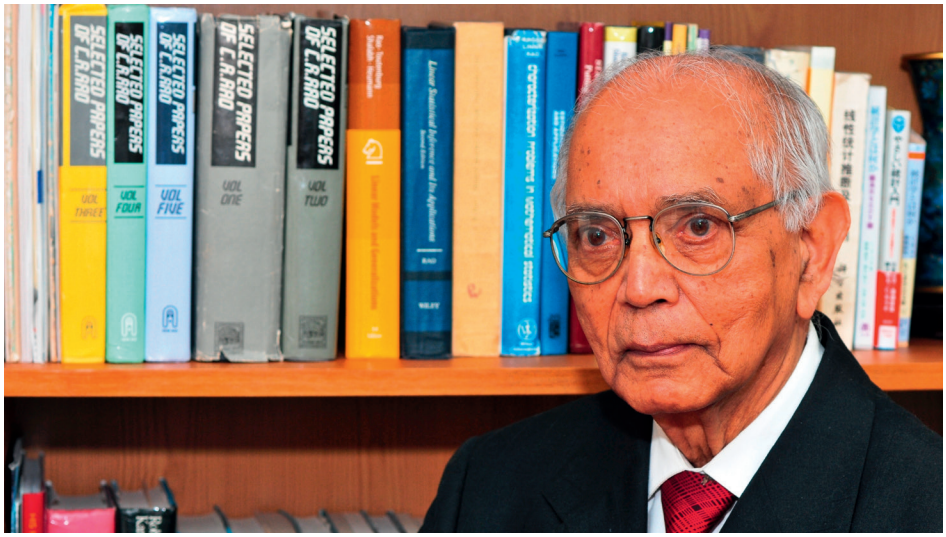
C. R. Rao's century

On 10 September 2020, C. R. Rao turns 100. **Bradley Efron**, **Shun-ichi Amari**, **Donald B. Rubin**, **Arni S. R. Srinivasa Rao** and **David R. Cox** reflect on his contributions to statistics



"Rao *really* was Fisher's student"

Bradley Efron, professor of statistics and biomedical data science, Stanford University

In 1965, I was a first-year Stanford postdoctoral student, awash in provocative ideas from the brilliant post-war school of statisticians – Robbins, Stein, Tukey, Cox, and, of course, C. R. Rao.

When the fat second edition of Rao's magisterial book on linear statistical inference arrived on my desk, it was a big event in the department, not just for me.[1] (The book is still in use, though it has gotten a little beat up.)

To say that Rao was R. A. Fisher's PhD student is true enough, but it doesn't get across the fact that Rao *really* was Fisher's student, in the sense of carrying on the Fisherian statistical tradition. "Inference" is the key word here. It evokes the scientist considering his or her most recent experimental data, and trying to assess the likelihood of various competing interpretations; it stands in at least partial opposition to the harder-edged Neyman–Wald decision-theoretic viewpoint. Rao's book is a testament to Fisherian data analysis. The author was, and is, the premier Fisherian of the post-war era.

A notable characterisation of Rao's work, and Fisher's too, is its reliance on geometric intuition, substituting what, for me, are vivid pictures in place of rote algebra and analysis. Perhaps the best example is the Fisher–Rao metric, which uses Fisher information to define distance between densities in a parametric probability space. Devised by the 25-year-old Rao in 1945,[2] it introduced differential geometry into statistical inference, opening up the burgeoning field now called information geometry.

I have some vested interest here. My 1975 paper on statistical curvature followed Rao's lead.[3] One of its goals was to buttress Fisher and Rao's theory of second-order efficiency, which claimed a preferred position for maximum likelihood estimation (MLE) among all other first-order efficient methods such as minimum chi-square.

Maximum likelihood is so central to modern practice – in the machine learning world as well as in traditional applications – it is easy to forget that its dominance was by no means pre-ordained in the post-war decision-theoretic environment. Indeed, in a 1980 paper, "Minimum chi-square, not maximum likelihood!", biostatistician Joseph Berkson took Rao and, to a lesser extent, me to task for glorifying MLE at the expense of its Pearsonian competitors.[4] But Rao's deeply argued advocacy played a central role in MLE's eventual success, and Berkson's arguments were what now seem a final salvo of opposition.

I'm picturing a very large cake with one hundred candles (and "one to grow on"), with a chorus of one thousand singing the birthday song. Thank you, C. R., for a long and especially inspirational career. Happy centenary!

"He is a godfather of Japanese statisticians"

Shun-ichi Amari, professor in the RIKEN Center for Brain Science

It is essential for statistics to study how far two probability distributions are separated – in other words, their distance. Professor Rao's first paper, published in 1945, is a monument.[2] It proposed the differential-geometric structure of a manifold of probability distributions, and is the origin of information geometry. However, it took a long time for its importance to be understood and popularised. Efron, studying unpublished calculations by Fisher, found that the statistical curvature of the manifold is indispensable for analysing the higher-order asymptotic efficiency of statistical inference,[3] but it was in Russia and Japan that the mathematical framework of the geometry inspired by Rao was developed further.

