*Ole E. Barndorff-Nielsen*

# STOCHASTICS IN SCIENCE

*Some autobiographical notes*

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# FOREWORD

This book is not about family and does not endeavour to express the extent to which was here is told has been dependent on the love and support from my family and most of all from my wife Bente. I am still hopelessly in love with her and hope that this will nevertheless shine through.

Throughout our life together, from the first storming loving attraction and all the way till our recent 60th wedding anniversary, she has made it possible for me to pursue my fascination of Stochastics, the common term for Probability and Mathematical Statistics, while at the same time devotedly foster family life, in all respects.

What excuse might there be for writing an autobiography (beyond a certain embarrassing amount of self-esteem?) In my case I can think of three. Firstly, the reading of biographies is a favourite pastime of mine. Secondly, my hope is that these memoirs will give the lay reader an impression of how varied and exciting life as a mathematician can be, both in strictly scientific terms and from a general human perspective. At the same time, there may be points that have an interest for some of my fellow scientists. Thirdly, I am fortunate in having had the opportunity to work with a great variety of scientific topics where mathematics goes hand in hand with real world problems.

A more narrow point is that I am the only person alive who can tell about the early days of the development of the Institute of Mathematics at Aarhus University.

Throughout the text glimpses are given of what may be considered the main thrusts of my theoretical work. Necessarily this will be partly couched in mathematical terms, but with as few technicalities as possible; no real impression can be given in the present context of the often highly advanced mathematical considerations lying behind some of the results.

In the range of research fields encompassed by the natural and technical sciences Stochastics has a modest role in that, by its very nature, it does not generate results that are spectacular in the public eye, such as breakthrough developments in physics or chemistry. Its role lies in finding systematic, mathematically based, methods for uncovering significant structures in systems where randomness strongly influences the possible observational outcomes. A hallmark of such a method is its versatility, that is its applicability to a wide range of problems.

I have taken the modest role of Stochastics in the Sciences as an excuse to include, at various places in the manuscript, some assessments of my work by other researchers.

The title of this book might have been “Stochastics in Science – and Opera”. My passion for opera, and in particular Richard Wagner’s works, stems from my father who introduced me to opera at an early stage of my life. And my life in Stochastics in Science have given many opportunities to pursue that passionate interest, as will transpire sporadically in the book.

Throughout my scientific life I have been privileged to study and work together with senior and junior scientists from around the World, many of whom are mentioned in the present notes. Hopefully these memoirs will convey an impression of my immense gratitude to them, as friends and fellow scientists.

In several cases of these contacts material of great historical interest has been involved. Some of this material, written by my collaborators, is included here with their permission, and has not been published elsewhere.

The book is not meant for systematic reading, and here and there are some minor overlaps in the account. Rather, it is my hope that readers may find some enjoyment in browsing the text according to his or her interests. It should be easy to distinguish the more technical material from that of broader human nature.

Acknowledgements: ...

Aarhus, ....2018
Ole Eiler Barndorff-Nielsen

*Umsonst sucht' ich,*
*und sehe nun wohl:*
*in der Welten Ring*
*nichts ist so reich,*
*als Ersatz zu muten dem Mann*
*für Weibes Wonne und Werth!*

Loge in Richard Wagner: Das Rheingold

# EARLY DAYS

# 1 FROM START TO CHAIR

## 1.1 ADOLESCENCE

I grew up in Copenhagen as an only child in a small loving family. My father Eiler Nielsen and mother Edith Barndorff were both from poor homes. My father had no brothers or sisters; my mother had three brothers, none of whom had any children.

I was baptised Ole Eiler Nielsen but on starting school it turned out that there were five other boys in the same class with the name of Ole Nielsen. My parents therefore decided to change my name by adding my mother's maiden name Barndorff with a hyphen to Nielsen.

Neither my father nor my mother had any advanced education. But both had language skills and my father was very good in doing fast ordinary calculations. They both obtained positions in a large insurance firm, 'Skandinavia' Grøn og Witzke, situated in the midst of Copenhagen at Kongens Nytorv, right across from the Royal Theater and Opera house.

**Kongens Nytorv 1907. Painting by Paul Gustav Fischer (1860-1934). In the background, on the right, the French Ambassy and to the left of that 'Skandinavia' Grøn & Witzke.**

Born 1935 my recollections from the Second World War are scarce. Denmark was occupied by the Nazi regime from 9 April 1940 till 4 May 1945. At first the occupation was conducted in a relative mild way compared to the other occupied European countries, but a resistance movement gradually grew up. The situation changed drastically in 1943 when the Germans moved to capture the Danish Jews in order to send them to one of the concentration camps. Somehow the Danish resistance movement got wind of this and it was possible overnight to sail most of the Jews to neutral Sweden in small vessels. In another move the Germans tried to arrest the Danish police force but many of the force were able to escape. One of them stayed undercover in our home for a few days until he managed to escape to Sweden.

While I was in secondary school, 1946-1948, my parents encouraged me to work diligently in order to attain access to Metropolitanskolen which was considered an elite school, of which there were no others. The history of Metropolitanskolen went back to 1209 where it was founded by Peder Sunesen, the Bishop of Roskilde. Until 1938 the school was located next to Copenhagen University.

I was admitted to Metropolitanskolen in 1949 and was taught there by devoted teachers in both classics and science. My favourite was the teacher of German Hr. Stubbe. He learned of my interest in opera, and in particular of Richard Wagner. On the side, in a minor way, my father sold classical records privately and we had a small selection of classical music at home, playable on an oldfashioned gramophone, wound by a crank and where the pickup weighed about half a pound.

**With my father**

Two of my favourite records were of the Overtures to Wagner's operas 'Die Meistersinger von Nürnberg' and 'Rienzi'. The latter sounded very loud, even for Wagner. These records and my father's enthusiasm were the beginning of my lifelong love of and interest in opera and especially in the life and works of Richard Wagner.

Hr. Stubbe arranged for me to give a talk about this interest of mine in one of our German classes. This included the 'Rienzi' record. Unfortunately, shortly after the opening bars of the overture, Rector Bang passed by outside and ordered a stop to the proceedings, no doubt to the relief of my fellow students.

## 1.2 STATE SERUM INSTITUTE AND EARLY STUDIES

My first working experience with statistics and stochastic modelling was around 1955, at which time I had a part time employment at the Biostatistics Department of the Danish State Serum Institute in Copenhagen. The Chief of the Department was Michael Weis Bentzon (1920-2018), whose background was in actuarial science. Affiliated to the Department as consultant was Georg Rasch who had founded the Department and had headed it from 1940 to 1956. After taking the degree of cand. mag. in mathematics at Copenhagen University, Rasch had worked from 1925 to 1940 as Scientific Assistant to Niels Erik Nørlund[2] and had written a "doktorafhandling" (Sc.D.)[3] in mathematics on product integration[4]. In connection with various problems in Physiology Rasch had become interested in statistical questions, and his knowledge and interest in the field had been greatly enhanced through a sabbatical stay in London at the Galton Laboratory in 1935.

The job as assistant at the Biostatistics Department was crucial for my scientific future. Having originally started at Copenhagen University by studying medicine, I soon realized that this was not for me. Looking for a line of study that included much mathematics, and preferably – and in contrast to medicine – had very few students, Actuarial Science seemed an ideal choice. The Professor of Actuarial Science William Simonsen had a scientific assistant Tøger Busk who had been asked by Weis Bentzon to look for a student that might be interested in a job as computing assistant. Tøger Busk suggested me for the job, which I got.

Tøger Busk (1917-1996), who was Professor of Numerical Analysis at the Danish Technical University 1965-1987, received in 1992 the Polish Military Cross for leading a branch of the Polish Secret Service in Denmark during the last four months of the Second World War, something that I would have foresworn if asked about it at any earlier stage, as outwardly Busk, who was a very kind man, gave the impression of being somewhat far from the realities of this World. But that would seem ideal for Secrete Service work.

The position at the State Serum Institute turned out, due to the exceptional research environment created by Michael Weis Bentzon and Georg Rasch, to be decisive for my whole future, and I stand in great gratitude for this. There was, naturally, much routine work to be done, but Bentzon and Rasch soon involved me in discussions of research problems connected, sometimes

Georg Rasch

in a rather loose sense, to problems of actual importance for the responsibilities of the Department, and after a while I spent most of my time there working on such questions.

It was an exciting time in science generally and particularly in Physics, the debate around the differing viewpoints of Einstein and Niels Bohr being still very lively, and in Biology, where the quest to understand the fundamental laws of biological life was the prime theme. The endeavours in this regard were centered on the Phage Group, an informal international research network that studied bacterial genetics and molecular biology.

Two of the Danish biologists participating in these endeavours were the molecular biologist Ole Maaløe[5] and the immunologist, and later Nobel Prize winner, Niels Kaj Jerne, both of whom were associated to the Danish State Serum Institute. Another member of the Phage Group was the American molecular biologist James Dewey Watson[6], who spent a year in Copenhagen, 1950 to 1951, studying with Ole Maaløe.

Jerne (1911-1994) was a renowned immunologist whose work was mostly of a theoretical kind. In many ways he was a highly exceptional human being and scientist. His career did not follow an ordinary pattern, neither career-wise nor in his private life, as described in great detail in a vivid – and long (755 pages) – biography written by Thomas Söderqvist and entitled "Hvilken kamp for at undslippe" ('What struggle to escape'). He held a long succession of leading scientific positions in Denmark and around the World. Jerne met originally much criticism for some of his path breaking ideas. He received the Nobel prize rather late in life (1984).

As will be seen from my description of Rasch's life his career was not unlike Jerne's. They both did not spend much time in studying the works of others but relied on following their own intuition and inspirations:

> "*Videnskaben består ikke bare i en akkumulerende serie af falsificerbare påstande, som eksperimentalisterne derpå søger at falsificere, men også, og især, i en forestillingsevne: en udvikling af begreber og nye perspektiver, der ændrer synsmåden og arten af påstande, diskussioner og eksperimenter.*
> Niels K. Jerne

Neither Jerne nor Rasch were prone to study large parts of the literature in detail. In some measure I have had the same attitude and, thinking back to my own career, I believe that it has stood me in rather good stead.

**Niels K. Jerne, Ole Maaløe og Georg Rasch at Jerne's thesis defense in september 1951.**

The work of the Phage Group was to a large extent centered on the study of infection of bacteria by bacteriophages, or phages for short. Maaløe and Jerne were in particular interested in obtaining a better understanding of the process of reproduction of phages inside a host cell. They consulted with Rasch and Bentzon[7] about this, and I was fortunate to be drawn into the discussions. This resulted in the wish to construct a stochastic model for describing the reproduction process.

A birth and death process with multiple birth types was set up and studied, and I could write a small paper with a solution to one of the problems raised; that formed part of the qualification for the mag. scient. degree, But the project was fairly soon set aside for others of more pressing interest. However, I learned a lot from this first introduction to stochastic process theory.

The job at the State Serum Institute was portent for me in several respects, in addition to those mentioned above: (i) Key elements of what it means to work as a scientist were instilled in me. (ii) The approach to discussing data from the viewpoint of stochastics[8] that puts careful and detailed analysis of the data on an equal footing with the mathematical modelling and inference, in symbiotic balance. This approach has been a hallmark of the Danish tradition in statistics, starting with Thorvald Nikolai Thiele (1838-1910), see the excellent biography by Steffen Lauritzen [Lauritzen (2002)], and carried on by Rasch, see [Rasch (1960)], Hald, see [Hald (1952)], and pupils of these masters. (iii) Readiness to seek to acquire and apply any type of mathematics that seems relevant.

Moreover, it was around this time that Georg Rasch developed the theory of measurement models which, in terms of applications, has had a major impact in psychometrics and social science[9] and which gave essential impetus to parts of the lively discussions on conditional inference that took place from the mid-Fifties through to the mid-Eighties. It was fascinating and inspiring to learn at first-hand how Rasch invented and reasoned around these models, in particular the so-called item analysis model which probabilistically specifies the distribution of correct and wrong answers in a questionnaire where a number of persons are asked to solve a number of problems. The model associates an 'ability parameter' to each of the persons and a 'difficulty parameter' to each of the problems.

I still vividly remember how one day Rasch came to the Biostatistical Department at the Serum Institute enthusiastically telling how the Norwegian economist Ragnar Frisch[10], who was well acquainted with Rasch, had stated his astonishment that on conditioning on the total scores for the persons the ability parameters drop out of the specification, and similarly for the difficulty parameters, something that Rasch had evidently not paid much attention to previously. But this gave the impetus for Rasch's philosophical development of what he called 'Objective Inference'. For my own part this was the origin of my subsequent and standing interest in 'conditional inference'. After my time at the Serum Institute I had occasion to work for a period at the Danish Pedagogical University[11], where Rasch was also a consultant, with such measurement problems.

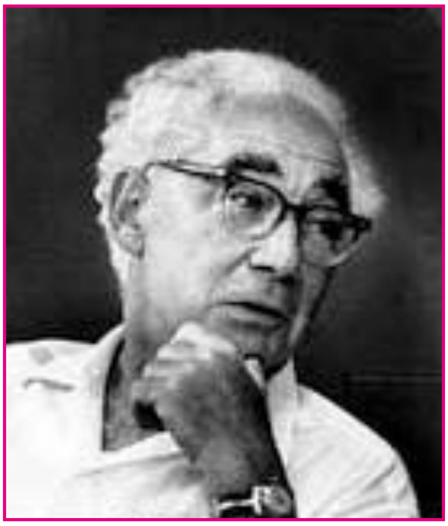
In the 1950-ies Mathematical Statistics in the Fisherian sense was well represented in Denmark primarily by Georg Rasch and by his former assistant at the State Serum Institute Anders Hjort Hald (1913-2007), who in 1948 became Professor of Statistics at the Social Science Faculty at Copenhagen University.

But in terms of education in statistics offered at the University of Copenhagen the only course was that given by Hald and based on his book from 1948, appearing in English and published by Wiley in 1952 under the title "Statistical Theory with Engineering Applications". As regards Probability there were no courses provided at Copenhagen University, though some important research work was undertaken at the Niels Bohr Institute by Professor Børge Jessen in collaboration with his assistant Erik Sparre Andersen, later Professor at the Institute of Mathematics at Aarhus University, and by Niels Arley, who was also associated to the Niels Bohr Institute and worked on applications of probability in cosmic radiation, atomic physics and radiobiology. Together with Rander K. Buch, Arley wrote a textbook with the title "Introduction to the Theory of Probability and Statistics".

In consequence of this state of affairs my mentors Rasch and Bentzon very strongly advised me to start studying Statistics and Probability as they saw a big future for these fields both in research and application-wise. That, however, would to a large extent have to be on my own, as should be clear from what was just said above, nothing like a degree in mathematical statistics and probability existed in Denmark. However, at that time it was possible for students having special interests to compose a study plan accordingly and to obtain not a candidate degree, but a degree called Magister Scientiarum.

A prerequisite was that the plan could meet with the approval of the relevant Professors. Accordingly, I consulted in turn with Professor Georg Rasch, Professor Anders Hald, Professor William Simonsen, Professor Børge Jessen, and Professor Einar Andersen, who was also Director of the Geodesics Institute. All of these were extremely supportive, and each recommended a number of works that I ought to study, so that I ended up with enough reading material to last for a life time. I therefore had to take some Merry-Go-Rounds to try to cut the job down to something realistic.

Quite extraordinarily, both Simonsen and Andersen set up special intensive courses for me and a couple of other students that took up of the same idea for obtaining a degree. And Rasch and Bentzon organized a study group on Volume I of William Feller's monumental work "An Introduction to Probability Theory and Its Applications" (Volume II was not published till some eight years later). The study group also comprised Arne Jensen, later Professor of Operations Research at the Danish Technical University, and Niels Iver Bech, later Director of the first Danish information technology centre 'Regnecentralen'.



