed his high school studies in Zürich. As with all people of his generation and some of ours, he came to statistics in a roundabout way. His first intellectual love was German literature, which he wished to pursue at the University of Zürich. His father pointed out to him that, with Hitler in power, his opportunities as a scholar in that field would be limited and that mathematics, for which he also had an affinity, would offer better career opportunities. Although mathematics was only his second preference, he had shown talent in it, for instance discovering (but not proving) Fermat's "little" theorem in high school. By a combination of luck and deliberate decision, he escaped the cauldron of the beginning of the Second World War in Europe and eventually made his way to the United States. He started undergraduate study in mathematics in the United Kingdom at the University of Cambridge but disliked the emphasis on mathematical physics. His birth in Strasbourg turned out to be very fortunate because he was able to enter the United States on the then-empty French quota with a passport issued by Lichtenstein. He ended up, in 1941, despite his lack of an undergraduate degree, as a graduate student in mathematics at the University of California, Berkeley. There again he was torn between a first love, pure mathematics and in particular mathematical logic, and applied mathematics in the form of statistics, which he cared for much less. As he said in an interview with M. H. DeGroot,[4]

> *One of the things that I disliked about statistics when I wanted to get out of it, was the applied flavor, their connection with the real world, instead of their being this ideal abstract stuff. I always had the feeling that whatever abilities I had were more in the abstract direction. But the curious thing is that over the years, I have gotten to like the applied aspect of statistics. I like to think about statistics in connection with real situations, not totally in the abstract. So my career in statistics has actually worked out better than I had any reason to expect. I think you will find generally that in my generation everybody came to statistics in a peculiar way, because the subject didn't really exist.*

Practical considerations, which were of great importance particularly at the beginning of the Second World War, led him to become a student of Jerzy Neyman, who in addition to being one of the founders of modern statistics was also the founder of the statistics group in the Mathematics Department and subsequently of the Statistics Department. After a stint as an aerial photograph analyst in Guam, he returned to Berkeley in 1945 and, on obtaining his Ph.D. in mathematics in 1947 joined the faculty of the Mathematics Department. While still a student, he introduced a fundamental idea that appeared in 1947.[5] This was the notion of a minimal complete class of decision procedures, which became a pillar of the theory developed by Abraham Wald.[6] At the same time, he began one of the many collaborations that marked his career, this time with Henry Scheffé, then a visiting Guggenheim Fellow. Although a paper in the *Proceedings of the National Academy of Sciences* (PNAS) was published in 1947, their work unifying a number of disparate phenomena observed previously, with the idea of the completeness of a sufficient statistic, was fully developed in their later 1950 paper.[7] These works already showed that Lehmann had brought his talents for abstraction to statistics. The work with Scheffé also exhibited a fundamental trait, his love of working with others. This love manifested itself in the many important collaborative papers and in the very large (43) and impressive group of Ph.D. theses that he supervised. We'll return to this topic after we further discuss his major research contributions. From 1947–59 Lehmann's main focus was in building up his integrated view of the decision theory of Wald and the more classical testing and confidence region point of view of Neyman and to some extent Fisher.

In addition to the work with Scheffé we have mentioned, Lehmann between 1948 and 1953 on his own and in a collaboration with Charles Stein developed the testing of composite hypotheses.[8] Characteristically, they developed a very general method of constructing tests that were optimal by accepted criteria and then applied the method in many important situations, with some expected and some quite surprising results. In their next major work,[9] they began using, for the first time, the powerful notion of invariance of a statistical problem under a group of transformations and developed a framework in which they established for the first time links between Fisher's permutation tests and the optimality principles of Neyman and Wald. Moreover, he learned of Stein's unpublished work on minimaxity and invariance with G. Hunt. In his 1959 book[10] and later in 1983,[11] he showed how this framework could provide a unified understanding of large classes of problems in the two pillars of statistics, testing and estimation.