Rao's initiation of information geometry is one of the many achievements for which he was awarded the US National Medal of Science. Information geometry has grown to become an important tool not only in statistics but also in artificial intelligence, data science, signal processing, physics and many other fields, since it elucidates the fundamental structure of the manifold of probabilities.

I first met Rao in 1984, at an international workshop in London, organised by Sir David Cox, on the differential geometry of statistics. Rao has guided me in the field of statistics ever since, and he is a godfather of other Japanese statisticians too, having invited young researchers to the Indian Statistical Institute to teach them modern statistics in the aftermath of World War II.

I remember in 1999, at a banquet of the International Statistical Institute in Helsinki,

Rao and I had a pleasant discussion over dinner. Later, a young Japanese researcher asked me who the gentleman sitting beside me was. When I told them it was Rao, the researcher said: "Oh! the famous legend, Professor Rao, he is still alive!" Indeed, he is – and celebrating his centennial year in good form.

"A jewel of statistics"

Donald B. Rubin, professor, Tsinghua University; senior research fellow, Temple University; and professor emeritus, Harvard University

I was "introduced" to Professor Rao by my Scottish PhD advisor, Bill (William G.) Cochran around 1968. Bill suggested that I turn to Rao's famous textbook on linear models for its straightforward mathematical clarity, at least relative to some other "math-stat" texts that were in use at the time. Being an official dinosaur, I still use it as a "go to" resource.

Shortly after that introduction, I was exposed to some of the massive contributions to statistics with Rao's name attached to them: Rao–Blackwell, Fisher–Rao information, Rao distance, Cramér–Rao bounds – a remarkable collection.

One aspect of Rao's work that did not strike me until years later was his focus on big ideas that had major effects on the subsequent directions of the field of statistics. He clearly had tremendous mathematical acuity, like his PhD advisor, Fisher, who also focused on big ideas. But, unlike Fisher, who often (to me, at least) seemed to be writing notes to himself, Rao was writing to communicate to others with great clarity.

I do not remember when I first met Rao in person, but I think it was when I was visiting a colleague at the Penn State Department of Statistics. Rao was clearly the "heart" of that department, surrounded by many talented individuals, but his presence created its vitality and helped to ensure its stature among US departments of statistics.

Despite his dominant reputation, Rao always seemed to be extremely modest and, moreover, helpful to younger colleagues. There were several times over the years when he arranged for me to participate in events; the most recent of these was when I was honoured to receive the Rao prize at Penn State in 2017.

A couple of comments made to me by some elder stars in our field show how highly Rao's contributions are regarded. The first is from John Tukey, with whom I would occasionally have lunch or coffee when I was an undergraduate and part-time lecturer at Princeton University in the early 1970s. I remember one conversation, when Tukey said what a bright light Rao was, and how much he admired his work – no small compliment from someone who could count polymath John von Neumann and Albert Einstein among his colleagues! Then, a few years later, when I was visiting UC Berkeley, I had a luncheon conversation with Jerzy Neyman, the founder of Berkeley's statistics department and a combatant of Fisher's. Neyman mentioned to me how much he would have liked to have Rao in that department.

When thinking about Rao, I am reminded of the corny Hollywood movie, *The Jewel of the Nile*. I think this is because Bill Cochran, who spent much time in India, once said of his younger colleague that he would become a "jewel of statistics". And he did.

"Rao's ideas continue to find new applications today"

Arni S. R. Srinivasa Rao, professor at the Medical College of Georgia, Augusta University

I first heard about Professor Rao's work and famous contributions in statistics (without knowing their meaning and impact) during my high school to undergraduate days in the late 1980s and early 1990s. My mathematics lecturer, Mr M. Perisastri, was five years younger than Rao and was very fond of him – in fact, his father was a teacher of Rao's at Andhra University.

I met Rao for the first time around 2007–8 when he visited India on an academic tour. I was an assistant professor at the Indian Statistical Institute (ISI) in Kolkata at the time. Then, in 2010, I served as a co-convener of Rao's 90th birthday celebration conferences in Kolkata, and I had an opportunity to interact with Rao and the late Mrs Bhargavi Rao (who died in 2017 at the age of 93). The Rao couple were admired and treated with utmost respect by all at the ISI Kolkata campus whenever they visited.