Feller's two Volumes Work have been characterised as one of the main contributions to the mathematical literature in the Twentieth Century. They were essential in establishing Probability as a main part of Mathematics and they still constitute a key reference for basic concepts and results in the area. They are remarkable for their emphasis on the applicability of Probability to an enormous range of fields, and the style of writing is exceptionally clear and informative.

On all accounts these qualities reflected Feller's personality as a widely intellectual and delightful human being. Before moving to USA, where he eventually became Professor at Princeton University, Feller (1906-1970) had spent time at the Universities in Stockholm (with Harald Cramér) and Copenhagen, and in 1959 he paid a brief visit to the Niels Bohr Institute where I had the good fortune to hear him give a lecture on a problem in Probability. This gave me a vivid memory and I particularly recall him saying, in leading up to give the proof of a result, that it could be established by reference to a well-known mathematical theorem but that that would be "like cracking a nut with an elevator". Indeed, this attitude is apparent also in the two Volumes mentioned above where the mathematical derivations are complete but without unnecessary generality.

All in all a privilege for a young man embarking on a life in Stochastics in Science.

## 1.3 THE DEVELOPMENT OF MATHEMATICAL STATISTICS AND PROBABILITY IN DENMARK

### Statistical analysis in Denmark – a long tradition

Statistical analyses where thorough attention to determine characteristic features of the data are combined with significant mathematics in order to model and assess the data have a long tradition in Denmark, starting in a way with Tycho Brahe and as documented by Anders Hald in a study "The life and works of some Danish statisticians", published by The Royal Danish Academy of Sciences and Letters.

Hald's discussion goes back to the end of 18th Century, bringing the account up till around 1950, and encompasses some twenty personalities, and ends around 1950. In the earlier part of the period the empirical data studied came primarily from astronomy, geodesy and insurance. Later biology, telephone traffic, social sciences and psychology became dominant. Some of the prominent names are H. C. Schumacher, who became a close friend of Gauss, C. C. G. Andrae, J.P. Gram, A. K. Erlang, J. F. Steffensen and T. N. Thiele. Many of the works of these researchers were well known abroad, particularly those of Gram, Erlang and Thiele. Thiele's life and work are described in the biography by Steffen Lauritzen 'Thiele: pioneer in statistics'.

Especially well known is Thiele's introduction of the concept of cumulants and his derivation of their basic properties (1889, 1899). Thiele used the term halvinvarianter (semi invariants) for these; the word cumulants was introduced by Ronald Fisher (1929). Fisher did not appreciate Thiele' contributions properly as remarked in correspondence between Ronald Fisher, Arne Fisher and Ragnar Frisch; and Fisher's ignorance was strongly exposed by Harold Hotelling, see 'Thiele: pioneer in statistics', p.3.

In Denmark, research and teaching in statistics in the middle of the 20th Century was greatly influenced by Georg Rasch. He studied mathematics at Copenhagen University and 1925-1934 he held a position at the Geodesic Institute whose Director was Einar Andersen. In 1935-36 Rasch studied at University College London, where he became acquainted with Fisher's pathbreaking work on statistical inference, and in 1935 he became statistical consultant at the Danish State Serum Institute.

In the study of a new data set Rasch insisted on a detailed graphical analysis before proceeding to mathematical/probabilistic modelling. This attitude was shared by Anders Hald, as reflected in his book 'Statistical Theory with Engineering Applications', and it has dominated the statistical scene in Denmark since then.

After the Second World War the development of Mathematical Statistics and Probability in Denmark was strongly hindered by the opposition from the Faculty of Science at Copenhagen University to having Georg Rasch as Professor. It is true that Rasch had a colourful personality and was not especially well organized in his work. But he was a true and devoted mathematical scientist, and as a researcher he was very original and inspiring.

In the end the Faculty felt forced to establish a Chair in Mathematical Statistics but offered it to Anders Hald who for a long time had declined to take up such a Professorship in respect of Rasch. Hald reluctantly accepted, taking the Chair in 1960, and in a short period build up a broad milieu in Mathematical Statistics[12]; but for a long time, Probability was not pursued on its own. Shortly after, Rasch took up the Chair left by Hald, in the Social Science Faculty.

However, the situation as regards Probability in Denmark changed decidedly with the establishment, in 1954, of the Institute of Mathematics in the Faculty of Science at Aarhus University. The Faculty was created to mark the 25th anniversary of the University, and the first Professor appointed, in Mathematics, was the very dynamic and untraditional Svend Bundgaard.

Bundgaard decided to build up the Institute by focusing on three areas: Topology, Functional Analysis, and Probability. He was able to obtain very considerable funds for fostering the developments by inviting mathematicians in these fields to spend long periods at the Institute, and in planning the building of the Institute he insisted and fought through a decision to have a top floor there with several apartments and guest rooms for housing visitors. This strategy, exceptional for those times, was very successful in attracting leading researchers from around the world and made the Institute an exciting place, not least for young people. And Bundgaard was not only a dynamic leader of the Institute of Mathematics but became a driving force in the building up of the Faculty of Science.

In 1958 Erik Sparre Andersen, known for path breaking work in the early phase of developments in combinatorial probability, was appointed Professor of Probability Theory at the Institute of Mathematics.

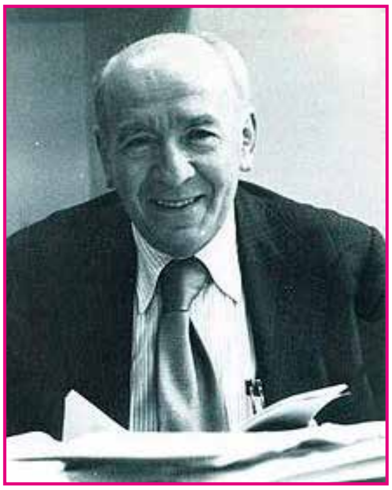
About the same time Sparre Andersen had been given a grant from the United States Air Force to foster further developments in his field of expertise, and he invited me to come to Aarhus on a regular basis to collaborate with him. Not only the Air Force but also the US Army and Navy were handing out such grants to leading researchers in the Western World, as a reaction to the launch of the first Russian Sputnik in 1957.

As a result, for a period of roughly two years I was travelling to Aarhus from Copenhagen by night boat (there was no bridge over Storebælt at that time) to spend a day and a half at the Institute.

I received much warm hospitality and it was a very exciting and inspiring time for a young person to sense the pioneering atmosphere at the Institute and to meet and listen to a great number of outstanding international mathematicians.

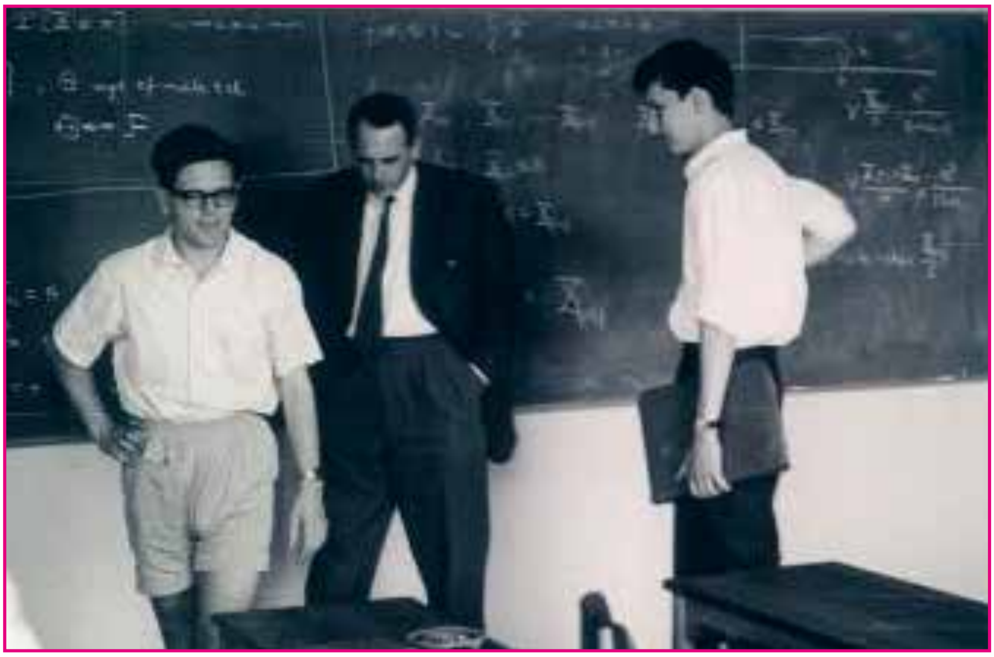
Among those who made the strongest impression on me were Harald Cramér (Swedish probabilist and statistician; 1893-1985) and Mark Kac (Polish mathematician; 1914-1984). Harald Cramér, who was Professor at Stockholm University, is considered as one of the giants in Mathematical Statistics. He had widespread interests and contributed with outstanding results also in Actuarial Science, Number Theory, and limit theory of Probability.

**Harald Cramér**



Mark Kac was of Polish origin but spent most of his scientific life in USA where he was Professor at Cornell University. His field was Mathematical Physics and one of his most well-known contributions is the Feynman – Kac formula. Richard Feynman was a very colourful person and one of the most outstanding physicists of the Twentieth Century. His life is described in the book ‘Genius: The Life and Science of Richard Feynman’, written by James Gleick.

**Marc Kac**



**After the defense for the mag. scient. Degree.**
**Standing, from left to right: Ebbe Thue Poulsen, Erik Sparre Andersen and the author.**

In comparison to Aarhus the milieu at the Institute of Mathematics in Copenhagen seemed to me somewhat dusty and dull and so when in 1960 I got an offer from the Institute in Aarhus to move there as Assistant Professor, I was not in much doubt and quickly accepted the position although I had a position as Instructor at the Institute in Copenhagen.

It then felt natural to seek to obtain the mag. scient. degree from Aarhus University and this was granted 24 June 1960.

SOLIDUM PETIT IN PROFUNDIS · UNIVERSITAS ARHUSIENSIS

In the summer of 1962 my wife Bente, our two children Lotte and Mikkel and I changed environment, for a two years period, to USA, the first year at the University of Minnesota and the second at Stanford University, under the auspices of Samuel Karlin (1924-2007). Scientifically that second year, which also included ample opportunity to benefit from the activities at the mathematics department at Berkeley University, was particularly exciting and inspiring for me, in particular because there I met Professor Kiyosi Ito (1915-2008), attended his lectures and got to know him and his family well.

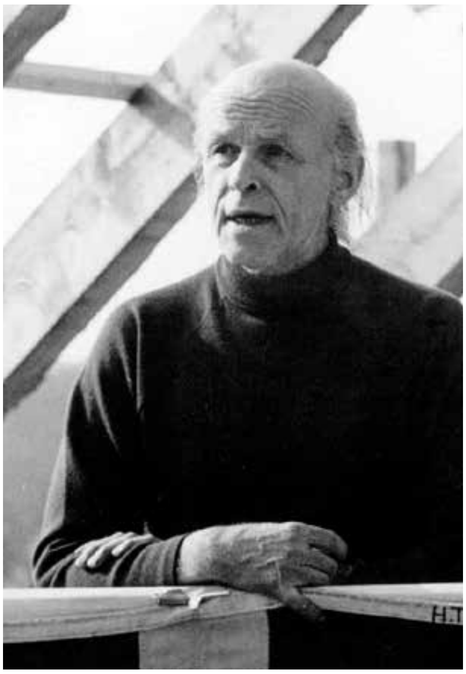
After our return to Aarhus, in terms of teaching my own work at that time and up till the mid-1980-ies was mainly directed towards building up the study in mathematical statistics at the Institute.

At the end of the 1960-ies I increasingly felt that Mathematical Statistics was not given the support and status within the Institute of Mathematics that would seem appropriate for a field of such independent scientific standing and importance. This viewpoint was not shared by Bundgaard, whom I perceived was rather regarding Mathematical Statistics as a somewhat inferior part of Mathematics, a mood that was not uncommon in the mathematical world around that time.

This was at the period where I was finishing my thesis 'Exponential Families and Conditioning' for obtaining the ScD degree and as a consequence of the above, and also because there was at Aarhus University no one with sufficient knowledge and insight to evaluate the work, I decided to submit the thesis to the University of Copenhagen.

At the end of 1970 and beginning of 1971 the dispute with Bundgaard, who by then was Dean of the Faculty of Science, came to a head and as a result I and my younger colleagues formulated a joint letter proposing that a Department of Theoretical Statistics be created within the Institute of Mathematics. In the meantime, Bundgaard became Rector of the University and the letter was submitted to the Faculty, the new Dean of which was Professor Jens Tyge Møller. A separate Department was established shortly after, as proposed.

The first of a long series of research reports and workshop proceedings or memoirs from the Department appeared in 1972 and the series continued till the year 2000. One of the issues in the series was Memoirs No. 1 from 1974, Proceedings of the 'Conference on Foundational Questions in Statistical Inference'. Many of the most active researchers interested in such questions participated in the Conference, including George Barnard, David R. Cox, Anthony W. F. Edwards and Steffen L. Lauritzen.



Svend Bundgaard

At the same time, education and research in mathematical statistics and probability theory was also being built up at the Institute of Mathematical Statistics at Copenhagen University. Throughout this period close contacts, with mutual visits, were runningly kept between the respective groups in Copenhagen and Aarhus, with the purpose of mutual criticism and inspiration. These collaborations were of particular value to the Aarhus group which at the beginning was much smaller and worked at a University with virtually no tradition for research and training in Mathematical Statistics.

Among the participants in these collaborations were Søren Johansen, Steffen Lauritzen, Niels Keiding, Søren Asmussen in Copenhagen and Karl Pedersen, Anders Holst Andersen and Jørgen Hoffmann Jørgensen in Aarhus.

When at Stanford I sensed that Ito and his family were not too happy staying at Stanford and right after returning to Aarhus I suggested to Svend Bundgaard that the Institute should try to invite Ito to move to Aarhus. This suggestion was followed up and Ito and his family spent the period 1966-1969 in Aarhus, after which they moved to Cornell University. In Aarhus, Ito's presence and lectures on stochastic processes was of enormous importance for the development of the milieu in probability and mathematical statistics. A lasting trace of some of those lectures is the book Kiyosi Ito: *Stochastic Processes* from 2004, published by Springer. That is an edited version of a set of lecture notes that Ito wrote at the Institute of Mathematics, Aarhus University. (The notes originally appeared in 1969 in the lecture notes series from the Institute.) The editing was carried out by Ken-Iti Sato and me, in close contact with Ito.

Kiyosi Ito

## 1.4 KIYOSI ITO

Kiyosi Ito (1915-2008) is one of the prominent Figures of Mathematics. He was born in Hokusei in the Mie Prefecture, Japan. Between 1938 and 1945 he worked for the Japanese National Statistical Bureau, and in 1952 he became Professor at the University of Tokyo, a position he held till his retirement in 1979. However, during that time he spent long periods abroad, visiting Cornell University, Stanford University, Aarhus University and the Institute of Advanced Study in Princeton.