He also began working with the major collaborator in his life, J. L. Hodges Jr. During World War II, in 1944, Hodges and Lehmann had spent time together in an Operations Analysis group in Guam. In 1947, the same year that Lehmann became a faculty member, Hodges came to the Berkeley Mathematics Department as a student and received his Ph.D. in 1949. Lehmann described their partnership as complementary thinkers, Hodges as "a problem solver" and himself as "a systems builder." They focused first on the other major area of statistics at the time, point estimation, and immediately made a brilliant contribution. They showed, by an unexpected differential inequality, that the mean of a Gaussian sample was admissible as an estimate of the population mean. Admissibility here meant that there was no estimate whose mean square error was larger than that of the mean, and strictly smaller for some values of the population mean. This result was expected, but no proof had been available until the discovery of this approach and the parallel but totally different argument of Colin Blyth, a Ph.D. student of Lehmann's. Indirectly, these arguments led to Stein's famous subsequent result that admissibility was only valid for 1- or 2-dimensional Gaussian samples, but not for 3-dimensional or higher ones. In turn, that led to the idea of regularization of procedures, which still plays a critical role in today's high dimensional statistical problems. A second paper showed by example the inadequacies of Wald's minimax point of view.[12] Yet another seemingly purely technical result is conceptually of importance to the current basic stochastic gradient descent algorithm.[13] Another conceptually important collaboration of theirs started the exploration of compromises between the Bayesian and frequentist points of view.[14] Two other joint papers on testing and a new area, multiple comparisons, completed their initial collaboration.[15,16] A new phase in their collaboration and in Lehmann's interests began with their intensive analysis of nonparametric methods.[17]

The term "nonparametric" came into prominent use in the 1950s. It refers to models that correspond to situations where we observe one or more samples from populations whose distribution is completely unknown. Such models had been implicitly considered by Fisher and others. Early work in this area was done by Kolmogorov, Smirnov, Pitman, Wilcoxon, Mann, Whitney, and Hoeffding.

Of central interest was the hypothesis that 2 samples come from the same population. Tests can be constructed such that the probability of a Type I error, that is the probability of falsely deciding against this hypothesis, does not depend on the common unknown distribution from which the samples have been drawn. It is such tests that were called nonparametric.

In the mid-1950s, Hodges and Lehmann turned their attention to the question of power for nonparametric tests. In fact, Hodges and Lehmann focused on what are now called semiparametric models. An example we pursue is the two-sample model, which specifies that the distribution of a sample from one population is obtained by a constant but unknown shift from a sample of another population. Hodges and Lehmann focused on statistical tests for the hypothesis that the shift was zero, or equivalently that the two samples came from the same population. That is, given some model for departures from the hypothesis, and a given nonparametric test, what is the probability of correctly deciding that the hypothesis is false? They did this in the context of the two-sample shift model we have been discussing. As an example, such a model may be appropriate when two treatments (one of which could be a placebo) are compared. For such samples from two populations, nonparametric tests had been constructed by using the combined ranks of the responses from the two samples. One such test was the Wilcoxon test, which was based on the sum of the ranks of one of the samples. In their 1956 paper,[17] Hodges and Lehmann studied the asymptotic power properties of this test using the concept of Pitman efficiency (ratio of sample sizes needed to achieve the same power) to compare the power of the Wilcoxon test to the widely used t-test, which is optimal given Gaussian assumptions for the two populations. They found, surprisingly, that for large sample sizes, despite the ranks inability to carry any information about each sample viewed on its own, the efficiency, when the underlying population was Gaussian, was 0.955. Moreover, it has a lower bound of 0.864 for any population and can be arbitrarily high. This result suggested that much can be gained using rank-based tests, and there is little to lose. In this paper, Hodges and Lehmann conjectured that if the ranks were transformed to make them approximately normally distributed under the hypothesis (called normal scores), then the lower bound on the efficiency would be 1 and would be greater than 1 for all distributions except the Gaussian distribution. This conjecture was verified by Chernoff and Savage in 1958. This discovery of the high efficiency of rank-based methods sparked a flurry of research activity, led by Hodges and Lehmann and followed by others, that lasted decades. Their 1956 results were shown to hold for many 2 sample problems and corresponding rank tests and to extend to many experimental frameworks beyond the two sample cases.