A few years ago, after some work of mine was highlighted in publications of the

Timeline: C. R. Rao

1920 **Born in the town of Hadagali, India**
1937 **Completed BA from A.V.N. College, Visakhapatnam, India**
1940 **Awarded MA in mathematics, Andhra University, India**
1941–79 **Worked at the Indian Statistical Institute**
1943 **Awarded MA in statistics, Calcutta University, India**
1945 **Published first paper from Kolkata, which later led to results including the Cramér–Rao bound, Rao metric and Rao–Blackwellisation**
1947 **Published foundational paper on orthogonal array**
1948 **Published foundational papers on multivariate analysis of variance and tests of significance in multivariate analysis**
1948 **Awarded PhD from Cambridge University, UK, under the guidance of R. A. Fisher**
1965 **Published breakthrough book, *Linear Statistical Inference and Its Applications***
1971–6 **President of the Indian Econometric Society**
1973–5 **President of the International Biometric Society**
1976–7 **President of the Institute of Mathematical Statistics, USA**
1977–9 **President of the International Statistical Institute**
1979–88 **University professor, University of Pittsburgh, USA**
1988–2001 **Eberly professor, Pennsylvania State University (PSU), USA**
1995 **Made a Fellow of the National Academy of Sciences, USA**
2001–10 **Director of the Center for Multivariate Analysis, PSU, USA**
2001 **Awarded Padma Vibhushan, India's second highest civilian award**
2002 **Awarded National Medal of Science, the USA's highest science award**
2003 **Awarded International Mahalanobis Prize by the International Statistical Institute**
2010 **Awarded Guy Medal in Gold by the Royal Statistical Society, UK**
2010– **Research professor, University of Buffalo, SUNY, USA**

American Mathematical Society and American Statistical Association, I was invited by Rao to coedit volumes of the *Handbook of Statistics*, which he has edited for almost four decades. That was extremely exciting for me, and we have so far edited five volumes together. His passion for the subject, and for the book series, remains unchanged.

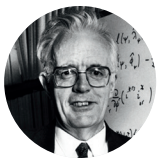
Rao's ideas continue to find new applications today. A forthcoming paper of mine, with Steven G. Krantz, applies the Fisher–Rao metric and Rao's distance measure to virtual tourism technology.[5] In essence, the space created by the Fisher–Rao metric and Rao's distances, combined with conformal mapping principles of mathematics, can help in maintaining consistent measures between various objects within two- and three-dimensional visuals. Other recent publications marking Rao's centenary highlight his significant contributions in areas such as econometrics,[6] sample surveys,[7] and distribution theory and inference.[8]

I was happy to see Rao most recently in Buffalo, New York, during the spring of 2019, and found him to be in good humour. He shows tremendous respect for his collaborators, colleagues, students and family, and I pray to God for continued blessings to Professor Rao, a living legend of statistics.

Above: A photo from C. R. Rao's 70th birthday celebration meetings, held in Switzerland in 1990. Professor Rao is pictured third from right; Sir David Cox is third from left.

"Rumours of a brilliant student working with Fisher reached me"

Sir David Cox, professor in the department of statistics, University of Oxford



It is a privilege to have the chance to congratulate Professor Rao on his centenary and more particularly to express my admiration for his remarkably rich, original, influential and wide-ranging contributions to our field. It would be absurd to aim to summarise these in a short article, so the following remarks are an outside observer's attempt to give an impression of his astonishing achievements in India.

Mahalanobis[9] had in the 1930s established the Indian Statistical Institute (ISI) in what was then called Calcutta, and this had a major impact in India and in establishing Indian-based statistics as a major force internationally. Mahalanobis had strong and fruitful relations with R. A. Fisher and bad relations with Karl Pearson. He sent Rao to Cambridge to work in Fisher's Department of Genetics, which was physically and otherwise a long way from the Statistical Laboratory, which was in the Faculties of Mathematics and Agriculture. At the time, in the mid- to late 1940s, I was working in industrial research. But, even there, rumours of a brilliant student working with Fisher reached me, though it was not until 15–20 years later that I first met Rao (although I had earlier encountered the formidable figure of Mahalanobis).

Starting in the late 1960s, I had the privilege of visiting the ISI several times, both in Calcutta and, later, in the splendid new campus in New Delhi, which, by some miracle of organisational mastery, Rao had established. These visits, and very particularly Rao's role in them, remain vividly in my mind. He also made a major contribution in Hyderabad, where there is, I think, in particular a road named after him.

Rao, of course, also made major contributions during the later period when he was primarily based in the USA and, these being quite recent, I shall not attempt to comment on them.

I will conclude with two more personal memories, recalled with warmth. One is of Rao's 70th birthday conference in Neuchâtel, Switzerland, organised by Professor Yadolah Dodge, where one evening his daughter danced with unforgettable style and dignity. The other was in Hyderabad where my wife and I were asked to go with Mrs Rao while she chose a new sari. This proved to be a multiple-choice problem of astonishingly high dimension. At a surely judiciously planned moment, Rao himself appeared, made a brief and characteristically perceptive comment, and all was resolved to general satisfaction. A metaphor?

Happy Birthday, Professor Rao, and thank you! ■

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Note

Professors Cox, Efron and Rubin were speakers at a special invited session of the JSM 2020 Virtual Conference, organised by Professor Arni S. R. Srinivasa Rao, in honour of Professor C. R. Rao's 100th birthday. See bit.ly/CRRao100 for details.