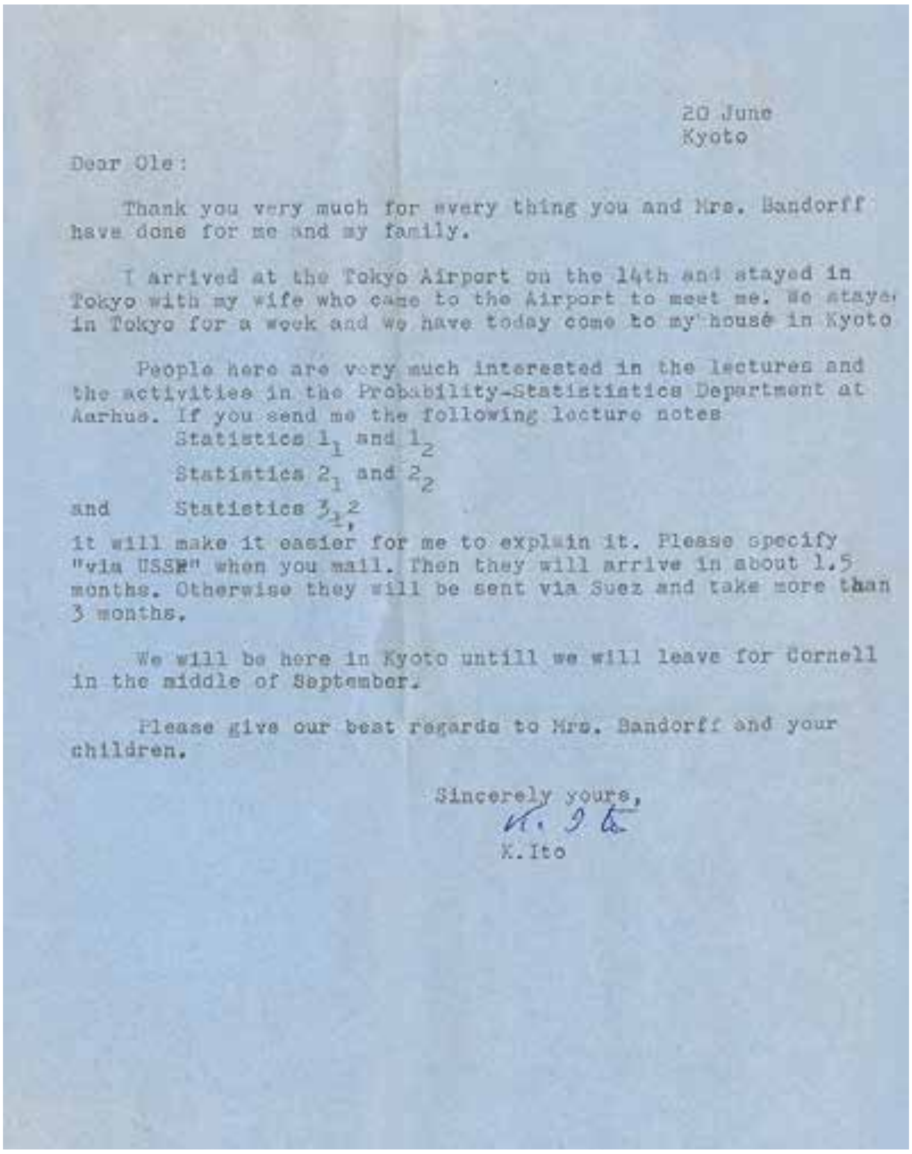
Ito developed a theory of Stochastic Calculus, analogous to the classical Analytic Calculus due originally to Newton and Leipniz. Analytic Calculus remains the main working horse of classical Physics and other traditional parts of the Natural Sciences. However, from around the beginning of the Twentieth Century it was increasingly understood that the basic assumptions of differentiability underlying Analytic Calculus in many cases do not correspond to reality and that the differences caused by randomness are essential.

In other words, a corresponding – general – theory of stochastic processes was called for, a theory that allows 'differentiation' and ''integration' in some suitable sense. Such a theory should in particular encompass processes that move in jumps but with infinitely many jumps in any finite time interval.

A very special, but basic, kind of such processes are the so-called Lévy processes. These are discussed more closely later in this book.

From around 1945, Ito build the theory of 'Stochastic Analysis', to meet those requirements. In his honour the theory is often referred to as 'Ito Calculus'.

The theory allows in particular to set up stochastic partial differential equations – or SPDE's. Such equations have manifold important applications, not only in Physics but also, for instance, in Mathematical Finance, where they constitute a major tool.



20 June
Kyoto

Dear Ole:

Thank you very much for every thing you and Mrs. Bandorff have done for me and my family.

I arrived at the Tokyo Airport on the 14th and stayed in Tokyo with my wife who came to the Airport to meet me. We stayed in Tokyo for a week and we have today come to my house in Kyoto

People here are very much interested in the lectures and the activities in the Probability-Statististics Department at Aarhus. If you send me the following lecture notes

Statistics $1_1$ and $1_2$

Statistics $2_1$ and $2_2$

and Statistics $3_1$, 2

it will make it easier for me to explain it. Please specify "via USSR" when you mail. Then they will arrive in about 1.5 months. Otherwise they will be sent via Suez and take more than 3 months.

We will be here in Kyoto untill we will leave for Cornell in the middle of September.

Please give our best regards to Mrs. Bandorff and your children.

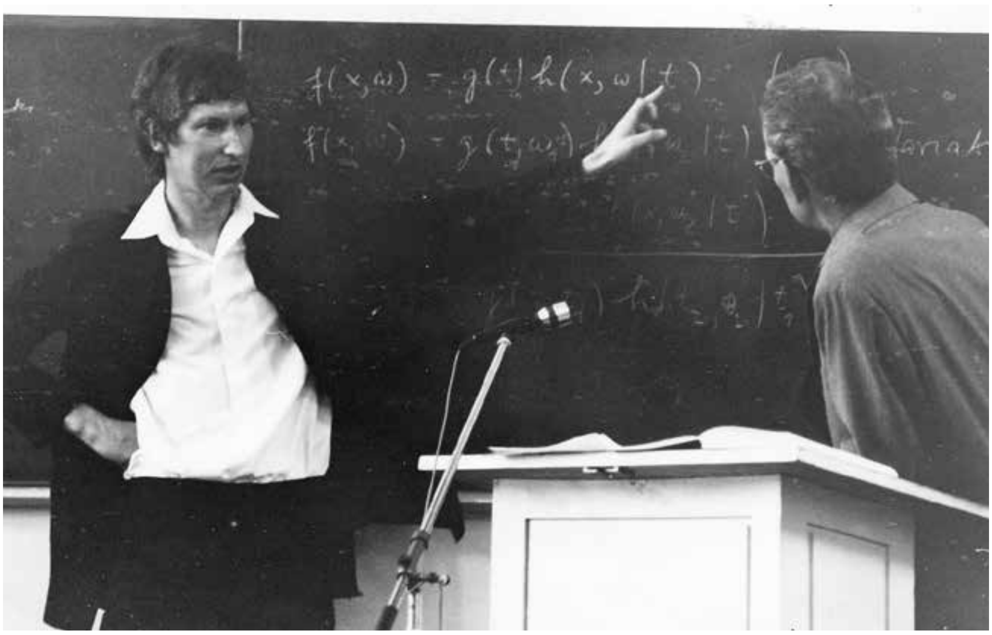
Sincerely yours,

K. Ito

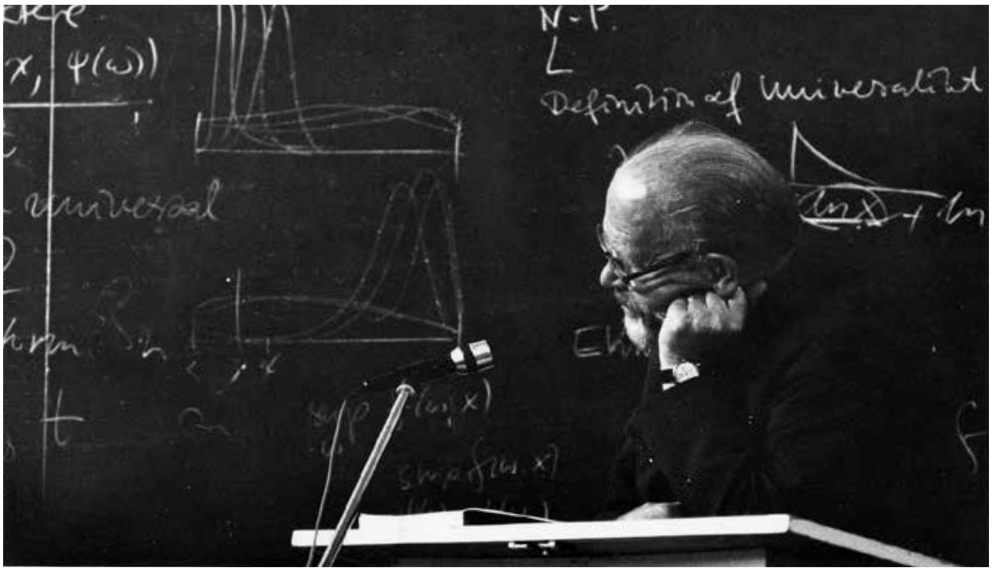
K. Ito

## 1.5 SC.D. DEFENSE

An account of some of my work in the subject area of exponential families was presented in an Sc.D. thesis entitled ‘Exponential Families and Conditioning’, submitted to the University of Copenhagen, rather than to Aarhus University where knowledge and expertise in likelihood-based inference was not otherwise represented in any substantial measure. The contents of the thesis are discussed in a report, reproduced in from the Committee evaluating my application for a Chair in Mathematics at Aarhus University. The application was successful and I became Professor in 1973.



**With Anders Hald**



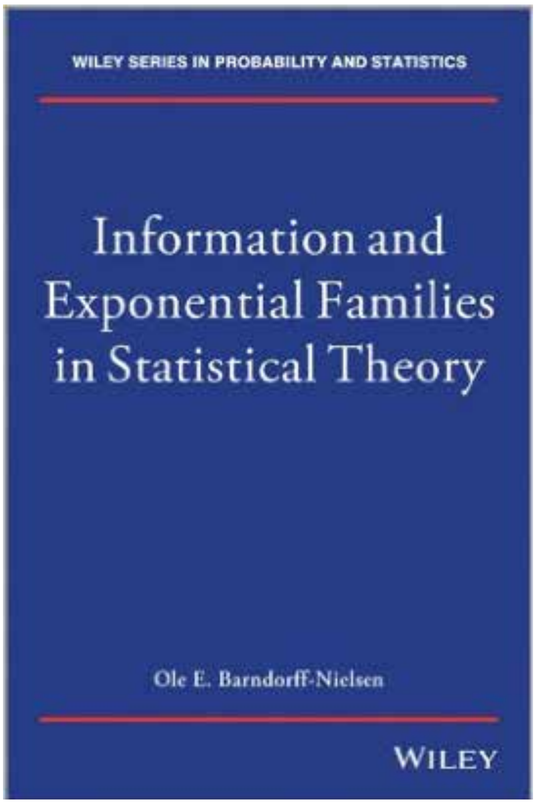
**Georg Rasch at the cathedra**

Anthony F. Edwards

The discussion at the defense stretched over about 6 hours with many participants.

Among the people who took an interest in the work described in my Sc.D. thesis was Professor David G. Kendall, Cambridge University, and he proposed to Wiley to publish an extended version of the thesis. This resulted in the book "Information and Exponential Families in Statistical Theory", which was published in 1978 and reissued in 2013. In that book the theory of exponential families is developed in detail, based in part on convex analysis, and related to various extensions of the classical concepts of sufficiency and ancillarity. The role of S-ancillarity vis-a-vis exponential families was studied in particular, in connection to a concept of cuts in exponential families.

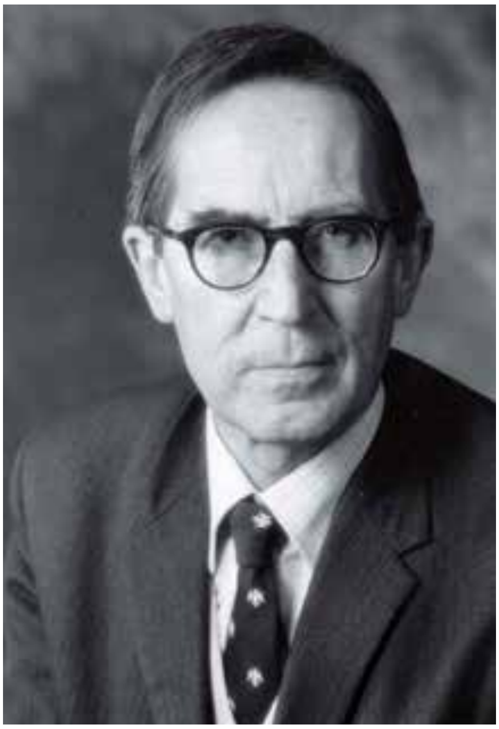
Kendall also invited me to spend a period at Churchill College, Cambridge, where I lived with my family August 1974 – February 1975 as Overseas Fellow. Kendall was later highly instrumental in establishing the ties to Ralph Alger Bagnold.



## 1.6 DAVID GEORGE KENDALL

David Kendall (1918-2007) was the first Professor of Mathematical Statistics at Cambridge University, where he held the Chair from 1962 till his retirement in 1985. As mentioned in the lengthy obituary of him in the 'Independent' he was considered as 'the father and grand old man of British probability.' He was elected Fellow of the Royal Society (FRS) in 1964.

In his youth his scientific interests were divided between Astronomy and Mathematics, and his first paper, entitled "Effect of Radiation Damping and Doppler broadening on the Atomic Absorption Coefficient", was published in Zeitschrift für Astrophysik in 1938. He was torn between the two subjects and did not know how to resolve the conflict but, as he later said 'Hitler resolved it for me'. When the Second World War started Kendall was called to work on rocket techniques with the Projectile Development Establishment and was thereby led to study randomness – in the sense of probability and statistics – with the view to increase the accuracy of the rockets. On the strength of his wartime work he became Lecturer in Mathematics at Oxford University where he stayed till his appointment to the Chair of Mathematical Statistics at Cambridge University.

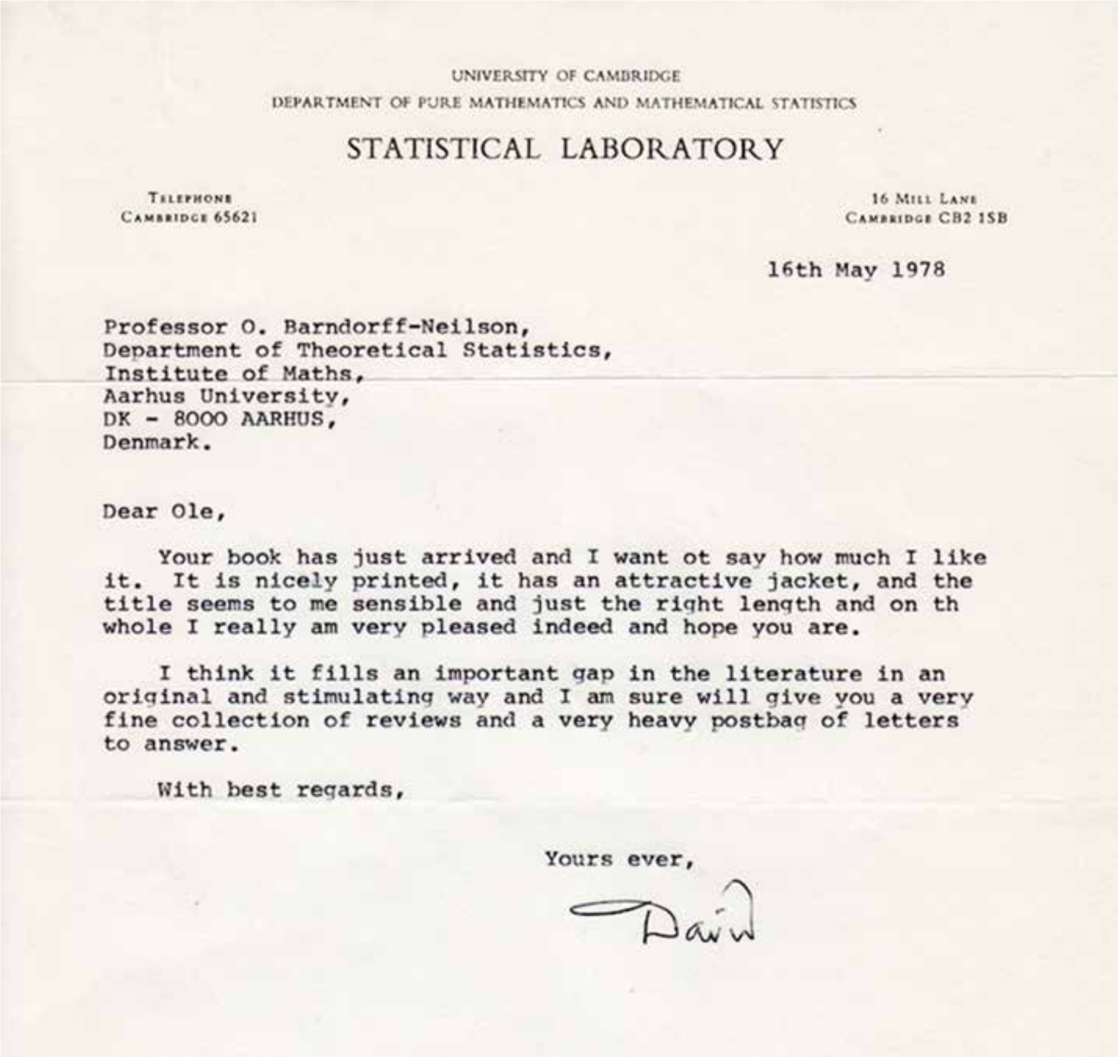


**David Georg Kendall**

From then on Kendall's scientific focus was on Probability and Statistics where he contributed with pathbreaking work to a wide range of topics, both in theoretical probability, particularly in the fields of population growths, queuing theory and Markov processes, and in applications of probability, such as to comet theory, epidemics and archaeology.