Particularly important extensions were made by them to estimation and confidence procedures. In 1963, Hodges and Lehmann proposed a highly novel method for constructing estimating equations corresponding to confidence bounds based on rank test statistics for the shift parameter in two sample experiments.[18] They showed that Pitman efficiency results established for rank tests carry over to efficiency, measured by the ratio of asymptotic mean square error, in estimation. In particular, the estimate based on the normal scores statistic has efficiency bounded below by 1. The estimate based on the Wilcoxon statistic has a simple intuitive form and is known as the Hodges-Lehmann Estimate. In 1963, Lehmann extended such results to confidence procedures and systematically in, a series of papers, extended them to linear models.[19–22] Many researchers, including Lehmann's students, have extended these basic ideas and results to more general frameworks. A 2010 book by Hettmansperger and McKean, *Robust Nonparametric Statistical Methods*,[23] covers many of the extensions of the basic original Hodges-Lehmann ideas, including major contributions by Jureckova.[24,25]

Hodges and Lehmann continued to work in this area. In 1970, they noted the surprising phenomenon that in some cases, for example, the two-sample problem when t test and the Z test designed for the comparison of Gaussian populations with the same known variance were compared, under the Z test assumptions, not only was the Pitman efficiency 1 but the difference in sample sizes needed to reach the same power stayed bounded.[26] They proposed that this phenomenon be investigated in connection to rank tests and related estimates. This was indeed done by Bickel and van Zwet in 1978.[27]

In 1966, Lehmann produced a highly influential paper on concepts of dependence, order relationships between distributions of pairs of random variables.[28] This idea, starting in 1974, turned into an element of a general theory for parameters characterizing basic properties of populations in the nonparametric framework. For instance, with Bickel, he abstracted properties shared by the mean and median as measures of the center of a population.[29] A surprising consequence was a confidence interval that, in the bounded case, with given probability contains all values of the measures of center abstracted by Lehmann.[30] As his system-building character dictated, he in 1975 published a wonderful book, *Nonparametrics: Statistical Methods Based on Ranks*,[31] encapsulating his and Hodges' research as well as work by others in this blossoming of nonparametric statistics. During the late 1970s, he returned to an area he had worked on briefly earlier with Hodges, multiple comparisons. He embarked on this with his statistician wife, Julie Shaffer.[32] This return reflected in part, his awareness of the direction statistics was following in the era of big data, where many simultaneous decisions based on the data had to be made.

The Books

Lehmann's *Testing Statistical Hypotheses* (TSH) was first published in 1959 and was succeeded by a second edition in 1986 and a third with Joe Romano in 2005. His *Theory of Point Estimation* (TPE), first published in 1983 but preceded by notes available since the late 1950s, was succeeded by a second edition with George Casella in 1998. Together, the books provided a unified theoretical foundation for mathematical statistics for two generations of graduate students. Both were firmly founded on the decision theoretic point of view of Wald, generalizing the principles introduced into testing by Neyman and