David Kendall had an enormous influence on the development of research and training of Stochastics, nationally and internationally, through personal contacts with many of the leading persons in the field and through the inspiration and support given to students and younger researchers. He traveled widely and was an avid mountaineer. He also had a very rich family life with six children, one of which, Wilfrid Kendall, followed in his father's footsteps and holds a professorship in Statistics.

All this and very much more is described in the Royal Society Memoir (Biogr. Mems Fell. Roy.S. 2009 55) written by John Kingman, himself a Fellow of the Royal Society and former student of Kendall's. This Memoir is a very vivid and comprehensive account of a warm and full human being.


UNIVERSITY OF CAMBRIDGE
DEPARTMENT OF PURE MATHEMATICS AND MATHEMATICAL STATISTICS

STATISTICAL LABORATORY

TELEPHONE
CAMBRIDGE 65621

16 MILL LANE
CAMBRIDGE CB2 1SB

16th May 1978

Professor O. Barndorff-Neilson,
Department of Theoretical Statistics,
Institute of Maths,
Aarhus University,
DK - 8000 AARHUS,
Denmark.

Dear Ole,

Your book has just arrived and I want ot say how much I like it. It is nicely printed, it has an attractive jacket, and the title seems to me sensible and just the right length and on th whole I really am very pleased indeed and hope you are.

I think it fills an important gap in the literature in an original and stimulating way and I am sure will give you a very fine collection of reviews and a very heavy postbag of letters to answer.

With best regards,

Yours ever,

David

Letter from Professor David Kendall who had suggested that my thesis might be published by Wiley.

# 2 STATISTICAL INFERENCE

## 2.1 THE INTERNATIONAL SCENE AROUND THE MID-TWENTIETH-CENTURY

Statistical inference is the area of statistical science that concerns principles for drawing testable conclusions from data. The basic such principles were introduced by Ronald A. Fisher in the period from the 1920-ies till the middle of the 1930-ies. Fisher (1890-1962) was a versatile scientific genius who worked not only in Statistics but also in genetics and evolutionary theory. In fact, much of his work in Statistics grew out of his research in Biology. Thus, for instance, his invention of the analysis of variance developed from his work on huge data sets from experiments on crops, carried out at Rothamsted Research Station where he worked in the period 1919-1933. He had an illustrious academic career, including a professorship at Cambridge University 1943-1957, where he was Fellow of Gonville and Caius College, and he became a Fellow of the Royal Society in 1929.

The central concept of statistical inference, introduced by Fisher, is that of likelihood. The so-called likelihood function L or its logarithm $l$=logL is a function of the data and of the mathematical model for the phenomenon generating the data. Such a model is typically specified only up to some unknown, possibly multidimensional, parameter and the likelihood function is the probability or probability density $p(x;\theta)$ of the data, considered as a function of the parameters $\theta$ for the observed set of data $x$, i.e. $l(\theta)$=log $p(x;\theta)$. Crucially, Fisher proposed that the value of θ that, for the given data set $x$, maximises the value of $l(\theta)$ must provide the best estimator of the actual value of $\theta$. That estimator is called the maximum estimator and is denoted by $\theta$.

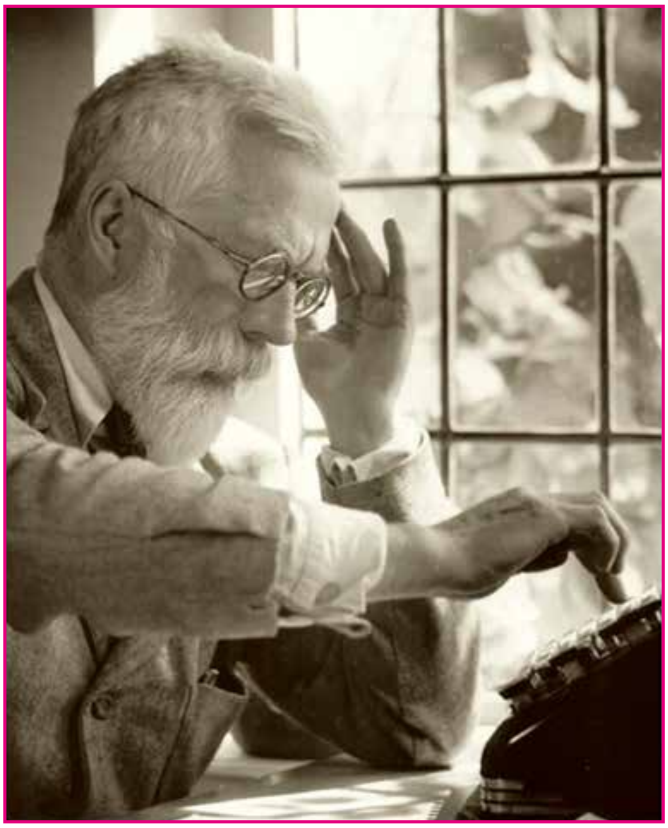
With $l(\theta)$ considered as the likelihood that the specific model determined by the parameter value $\theta$ is the true model it is natural to introduce the relative log likelihood $w=2(l(\theta)-l(\theta))$ and to view large values of $w$ as unlikely. It turns out that the probability law of $w$ is under rather general conditions, asymptotically as the number of observations tends to infinity, equal to a fully known probability distribution, the chi squared law (the number of degrees of freedom being equal to the dimension of the parameter space for $\theta$). The quantity $w$ is also referred to as the Neyman-Pearson likelihood ratio statistic.

Two other essential ideas, also introduced by Fisher, are those of sufficiency and ancillarity. Conceptually these are closely linked to likelihood and give guidance to determine which aspect of the data encapsulates the information that is relevant for the question studied. Closely related to ancillarity is the concept of conditional inference where conclusions are drawn conditional on part of the data, thereby giving the inference a proper frame of reference.

The likelihood function is generally considered to contain all the information that is available in a given data set about the concrete question studied, provided the correct model is contained in the considered class of models.

However, it may be that part of the data does not in itself contain any information on the parameters of the model in which case that part is said to be *ancillary*; and in that case any conclusions based on the given model and data should be phrased conditionally on that fact. On the other hand, if part of the data, $t$ say, is such that given $t$ the rest of the data does not contain information on the parameters then that part is said to be *sufficient* for the inference and further reasoning should be confined to that and thus to the likelihood function calculated solely from the sufficient part.

In this connection Fisher raised the question under which type of model there exists a statistic, that is a function $t$ of the data $x$, such that $t(x)$ is sufficient for $\theta$ though not a one-to-one function of $x$, the latter meaning that $t$ represents a genuine data reduction. Among such statistics $t$ there may be one that gives the highest sufficient reduction possible, in which case it is called a minimal sufficient statistic. Fisher claimed that the only models allowing sufficient reduction in this sense are what is now called exponential models which are a special class of families of probability laws. This class was the subject of my works in the early part of my research life, as indicated above.



Sir Ronald Aylmer Fisher (1898-1962)

Fisher's style of writing was not in strict mathematical terms and proofs of mathematical properties of the concepts of maximum likelihood, sufficiency, ancillarity and exponential families that he presented were generally lacking in rigour or incomplete. But he backed his ideas by illuminating examples. This style made it difficult to grasp the depth and applicability of the concepts. Initially his ideas were thus met with much scepticism and it took a considerable time before his contributions became widely and fully appreciated and provided with general mathematical formulations and proofs. This aspect of Fisher's work is well described in [Lehmann (1988)], which also gives an account of Fisher's disputes with Jerzy Neyman.

A key issue in that dispute was the concept of power of a statistical test, introduced in a seminal paper by Jerzy Neyman and Egon Pearson [Neyman and Pearson (1933)]. This in effect gave a deductive underpinning, in probabilistic terms and of crucial importance for the development of statistical inference, of Fisher's use of the likelihood ratio statistic. Much to the resentment of Fisher who's reasoning was intuitively based and who considered the optimality of the likelihood ratio argument obvious on grounds of the nature of inductive arguments, as he saw it. Fisher was opposed to the implied emphasis on the operational aspect of Neyman's and Pearson's argument and in my view that point has a degree of validity, meaning in particular that the so-called decision theory with its associated concept of utility, developed primarily by Abraham Wald from the Neyman-Pearson way of reasoning, is anathema to statistical inference (as is the Bayesian approach.)

Putting it in the briefest possible terms, Fisher advocated a mainly intuitive, non-mathematical approach to building a theory of statistical inference while Neuman emphasised the need to formulate a theory on mathematically precisely formulated principles followed by full mathematical proofs of properties derived from those principles. While neither of these two giants fully recognised the other's view and achievements, today it is generally recognised that statistics as a science must be based on an integrated view (quite apart from the fact that further developments in a purely Fisherian mode would require a new genius of his stature).

Jerzy Neyman (1894-1981) was Polish of origin and spent the first half of his life in Warsaw and at University College, London, and the second half at the University of California, Berkeley. He had enormous success in establishing mathematical statistics and probability as major scientific areas in the United States, and at Berkeley University in particular. This both through his own research and through the series of Symposia on Mathematical Statistics and Probability that he organised at Berkeley in the period 1945 to 1970. These Synposia helped linking probability theory closer to mathematical statistics, and the Symposia programmes comprised applications to Psychology, Astronomy, Economics, Meteorology and other areas.

Lehmann's book [Lehmann (1988)] provides a remarkable and greatly illuminating panorama of the activities in mathematical statistics that took place in USA during the first three quarters of the Twenties Century, giving thorough descriptions of main scientific issues and fine impressions of some of the major personalities involved but, as the author himself acknowledges, relatively little is said about developments elsewhere.

It is pertinent here to mention that for a considerable time, up to around 1975, there was, schematically speaking, a degree of unbalance in how statistics was taught and developed in USA and most European countries on the one hand and in Britain, India and the Nordic Countries on the other, the decision theoretic approach being prevalent in the former case, a more Fisherian coloured philosophy considered a core element in the other. In India C. R. Rao was a leading figure in this, as was Harald Cramér in Sweden.

The history of the development of the main ideas of likelihood, inference and their limitations, as well as the Fisher-Neyman dispute is excellently discussed in a paper by Stephen Stigler [Stigler (2007)]. Among the many papers and comments that Fisher wrote on the subject the one from 1922 [Fisher (1922)], stands out as particularly monumental.

In the period beginning around 1950 there was internationally a surge of interest in the theory of statistical inference generally and in particular on basic principles of statistical inference in line with Fisher's work. In regard to likelihood-based inference, with the associated concepts of sufficiency, ancillary and conditioning, three early papers, by [Barnard (1949)], [Cox (1958)] and [Birnbaum (1962)], were decisive for the renewed interest in this area. Other influential contributions were by Basu, Edwards and Fraser.

[Sverdrup (1966)], cf. also [Sandved (1967)], defined the ancillarity concept which is now known as S-ancillarity. Rasch's measurement model, introduced in the late 1950-ies and discussed above, and his insistence on the property that part of the parameter dependence may be eliminated by conditioning provides in fact a prime example of the principle and use of S-ancillarity.

Among the meetings on Statistical Inference held in Aarhus at the time was a Conference on 'Foundational Questions in Statistical Inference' which took place 7-12 May 1973, and in which a major part of the researchers working on aspects of the title of the Conference were present.

**From an excursion to Northern Jutland during a conference on Likelihood Inference. Second from left: George Barnard; Third: Preben Blæsild; Fourth: Anthony Edwards; Fifth: Freddy Bugge Christiansen**

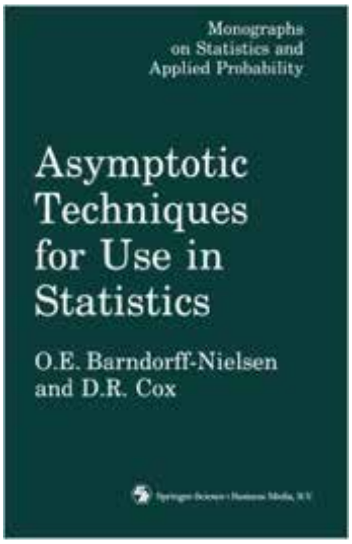
## 2.2 DAVID ROXBEE COX

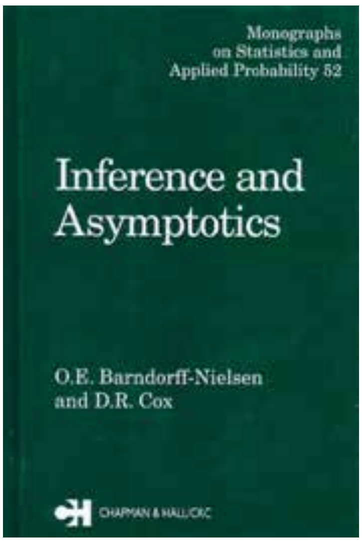
David Roxbee Cox

By the time I finished my University studies the leading figure internationally in carrying forward the science of inference in Fisher's spirit was David Roxbee Cox, then Professor at Imperial College London. I followed his writings avidly and accumulated a list of questions that I dearly wanted to raise with him. In view of his scientific standing and the fact that he was about ten years my senior it was with some trepidation that during a visit to London around 1963 I mustered the courage to walk into the Huxley building of Imperial College asking a secretary if I might have a word with him.

I was received in a completely forthcoming way and this our first discussion of questions on inference principles developed into a long-term collaboration and friendship, resulting in a series of joint papers and books.

David Cox (Sir, FRS) has been the most influential statistician for more than half a century. He was Professor at Imperical College London 1966 to 1988 at which time he moved to Oxford as Warden of Nuffield College and member of the Department of Statistics. In addition to his numerous scientific papers and books he has been highly influential in the developments of statistics internationally, serving as Editor of Biometrika and as President of the Bernoulli Society, to mention but a few of his many contributions in this regard. Among the characteristic traits of David's personality are his equanimity and sense of dry humour. This may be illustrated by an event that took place in connection with the World Statistics Congress of the International Statistical



Institute held in Amsterdam in 1985. Stepping off the airport bus on arrival he put down his briefcase and suitcase on the ground, only to discover a moment later that the briefcase was stolen. When asked later what was lost his answer was: “No valuables only scientific papers”. In fact, one of his notebooks containing some of his research ideas from the year passed was gone.

A case of artificial intelligence: In these early days of search on the net, a student of Peter Jupp was trying to locate a joint paper by David Cox and me. However, after typing “Barndorff-Nielsen and Cox”, the computer suggested that it should have been “Barnyard and Ox”.

## 2.3 A JOINT PASSION FOR INFERENCE AND OPERA

The common interest in inference with David Cox led to a close personal relationship, spurred also by our shared enthusiasm for opera, in particular the works of Richard Wagner.

It was David Cox who really opened my eyes for the depths of Wagner’s work and world through suggesting to me to read Robert Donington’s book ‘Wagner’s Ring and Its Symbols’. No doubt, part of my fascination of Wagner’s work also lies in the Nordic roots of ‘Der Ring der Nibelungen’, in particular the ‘Völsunga Saga’ and Snorri Sturlason’s ‘Edda’.

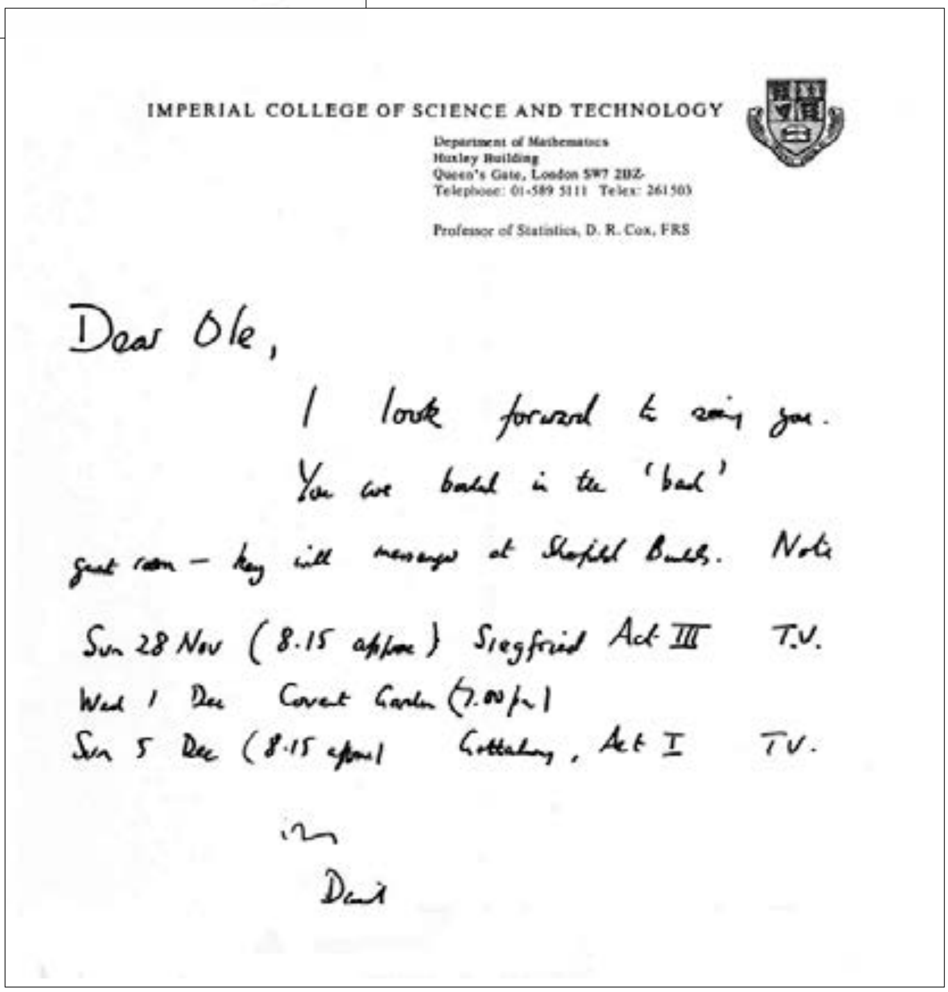
In 1981-1982, a new Concert Hall was built in Aarhus and to mark this it was decided to strive for developing the possibility to present the whole of Wagner’s Ring there. This was achieved through a series of steps giving ‘Die Walküre’ in 1983/84, ‘Rheingold’ in 1984/85, ‘Siegfried’ in 1985/86, ’Götterdämmerung in 1986/87 and finally the whole Ring in 1987/88. This was in large measure a joint Nordic enterprise and it met with a tremendous success, both nationally and internationally.

**Inference or Opera?**


Remarkably, in 1996 the Ring was again given there, in a new production and again in World quality, regarding singers and orchestra as well as staging.

On this occasion David and Joyce Cox visited Bente and me in Aarhus and we spent a whole week in August-September 1996 attending the performances of the four operas of the Ring cycle, much of the time in between discussing what we have heard and seen.

At the end of 2013 I sent David Cox an early version of my memoirs, asking for his possible comments.



IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY

Department of Mathematics
Huxley Building
Queen's Gate, London SW7 2BZ
Telephone: 01-589 5111 Telex: 261503

Professor of Statistics, D. R. Cox, FRS

Dear Ole,

I look forward to seeing you. You are booked in the 'bad' guest room – key with messenger at Sherfield Build. Note

Sun 28 Nov (8.15 approx) Siegfried Act III T.V.
Wed 1 Dec Covent Garden (7.00 pm)
Sun 5 Dec (8.15 approx) Götterdämmerung, Act I TV.

[illegible]

David

From some happy occasion

On 1/15/14 4:10 PM, "David Cox" wrote:

Dear Ole,

It is great news that you and Bente are coming in July. Thank you for the reminiscences. I recall our first meeting with great clarity and pleasure. (You had a colleague with you who said not a word throughout, so far as I recall; Blaesild??)

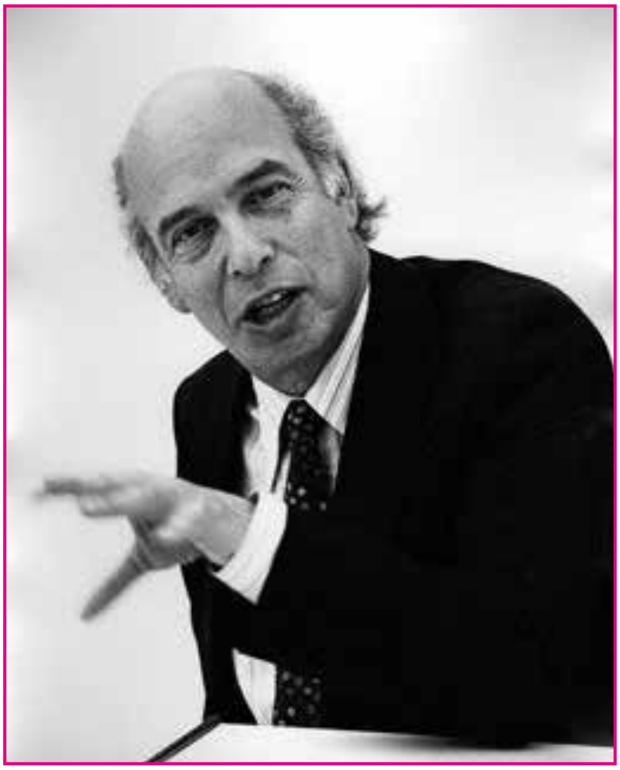
You asked a long series of penetrating and highly relevant questions about principles of inference and I struggled to answer as best as I could and to make suggestions of things to read, all of which you seemed already to have read. But as you say it led to a long and for me highly rewarding and enjoyable collaboration. Of course I recall also the Ring cycle at Aarhus. Joyce and I had done our preparation carefully and the fact that it was in German (which we don't understand) with Danish surtitles (which we don't either) did not stand in the way of a deeply memorable few days.

I assume that in your recollections you will cover the meeting you organized on inference in Aarhus fairly soon after we first met.

Very best wishes and to Bente from us both, David

## 2.4 CLAUS MOSER

Among the many scientists with whom I became acquainted through my relation to David Cox was Sir, later Lord, Claus Moser (1922-2017). David introduced me to him at a dinner of the Royal Statistical Society Dinner Club.



Claus Moser

Claus Moser was a quite extraordinary human being, of great wit and charm. Born in 1922, he was of a Jewish family that fled Germany to England in 1936. There he had an illustrious career, including a Professorship at the London School of Economics, the Directorship of the Central Statistical Office, President of the British Association for the Advancement of Science, Chairmanship of the British Museum Development Trust, Chancellor of Keele University, where the Claus Moser Research Centre, dedicated to research in the Humanities and Social Sciences, was established in 1997 and officially opened in 2008.

But most relevant here, he was Chairman of the Royal Opera House, Covent Garden, 1974-1987, and at the dinner we had a lively conversation around our shared enthusiasm for opera. A couple of years earlier a Society of people interested in opera had been started in Denmark, and I had joined this Society. I wished to write an article for the newsletter of that Society about the similar Covent Garden 'Friends of the Opera' and asked Moser about that organisation. In a most friendly manner, he arranged for me to visit Covent Garden where I was shown around the Opera house. That gave me a lasting fascination of the traditions of that institution.

From Sir Claus Moser KCB *Chairman*

# Royal Opera House

Covent Garden London WC2E 7QA
Telephone: 01-240 1200
Cables: Amidst London WC2

*as from*
New Court
St. Swithin's Lane
London EC4P 4DU
Telephone 01 280 5000

13th January, 1986.

Dear Professor Barndorff-Nielsen,

Many thanks for your letter of the 6th January. It was a great pleasure to hear from you again after all this time.

I was also most interested to hear bout operatic developments at your end. It will indeed be very nice if you were to write about the Royal Opera House. I suggest that the best person for you to see is the Hon. Kensington Davison, who in fact, runs The Friends of the Royal Opera House and is enormously knowledgeable about all aspects of the House.

I have spoken to Ken and he will be happy to see you. I suggest you ring him when you get to London.

Yours ever,

Claus

Sir Claus Moser

Professor O.E. Barndorff-Nielsen,
Department of Theoretical Statistics,
Institute of Mathematics,
Aarhur University,
DK-8000 Aarhus,
DENMARK.

*Royal Opera House, Covent Garden Ltd – Registered in London, No. 480523 Registered Office: Royal Opera House, Covent Garden, London WC2E 7QA*

## 2.5 ROYAL STATISTICAL SOCIETY RECEPTION AND DINNER

The Royal Statistical Society is the oldest and the leading statistical society in the World, famous as the home for a long sequence of illustrious contributions to statistical science by prominent researchers.

In 1984 the Royal Statistical Society held its 150th Anniversary Conference. The Conference took place in the Whitehall Banqueting House and the Queen and her Consort participated. It was a grand occasion, of which the documents shown here give an impression.

ROYAL STATISTICAL SOCIETY

Reception, Whitehall Banqueting House, 5 April 1984

I. Details are enclosed of the arrangements for the Reception

II. Some guidelines for anyone introduced to HM The Queen or HRH The Duke of Edinburgh

1. If you are formally introduced to either HM The Queen or HRH The Duke of Edinburgh, shake hands and
   Ladies: give a small curtsey
   Men: bow the head.

   To HM The Queen, say "How do you do, Your Majesty" or simply "How do you do". The same applies to the Duke of Edinburgh but with "Your Highness".
   There may be no need to say anything further. If HM or HRH asks a question, make an appropriate (fairly short) reply.

2. If you are not formally introduced, but are drawn into conversation with HM or HRH, simply respond as is appropriate; you will not necessarily have to shake hands or curtsey/bow.

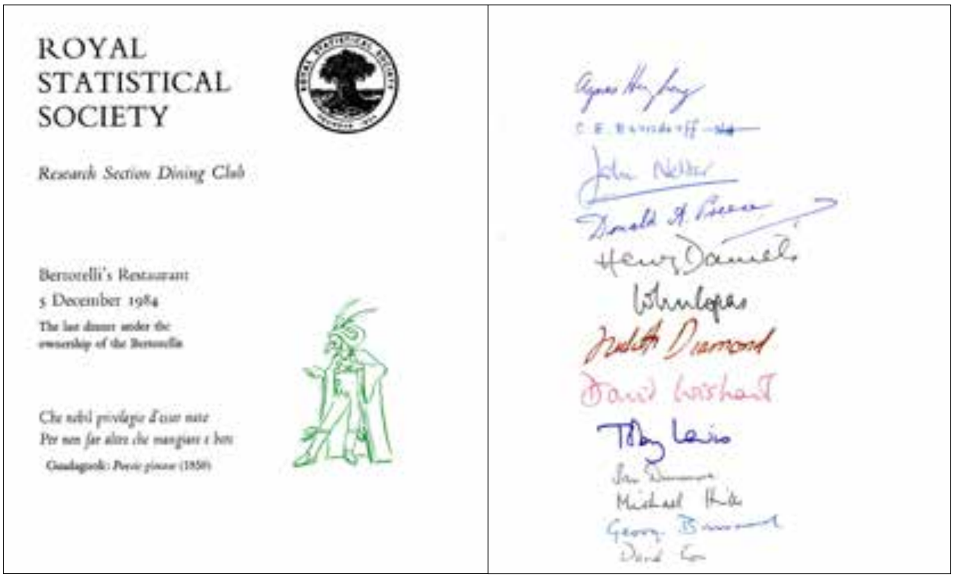
3. After meeting HM or HRH please remain in your place until the royal party has moved on.

4. Drinks will be offered to the royal party after some formal presentations and before the "walk-about". It will be in order for anyone to be holding a glass once the royal party has been offered drinks.

5. Dress is informal (short dresses/lounge suits); gloves for the ladies are optional.

A memorable point was that as you entered the Banqueting House you had before you a broad staircase leading to the reception rooms on the second floor. At the foot of the staircase you could pick up a small brochure telling about the House, and it began with the words 'In the ceiling there is a rather uninteresting painting'.

The reception was followed by a dinner organised by the Society's Research Section Dining Club.




## 2.6 CAMBRIDGE

Apart from the two years in USA 1962-64, one of my long-term visits abroad were to Cambridge, August 1974 – February 1975, as Overseas Fellow at Churchill College and visitor at the Statistical Laboratory, Cambridge University. This was at the invitation of David Kendall.

Bente, I and our children stayed in one of the very comfortable houses reserved for visiting Fellows and nicely located on the College grounds.

**Main entrance of Churchill College and Dhruva Mistry sculpture**

During the stay in Cambridge, I was invited by Anthony W. Edwards, author of the book *Likelihood*, to dine with him at High Table in Gonville and Caius College, one of the oldest Colleges in Cambridge, the College also of Ronald Fisher, and where Ralph Bagnold took his degree under Professor Stratton after the First World War.

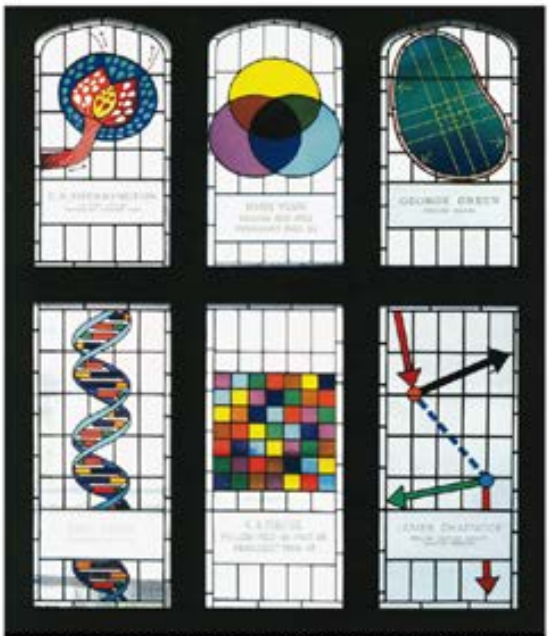
# Historical Notes: Mathematical Stained Glass

**A.W.F. Edwards** FIMA, Gonville and Caius College, Cambridge

**Apart from Trinity College,** Cambridge, Gonville and Caius College has more Nobel Laureates to its name (nine scientists and four economists) than any other Oxford or Cambridge college. Of course, some Caius luminaries are before the era of Nobel prizes, such as William Harvey, but there being no Nobel for mathematics it is particularly gratifying that in the six celebratory stained-glass windows in the Caius Hall there are as many mathematicians – George Green, John Venn and Sir Ronald Fisher – as there are Nobel Laureates – Sir Charles Sherrington, Francis Crick and Sir James Chadwick.