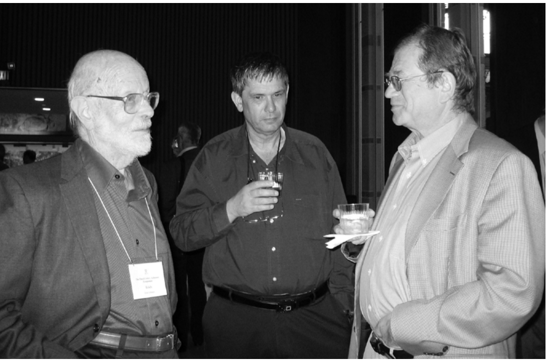
Pearson. But they grew in richness and in size with the growth of the field based on new types of data and questions and new and revived older approaches to dealing with these questions. Nonparametric methods and models, large sample approximations, Bayes and empirical Bayes approaches, robustness of solutions, and multiple comparisons came to the fore as the amounts of data, their complexity, and simple assumptions about their generation became untenable. Nonparametric and robust approaches reflected our lack of knowledge of the generating process, and their analysis was only possible through asymptotic approximations. Bayes and empirical Bayes approaches could be viewed as moving us from classes of methods such as maximum likelihood, which tend to fail in high-dimensional data and parameter spaces, by implicitly reducing the dimension of the data space and probability models. While his research work touched on only some of these ideas, he was aware of their importance and incorporated them into his canonical books. The range of the books and their size grew proportionately. TSH grew from 350 pages in 1959 to nearly 800 pages in 2006, TPE from 500 to 600 pages. Of major importance also was his book *Nonparametrics: Statistical Methods Based on Ranks*, which we have already mentioned.

Lehmann's Philosophy of Statistics

His views evolved considerably during his life. As he writes in his memoir, "...my first statistical education as a student of Neyman was supplemented by the later influence of Fisher's ideas." His intellectual inclinations resonated with Wald's rigorous mathematical formulation of the foundations of the field, and the subjective as opposed to frequentist view of probability did not appeal to him. But he felt perfectly comfortable with Bayesian or other methods as a way of generating procedures. He realized with time Fisher's profound, if not entirely rigorous, creation of the foundations of the field.

Lehmann's Students

Lehmann loved to interact with people. This is attested to not only by his large number of collaborative publications and long list of Ph.D. students, but also by an article[33] and a book[34] on the people who influenced him and more generally the inhabitants of his long intellectual life. Lehmann mentored forty-three Ph.D. students, a number of whom also followed academic careers and in turn had Ph.D. students. His academic descendants are in the thousands. Lehmann students came from all over the world. To quote from his memoir: "Most were American, but many came from Asia, particularly India, Taiwan and Korea, others came from Europe: Norway, France, and Germany: and still others came from Israel."



Erich Lehmann, Ritov, Bickel at Lehmann fest in Rice University, Houston TX, 2007. (Photo source the Lehmann family.)

In his memoir, Lehmann also details his personal gains in working with students. He writes, "I found working with Ph.D. students very rewarding. They greatly enriched my research, and several of them became lifelong friends." He goes on to describe in detail how his first student, Colin Blyth, "profoundly affected my life and carrier." Blyth did this by turning Lehmann's class lectures on *Testing Statistical Hypotheses* into written material, which later was developed by Lehmann into his influential TSH. He goes on to say, "Some of my students became lifelong friends and occasionally, collaborators. I wrote joint papers with Wei-Yin Loh and Fritz Scholz, and for many years collaborated with Peter Bickel. A close relationship also developed with my last student, Javier Rojo." In appreciation of his talent, support and friendship, students and colleagues published a festschrift[35] and organized four symposia in his honor.[36–39]

His career was crowned with many honors, including election to the American Academy of Arts and Sciences in 1975 and the National Academy of Sciences in 1978, essentially all the honors and prizes of the statistics world and two honorary doctorates, one from the University of Leiden in 1985 and a second from the University of Chicago in 1991. Although he appreciated the honors, he remained his quiet and modest self. His scientific and personal legacy lives on.

Lehmann's 65th birthday celebration, 1982. (Photo source the Lehmann family.)

Note from P. J. Bickel

Unfortunately my coauthor, K.A. Doksum, became ill and passed while our memoir was in production. Although our collaboration this time was at a distance, we complemented each other perfectly on this work in our shared affection and respect for our thesis adviser. He will certainly be missed.

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