George Green's (1793–1841) window is a diagram, necessarily two-dimensional, of Green's Theorem in the vector calculus of three-dimensional space. Green was self-taught, publishing his famous 'Essay on the application of mathematical analysis to the Theories of Electricity and Magnetism' privately in 1828 before he came up in 1834 as a scholar to read the Mathematical Tripos. He was classed fourth wrangler in 1837 (Sylvester was second) and elected a Fellow in 1839.

John Venn (1834–1923) was sixth (equal) wrangler in 1857 and a Fellow from 1857 until his death. His book *The Logic of Chance* had a profound effect on the development of repeated-sampling theories of statistical inference; it also contained the first diagram of a random walk on a two-dimensional lattice, with comments on its fractal limit. He gave the first ever lecture course in the 'Theory of Statistics' in England in 1890, as part of the Moral Sciences Tripos. His eponymous diagram appeared in another book, *Symbolic Logic*, in 1881.



Caius window pairs (left to right): Sherrington (top) and Crick; Venn and Fisher; Green and Chadwick.

In Venn's window the colours of the intersections of the three sets are correctly rendered by overlapping the layers of glass, a technique known in stained-glazing circles as 'plating'.

R.A. Fisher (1890–1962), statistician and geneticist, graduated with a first in mathematics in 1912 (the order of merit in the Mathematical Tripos having been recently abandoned). His window is reproduced from the dust jacket of his path-breaking book *The Design of Experiments* (1935). He spent a postgraduate year at the Cavendish Laboratory, and in 1920 was elected a non-resident Research Fellow while Chief Statistician at Rothamsted Experimental Station, where he pointed out that Latin Square arrangements of field trials have to be randomised in order for the experiments to be validly interpreted. After moving to the Galton Professorship of Eugenics (Human Genetics) at University College, London, he returned to Caius as a Professorial Fellow in 1943 on his election as Arthur Balfour Professor of Genetics.

There is a mystery about the origin of the particular $7 \times 7$ Latin Square of the dust jacket. It does not appear in the book itself, nor has it ever been found in any of his papers. Perhaps the publisher's art department was simply instructed to think up a colourful one – it is not difficult once one knows the rules. Happily, seven colours is also the number required to colour a three-set Venn diagram, and with a little artistic licence it was possible to match the colours in the two windows.

The Venn and Fisher windows were the first ones to be installed, in 1989 ready for the Fisher Centenary. Unintentionally they also pay homage to Leonhard Euler, for not only are Latin Squares an invention of his, but Venn explicitly published his diagrams as an improvement on Euler's. Euler had indeed represented the classes in his propositions by circles, but had enumerated all the ways in which two or then three classes could overlap by their corresponding diagrams. Venn saw that it was better to use a single diagram, as in his window for three classes, and then shade out any overlap which was not represented in the problem under discussion.

Amusingly, in Euler's enumeration of his diagrams for three classes he made a mistake right at the end and never arrived at Venn's three-circle form. When he added a third set $C$ to a two-set diagram with overlapping sets $A$ and $B$ so as to overlap $B$ partially, he drew the cases $C$ wholly in $B$ and $C$ wholly outside $B$, but muddled the case $C$ partially in $B$, which would have given him Venn's three overlapping circles. This mistake is corrected, with only a footnote, in his collected works, leading to the frequent assertion that Euler included Venn's form.

In the three Nobel windows, Sir Charles Sherrington (1932), physiologist, has a rendering of one of his own diagrams showing 'Two excitatory afferents with their field of supraliminal effect in the motorneurone pool of a muscle'. Sir James Chadwick gained his Nobel (1935) for discovering the neutron. It is said that his window, showing an alpha particle bombarding a beryllium atom, does not recognise the conservation of momentum, but nobody seems to mind. Francis Crick (1962) gave his blessing to the window showing the Watson–Crick DNA double-helix on condition that Watson agreed – and that the window was not visible from outside at night. For in that case the DNA would be coiling the wrong way.

*The windows were designed by Maria Ulatowska McClafferty from drafts by Anthony Edwards, and photographed by Derek Ingram.*

At the dinner table Anthony arranged for me to be seated next to a younger person who had a speech impediment. He told me that he was studying Black Holes; however, due to his impediment it was difficult to bring the conversation much further. Later I learned that my table Fellow was Stephen Hawking (1942-2018), of whom I did not know much at the time. It turned out much later that he and I had a common interest beyond Physics and Mathematics[13].

As is the tradition, after the dinner some of the Fellows and their guests gathered in another room for coffee and port-wine. The port-wine, which was passed around by the Master of the College, was exquisite and I reached for the bottle to have a second glass, causing a scandal. I was sternly admonished that a refill was possible only if the Master decided to pass the carafe around again – which he courteously did. At Churchill College, where the focus is on science and technology, the dinners were much less formal, though one had to wear a gown.

The College has a splendid window commemorating some of its most illustrious Fellows. The picture of the window is included here by courtesy of the Master and Fellows of Gonville and Caius College.

## 2.7 CAMBRIDGE LINKS

This first stay in Cambridge was extremely fruitful for me, providing ample opportunities for discussions with British statisticians and probabilists, particularly in London as well as Cambridge. Among these were George Barnard, Anthony Edwards and Geoffrey Watson.

### 2.7.1 GEORGE ALFRED BARNARD

George Barnard (1915-2002) F.R.S. was pioneering the resurge of interest in the basic aspects of the likelihood concept that took place from around 1950 and onwards, beginning with his paper on 'Statistical Inference' published in 1949 in the Journal of the Royal Statistical Society B, p. 115-149. A later highly influential paper appeared 1962 in Journal of the Royal Statistical Society A, p. 321-372 under the title 'Likelihood Inference and Time Series' and was written jointly with G.N. Jenkins and C.B. Winsten.

Barnard was Professor at Imperial College, London, from 1954 till 1966, at what time he moved to a Chair at the newly established University of Essex, retiring in 1975. He told me that his move to Essex was partly motivated by his political convictions which were strongly leftist. After his retirement from the University of Essex he continued being active in research, spending much of his time at the University of Waterloo.

He served as President of several scientific societies, including the Institute of Mathematics and Its Applications (1970-1971) and the Royal Statistical Society (1971-1972). In 1996 he wrote a review (Journal of the Royal Statistical Society A, p. 178-179) of my book with David Cox on 'Inference and Asymptotics' in which he said "A great virtue of the book is that it raises perhaps as many questions as it answers".

### 2.7.2 ANTHONY W. F. EDWARDS

Anthony Edwards is a Life Fellow of Gonville and Caius and till his retirement he held a Chair in Biometry at Cambridge University.

He was one of the main contributors to the discussions of likelihood based inference that took place around the time discussed here and his book *Likelihood* was instrumental in generating interest in the area more broadly.

Ronald Fisher was the mentor of Anthony Edwards and, like Fisher, the main research areas of Edwards have been statistical inference from the likelihood point of view, on the one hand, and genetics and evolutionary biology on the other.

In 2015 Edwards was elected Fellow of the Royal Society for his contributions to evolutionary biology.

### 2.7.3 GEOFFREY STUART WATSON

My stay in Cambridge also led to my acquaintance and friendship with Geoffrey Stuart Watson (1921-1998) with whom I shared the interest of applications of Mathematical Statistics to fields of Natural Science.

In particular we had a common interest in directional statistics. One of his main contributions to the literature is the monograph *Statistics on Spheres*.

A charming and wide-ranging account of his life is available in 'A Conversation with Geoff Watson', published in *Statistical Science 13* (1998), 75-93.

We also shared a devotion to opera. One of his daughters became opera singer and another worked for the Royal Opera House in London.

# 3 LIKELIHOOD AND INFERENCE

Internationally, the discussions of inference principles were followed by endeavours to sharpen the techniques of maximum likelihood and likelihood ratio testing by incorporating the ideas of conditioning, ancillarity and partial sufficiency and by the use of refined asymptotic analysis.

As far as my own contributions are concerned the main results was the introduction of the $p^*$ formula and the modified directed likelihood $r^*$.

It is important to have an assessment of the distribution of the maximum likelihood estimator $\hat{\theta}$. Fromthis onemay derive an assessment of the distribution of the likelihood ratio statistic $r$. The limiting distribution of $\hat{\theta}$, as the number of observations becomes large, is Gaussian under mild regularity conditions. However as an approximation to the actual distribution of $\hat{\theta}$ this may not be very accurate. The same goes for the limiting chi squared distribution of the likelihood ratio statistic $r$.

The $p^*$ formula provides in general a very accurate formula for the (conditional) distribution of $\hat{\theta}$. In many cases of importance where the model is of exponential or transformation type it, in fact, provides the exact distribution of $\hat{\theta}$. But in addition it respects the principle that conclusions should be drawn conditionally on ancillary quantities. The same holds for $r^*$. Moreover, the formulae for $p^*$ and $r^*$ satisfy important invariance properties, and they allow for extensions to allow for partial inference about separate components of the parameter.

The theory of $p^*$ and $r^*$ was built on aspects of differential geometry and on advanced methods of asymptotics, especially that of large deviations, and is described in my paper [BN (1983)] and the monograph *Parametric Statistical Models and Likelihood.*

It was David Cox who proposed to call $p^*$ 'The Magical Mystery Formula'.

THE DEPARTMENT OF STATISTICS

presents

Ole Barndorff-Nielsen

in

The Magical Mystery Formula

$$p(\hat{\theta}:\theta|a) = c\,|\hat{j}\,|^{1/2}\bar{L}$$

Place: Eckhart 120
Time: Monday 7th April at 3.00 p.m.

**Invited talk given at Imperial College**
**"The Magical Mystery Tour is hoping to take you away"**
**– Beatles album *The Magical Mystery Tour* 1967**

The book on 'Inference and Asymptotics' from 1994, written jointly with David Cox, is a systematic account of what happend, up till that time, on the scene internationally in the area of statistical – mostly likelihood based – inference and related results from asymptotic analysis, particularly saddle point approximations and large deviations.

In compact form the basic formula for $p^*$ is

$$p^* = c|\hat{\jmath}|^{1/2} e^{l-\hat{l}}$$

where $j$ denotes the Fisher information matrix, which consists of the partial derivatives of the log likelihood function $l$, and where the $\wedge$ symbol above $j$ and $l$ indicates that these functions are to be evaluated at the maximum likelihood estimate $\hat{\theta}$.

The refined likelihood statistic $r^*$ is defined by

$$r^* = r - r^{-1} \log(r/u).$$

Here $r$ is short for the *directed likelihood statistic* relative to a parameter component $\psi$ and $u$ is a certain statistic defined in terms of likelihood quantities. More specifically, in defining $r^*$ one thinks of the full parameter $\theta$ as being of the form $((\psi, \chi)$ with $\psi$ one-dimensional and let

$$r = \operatorname{sgn}(\hat{\psi} - \psi) 2 \big( l(\hat{\psi}, \hat{\chi}) - l(\psi, \hat{\chi}_\psi) \big)^{1/2}$$

where $\hat{\chi}_\psi$ is the maximum likelihood estimate of $\chi$ assuming that $\psi$ is known.

In the simplest case where $\theta = \psi$ the expression for $u$ takes the form

$$u = \hat{\jmath}^{-1/2} \big( l_{/\hat{\psi}}(\hat{\psi}, a) - l_{/\hat{\psi}}(\psi, a) \big)$$

where $l_{/\hat{\psi}}$ stands for the partial derivative of $l$ with respect to $\hat{\psi}$ – note: not $\psi$ – when $l$ is considered as a function of $\psi$, $\hat{\psi}$ and ancillary $a$. In general, the formula for $u$ is

$$u = \frac{|l_{;\theta}(\hat{\theta}) - l_{;\theta}(\hat{\theta}_\psi)\; l_{\chi;\hat{\theta}}(\hat{\theta}_\psi)|}{|j_{\chi\chi}(\hat{\theta}_\psi)|^{1/2} |j(\hat{\theta})|^{1/2}}.$$

The roles of r and $r^*$ are primarily to provide intervals that with chosen degree of confidence can be assumed to contain the true value of the parameter $\psi$.

## ENDNOTES

1 Georg Rasch, 1901-1980, later Professor of Statistics at the Social Science Faculty at Copenhagen University. For excellent accounts of the life, personality and work of Rasch, see [Andersen (1983)] and [Olsen (2003)]. Scientifically he is primarily known for his invention of the so called 'Rasch objective measurement model' which is widely used, particularly in Psychology and related fields of the Social Sciences.

2 Niels Erik Nørlund, 1885-1981. Director of the Geodesical Institute and from 1923 Professor at the Mathematical Institute, Copenhagen University.

3 The Danish 'doktorafhandling' is roughly equivalent to an Anglo-Saxon thesis for a Sc.D. degree, the designation of the Danish degree being Dr. Phil.

4 See [Rasch (1930)], [Rasch (1934)], and also [Dollard and Friedman (1979)].

5 Ole Urban Maaløe, 1914-1988. Following his position at the State Serum Institute, Maaløe was called to the newly established chair in microbiology at Copenhagen University 1958.

6 Later, in 1953, together with the British physicist Francis Crick, James Watson discovered the structure of DNA, the achievement for which they were awarded the Nobel Prize in 1962.

7 In [Söderquist (1998)] the collaborations between Jerne and Maaløe on the one hand and Rasch and Bentzon on the other are discussed at several places. In this connection it is natural to mention that prior to meeting Rasch Jerne had never encountered the fields of statistics and probability, but he was soon convinced about the significance of these fields and spend much time with Fisher's books.

8 Here and elsewhere I use the word stochastics in its modern sense of covering all aspects of probability theory and mathematical/theoretical statistics.

9 See Rasch [Rasch (1960)], in particular Chapters V-VII, [Olsen (2003)] and [Andersen (1980)]. Cf. also http://rasch.org.

10 Ragnar Frisch (1895-1973) was co-recipient, with Jan Tinbergen, of the First Nobel Memorial Prize in Economic Sciences.

11 Recently – in 2007 – included in Aarhus University as The Danish School of Education.

12 An excellent brief account of the professional life of Anders H. Hald has been given by Steffen Lauritzen in Journal of the Royal Statistical Society A 171, (2008) 1029-1030. In terms of publications Hald is mostly known for his book on Statistical Theory with Engineering Applications and for his later writings on the history of statistics.

13 I cannot resist here to mention that as a member of the English Wagner Society I received an email on the occasion of Hawking's death, with a link to an article about one of his final interviews. There, Hawking was asked how he would like to spend his last days and replied: "Oh my last day, it would be being with my family and listening to Wagner. While sipping champagne in the summer sun."

14 Springer Series in Statistics

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# SAND TRACES

# 4 THE BEGINNING

## 4.1 FIRST STEPS

One day around 1969 I was approached by Professor of Geology Jens Tyge Møller who asked me to look into what he and some of his colleagues thought might be a serious problem with one of their laboratory procedures. They were studying the size distributions of windblown sand. Samples of sand were collected at the Danish West coast, brought to the laboratory and the size distribution was determined by sieving each sample through a stack of sieves of decreasing mesh size. Each of the fractions of the sample thus obtained was then weighted and assuming as an approximation, that the single grains were spherical a histogram of the size distributions was obtained. The general view internationally was that distributions must be Gaussian but Tyge Møller and his colleagues found invariably plots of the distributions on normal probability paper showed not a straight line but an S shaped curve. But since things “have to be Gaussian” the curve was considered as representing the concatenation of three normal distributions, corresponding to the three characteristic types of sand movement in wind: creep, saltation and suspension. To test this we collected, sometime later, samples of suspended sand particles, again on the West coast and found that they also exhibited an S shape.

When first contacting me, Tyge Møller brought with him a copy of a book “The Physics of Blown Sands and Desert Dunes” by one Ralph Alger Bagnold. This book showed examples of the empirical distributions of sand samples from the Libyan Desert and the author remarked that the distributional shape was far from Gaussian and that histograms, plotted in log-log scales, strongly indicated a hyperbolic shape rather than the Gaussian parabolic. An example of Bagnold’s illustration of this is shown by the right-hand Figure of the 4th display of Section 5.3 below.

To represent this observation in statistical terms I wrote down the formula for hyperbolic curves of the kind suggested, raised the expression by the negative exponential function and looked for a way to represent the integral of the resulting expression, in order to normalize by this to obtain a “hyperbolic” probability density. It turned out that the integral was expressible in terms of one of the Bessel functions of type K.

The probability density was thus determined in an explicit form, well amenable for inference purposes and providing remarkable fit to Bagnold’s empirical distributions. A further analysis showed, surprisingly, that in spite of the purely fitting origin of the distribution it was interpretable probabilistically in several important ways, in particular as a variance mean mixture of the Gaussian distributions and in terms of a stochastic integral.

Shortly after having completed a manuscript describing some of these findings, with focus on the statistical aspects and discussion of several types of empirical data, particularly regarding distributions of particles in various kinds of data, including sand samples collected at several locations in Denmark and analysed in the laboratory of the Institute of Geology, I received an invitation to a workshop at the Mathematical Research Institute Oberwolfach (MFO) where I presented the findings. Present among the group of participants was Professor David G. Kendall (later Sir Kendall) from the Statistical Laboratory, Cambridge University, who told me that he was acquainted with Ralph Bagnold through their Fellowships in the Royal Society of London. He also proposed to endorse publication of my paper in the Proceedings of that Society.

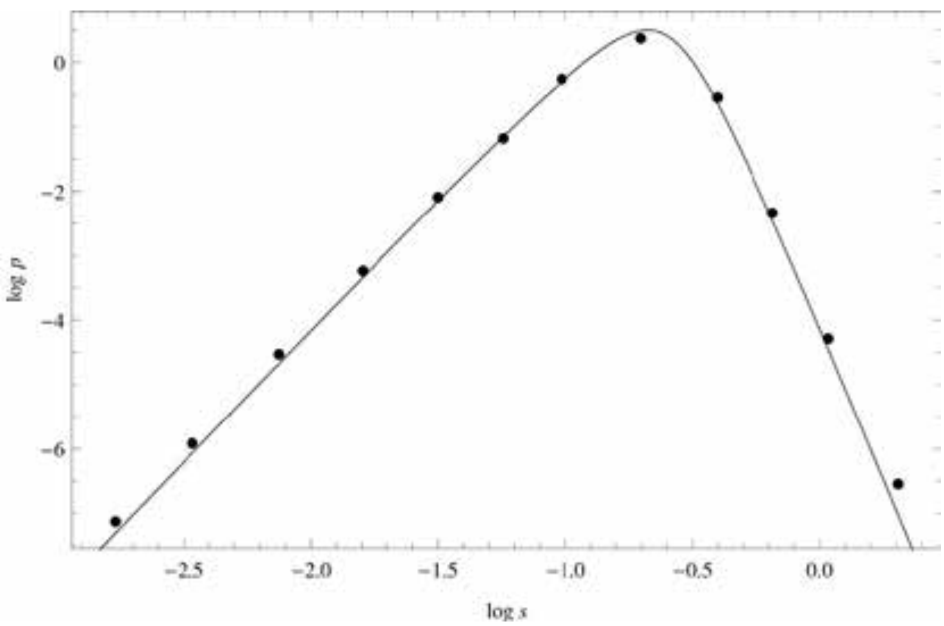
*Proc. R. Soc. Lond.* A. **353**, 401–419 (1977)
*Printed in Great Britain*

Exponentially decreasing distributions for the logarithm of particle size

By O. Barndorff-Nielsen
*Aarhus University, Denmark*

(*Communicated by D. G. Kendall, F.R.S. – Received 2 July 1976*)

The family of continuous type distributions such that the logarithm of the probability (density) function is a hyperbola (or, in several dimensions, a hyperboloid) is introduced and investigated. It is, among other things, shown that a distribution of this kind is a mixture of normal distributions. As to applications, the paper focuses on the mass-size distribution of aeolian sand deposits, with particular reference to the findings of R. A. Bagnold. The distribution family seems, however, to be of some potential usefulness in other concrete contexts too.




**Agreement between the hyperbolic law and the particle size distribution of one of the sand samples studied by Bagnold**

David Kendall further mentioned that he thought that Bagnold might still be alive, though it seemed uncertain in view of his advanced age – Bagnold was in fact born in 1896. I managed to trace his last known address and I sent a copy of my paper there, without knowing whether he was in fact alive and if so whether he would be in health and have interest in what I had written.

To my great delight, shortly after having sent my paper to Ralph Bagnold I received, virtually by return mail, a long reply from him, with suggestions for further work and saying that he was very pleased that after 30 years someone had taken up his old suggestion of the importance of the hyperbolic shape.

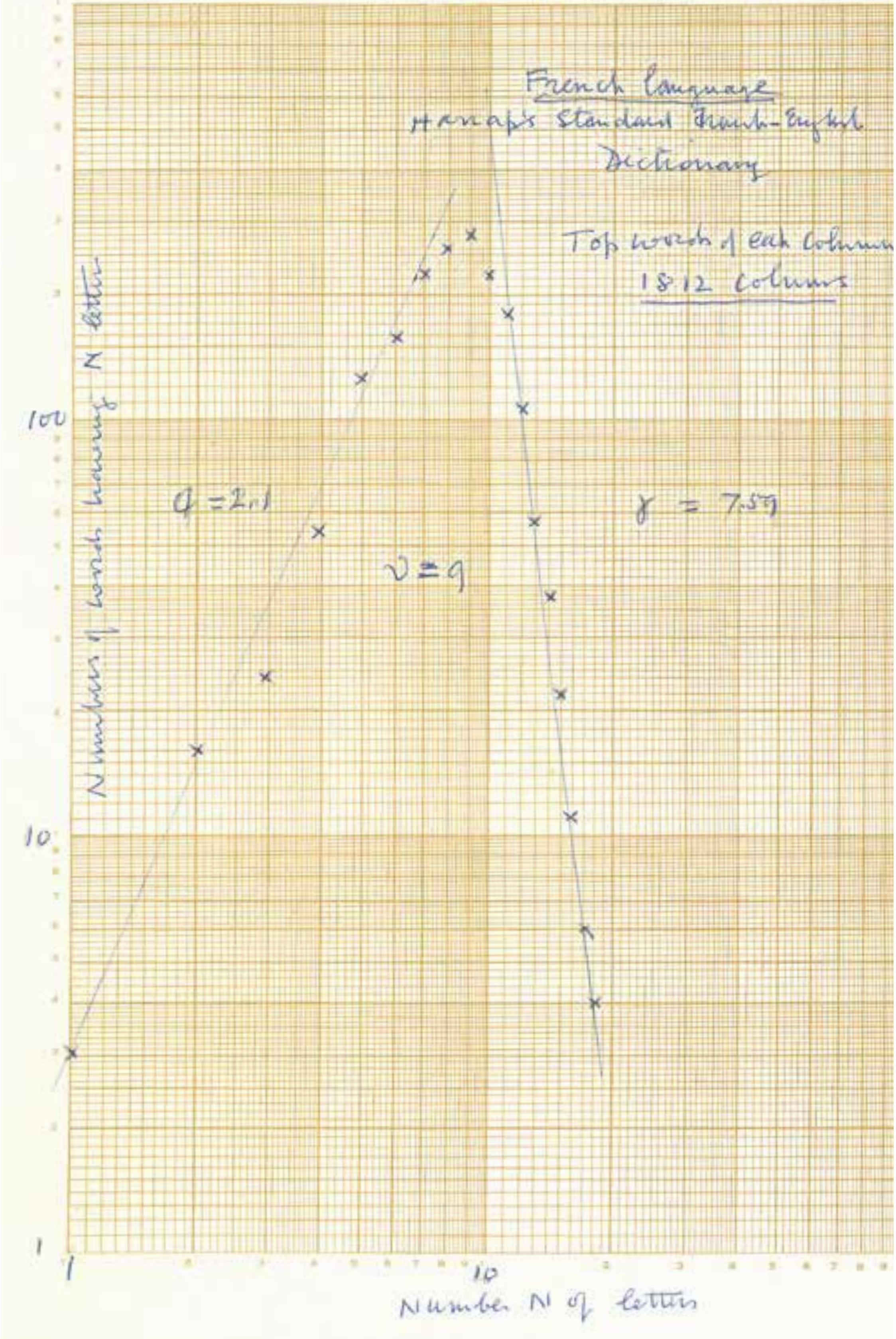
This first letter from Ralph Bagnold was the beginning of a delightful collaboration and friendship. This included a number of visits, of Bagnold to Aarhus and of me to his home in Edenbridge, Kent.

His visits and participation in the buildup of the study of sand transport by a group of geologists and statisticians at the Faculty of Science at Aarhus University engendered widespread appreciation and in 1978 he was awarded a Doctorate Honoris Causa by the University of Aarhus. One of the Faculty members taking a strong interest in Bagnold's life history and scientific work was the then Professor of the Institute of the History of Science at the Faculty, Olaf Pedersen. Motivated by these interests an interview of Bagnold by Olaf Pedersen was recorded by a film, entitled 'Blown Sand' and available on the internet.

We started to analyse a great number of empirically observed distributions of very different origin and character. Together we wrote a paper on "The pattern of natural size distributions", [BN, Bagnold (1980)], where it was shown how the hyperbolic distribution provided excellent fits to samples from river bed sediments, glacio-fluvial sediments, marine sediments and to sand in saltation.

Our early contacts revived Bagnold's interest in the subject of size distributions and in a paper 'The nature and correlation of random distributions', published in *Proceedings of the Royal Society of London'* (1983) he studied a number of such distributions – the frequency of words of given length (the 'size') in 7 different languages (including French, Hungarian, Anglo-Saxon and Sanskrit) – finding in all cases that the hyperbolic structure provided a strikingly good description.

Bagnold also asked me for an example of how the normal law might occur in reality. I mentioned Galton's apparatus whereupon – characteristically – he constructed such an apparatus and tested the claim finding, on the basis of a large number of trials, that the tails of the distribution were strictly log-linear, as for a hyperbolic distribution, in stark contrast to normality.




**The agreement between the hyperbolic law and the distribution of the lengths of words in the French language. Calculation and drawing by Ralph Bagnold**

## 4.2 ROYAL SOCIETY EVENT

The Royal Society

Thursday 12 April 1984 at 4.30 p.m.

*at 6 Carlton House Terrace, London SW1Y 5AG*

A REVIEW LECTURE

**Hyperbolic distributions in theory and in the physical world: for example sands, the relativistic ideal gas, turbulence, word lengths, palaeomagnetism**

By Professor R.A. Bagnold, F.R.S., and Professor O.E. Barndorff-Nielsen

Since by implied definition an empirical random distribution, in the statistical sense, is one not ordered by man, it must be regarded as a phenomenon that obeys some natural law capable of being disclosed by a study of the facts. A recent closer study of existing data from a wide variety of fields has indicated that in Nature distributions tend to be of a 'hyperbolic' type rather than the, generally assumed, normal type. There is an accompanying new concept whereby strictly random distributions can be defined in absolute terms by a pair of mutually independent 'chance' parameters.

The statistical theory and applications of hyperbolic distributions, and of certain related distributional types, will be discussed.

At present, the most abundant source of relevant statistics is that of the natural size distributions of granular material; and since the formative conditions for such distributions can to some extent be controlled or selected this source provides a promising experimental field in which to analyse the conditions that determine the varying values of the distribution parameters. Experiments in this direction will be described.

*Review lectures are intended to provide a review of the current development of a subject and are aimed at an audience of scientists not necessarily involved in the subject being reviewed.*

**Tea will be served from 3.45 p.m.**

All interested in the subject are welcome at the meeting. Tickets for admission are not issued but those who wish to attend are asked to inform the Executive Secretary, The Royal Society, 6 Carlton House Terrace, London SW1Y 5AG (reference: RLS/CAJ).

Telephone enquiries to: 01-839 5561, ext 278 or 277 Telex: 917876

At the instigation of Professor David Kendall, Bagnold and I were invited to give a joint Review Lecture on "Hyperbolic distributions in theory and in the physical world" to the Royal Society of London.

**The event at the Royal Society was followed by a dinner at the Atheneum.**

The Athenaeum is a gentlemen's club in London, founded in 1824, that has had many well known persons as members, today including lady members. The distinctive clubhouse (located at 107 Pall Mall at the corner of Waterloo Place) was designed by Decimus Burton in the Neoclassical style with a Doric portico, above which is a statue of the classical goddess of wisdom, Athene. The bas-relief frieze is a copy of the frieze round the Parthenon in Athens. The club's facilities include the extensive library, a dining room known as the Coffee Room, a Morning Room, a Drawing Room on the first floor, a Smoking Room (where smoking is not permitted) and a suite of bedrooms.

In the course of our preparation for the Royal Society Lecture I had, over the phone, to inform Bagnold about a paper to which we wanted to refer and that had appeared in the German probability journal Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete (which has since changed its name to Probability Theory and Related Fields). Not surprisingly I had to spell the title of the journal for him and having gotten through the third word he took a deep breath and commented "that is one Hell of a word".

afdeling for teoretisk statistik

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3 May 1984 OBN/ow

Professor David G. Kendall
Department of Pure Mathematics
and Mathematical Statistics
Statistical Laboratory
University of Cambridge
16 Mill Lane
Cambridge CB2 1SB
England

Dear David,

This is to thank you, again, most warmly for occasioning the Review Lecture with Ralph Bagnold at the Royal Society. I enjoyed this event and the subsequent dinner at the Athenaeum tremendously. In particular, I was extremely impressed by the far-ranging scientific discussion that took place, for more than 2½ hours, in the Royal Society Club, after the dinner.

Thank you also for your willingness to continue to serve as Review Correspondent for the period 1985-88.

With best wishes,

Yours Ever,

O.E. Barndorff-Nielsen

Among the topics discussed in the Royal Society Lecture was a reference to the ideal relativistic gas. The inclusion of this came about through my realising, quite by chance and some considerable time after the first contacts to Bagnold, that in relativity theory Maxwell's law – a three-dimensional isotropic Gaussian distribution – is not valid and must be replaced by what is in fact a three-dimensional hyperbolic law which is in general non-isotropic. A brief account of this connection was given in a paper published in the Scandinavian Journal of Statistics [BN (1982)]. More details on this are discussed in Section 8.

As an aftermath to the meeting in The Royal Society I was invited to participate in the 324th Anniversary Dinner of the Society.

The President of The Royal Society
requests the honour of the company of
Professor O.E. Barndorff-Nielsen
at the 324th Anniversary Dinner of the Society
to be held at The Park Lane Hotel, Piccadilly
(The Ballroom Suite)
on Friday 30 November 1984
Seven o'clock for half past Seven

R.S.V.P. to
The President of The Royal Society
6 Carlton House Terrace
London SW1Y 5AG

Evening Dress or Dinner Jacket
Decorations
This card should be produced

TOASTS

**The Queen**
Proposed by THE PRESIDENT

★★

**Queen Elizabeth the Queen Mother,**
**the Prince Philip, Duke of Edinburgh,**
**the Prince and Princess of Wales,**
**and the other members of the Royal Family**
Proposed by THE PRESIDENT

★★

**The Royal Society**
Proposed by
The Rt Hon. the Lord Carrington, P.C., K.C.M.G., M.C.

Response by THE PRESIDENT

★★

**The Guests**
Proposed by
Professor R.J. Elliott, Sec. R.S.

Response by
Professor J. Szentágothai, For.Mem.R.S.
President
Hungarian Academy of Sciences

★★

The Society has arranged to provide coffee to those Fellows and guests who wish to remain after the speeches.

# 5 RALPH ALGER BAGNOLD

## 5.1 FORMING YEARS

Brigadier Ralph Alger Bagnold, FRS, OBE was born 3 April 1896 and died 28 May 1990. He was from a long-lived family with military traditions in the UK army. His grandfather was Major-General Michael Edward Bagnold (1786-1857). And his father, Colonel Arthur Henry Bagnold (1854-1943) participated in the expedition 1884-85 that tried to rescue General Gordon in Khartoum.

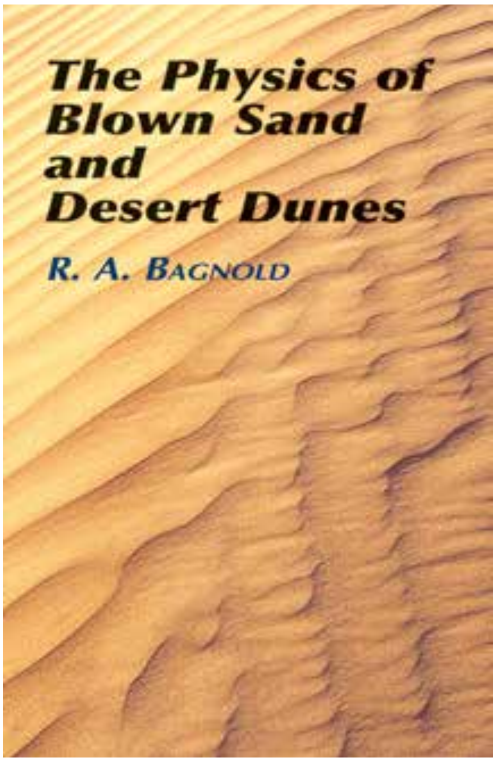
Ralph Bagnold himself participated in both World Wars. Much of the period between the wars was spent in Egypt where he started to explore the Egypto-Libyan Desert, together with some friends. There arose his scientific interest in the physics of transport and deposition of sand.

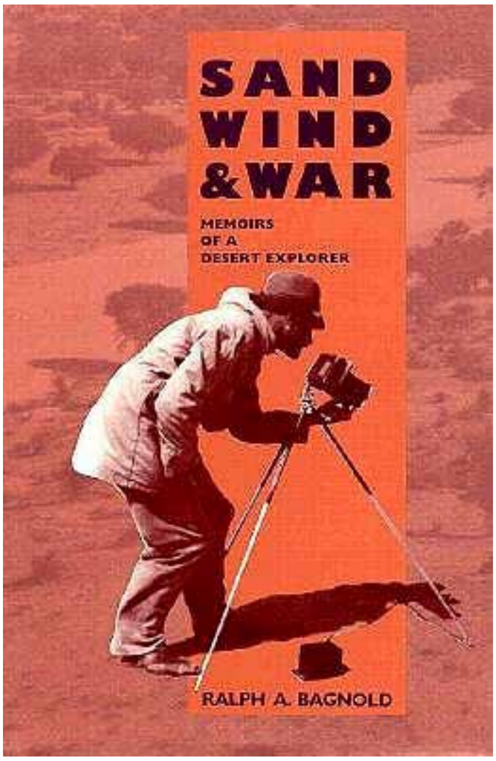
At the end of WW II, by the instigation of Frederick I.M. Stratton, who also fought in WW I and who was a Fellow of Gonville and Caius College at Cambridge University – which was also R.A. Fisher's College – Bagnold was admitted to the University and to Gonville and Caius, where he took a degree in Engineering.

## 5.2 SOLDIER, EXPLORER AND SCIENTIST: A BRIEF

Bagnold's explorations of the desert and his fascination with blown sands began when in the late 1920-ties and early 1930-ties he was stationed with the British Army in Egypt. He and some of his fellows there, having rather ample time on their own, started to venture into the desert East and West of the Nile, in an old Ford T motor car, learning how to navigate in the sands, for the purpose of which Bagnold invented a sun compass.

The adjacent map, compiled by Michael Welland and kindly put at my disposal here, shows the routes of Bagnold's exploratory travels in the time between the two World Wars. Michael Welland (1946-2017) was a British Geologist known for his highly knowledgeable and widespanning accounts

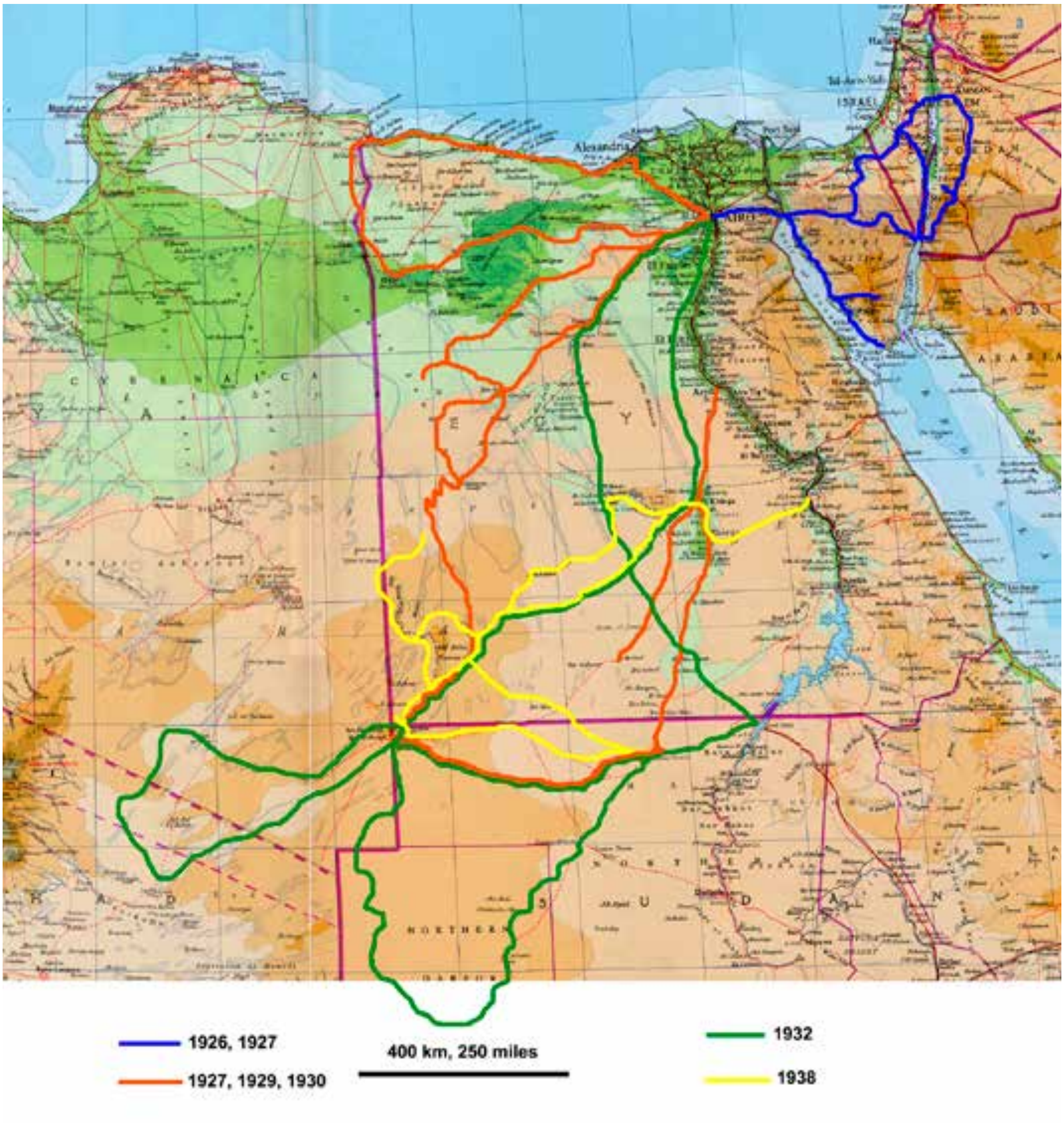



of the fascinating world of sands, in their geological roles, their influence on life on Earth, and in their historical importance. Particularly well known are his books 'Sand: a journey through science and the imagination' (Oxford University Press) which won the 2010 John Burroughs Medal, and of which the review in Nature said 'A popular book by a long-time professional geologist could easily have been worthy but dry. Yet Sand is serious and entertaining; it is the work of someone who has been in love with the stuff since he built sandcastles as a child. Nothing like it has been published before, even by the larger-than-life pioneer of sand studies, Ralph Bagnold, who serves as Welland's inspiration'. That book was followed by 'The Desert: Land of Lost Borders' (University of Chicago Press, 2015)[1]. Michael Welland was also the author of 'The History of the Inns and Public Houses of Wells' (Wells Local History Group, 2012).

Bagnold became so intrigued with the phenomena that he observed and studied that he decided to take leave from the Army, went back to his home at Shooters Hill and built a private wind tunnel to carry his investigations further.  This led to his writing the marvelous book on "The Physics of Blown Sands and Desert Dunes" which appeared in 1941, about the time when he was elected a Fellow of The Royal Society of London.

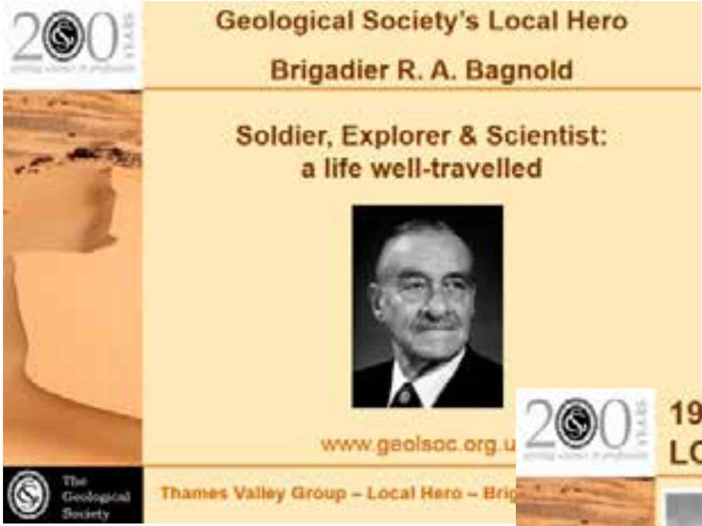
At the beginning of the Second World War Bagnold was called up for service in the British Army again and was to be posted in India. But by a fortunate turn of events he was transferred to Egypt which gave him the opportunity to propose to the Commander in Chief there, General Wawell, that a 'Robin Hood'-like army group be formed which, building on Bagnold's knowledge of the Desert, was able to raid the outposts of the Italian army in North Africa, with quite disproportionate effects. This was the renowned

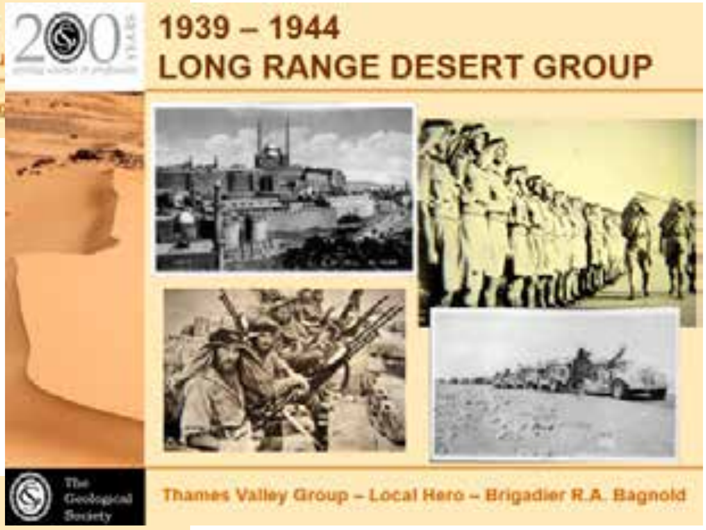



'Long Range Desert Group', or LRDG, whose history and achievements have been amply documented since. Much information on both his scientific work and his role in the formation and life of the LRDG is given in Section 5.4, in Bagnold's own hand.

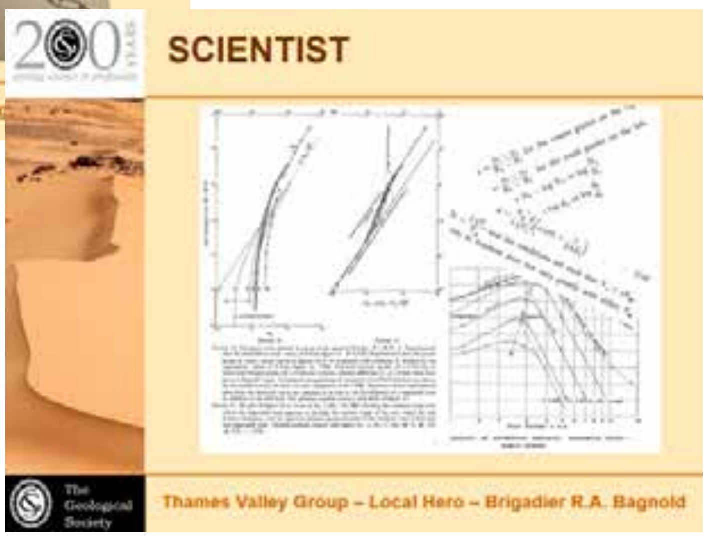
## 5.3 200 YEARS ANNIVERSARY OF THE GEOLOGICAL SOCIETY

In connection to the 200 Years Anniversary of the British Geological Society Michael Welland created a picture exhibition about Bagnold's life and achievements. Some of the pictures are shown below, courtesy of Michael Welland.


Geological Society's Local Hero
Brigadier R. A. Bagnold
Soldier, Explorer & Scientist:
a life well-travelled
The Geological Society


1939 – 1944
LONG RANGE DESERT GROUP
The Geological Society
Thames Valley Group – Local Hero – Brigadier R.A. Bagnold


SCIENTIST
AIR
The Geological Society


SCIENTIST
The Geological Society
Thames Valley Group – Local Hero – Brigadier R.A. Bagnold

**From: Michael Welland**
**Sent: 10 October 2006 11:21**
**To: Ole E. Barndorff-Nielsen**
**Subject: Ralph Bagnold**

Dear Professor Barndorff-Nielsen,
I apologise for writing to you "out of the blue", but I am hoping you can help me with a project relating to Ralph Bagnold. I am a geologist, based in London, and am trying to set up a project associated with the bicentennial programmes of the Geological Society here. Some of the planned events for next year will relate to 'geological heroes', an attempt to involve the public in awareness of geologists who have made significant contributions
(see http://www.geolsoc.org.uk/template.cfm?name=Bicentenary). Having developed a fascination with Bagnold as a scientist and an extraordinary individual, I am trying to assemble a proposal for an event about his life and work. I am familiar with some of the work that you did with Bagnold, and have come across reference to the film that was made at Aarhus. I have not been able to download the film from the web and, first of all, am writing to ask if there are other ways of obtaining a copy for potential use by the Geological Society. And secondly, I would very much appreciate any ideas that you may have from your personal experience on ways to put this celebration together.

Thank you in advance for your help

Best regards
Michael Welland

## 5.4 SOME UNPUBLISHED WRITINGS

This Section presents two pieces of Ralph Bagnold's writings that have never been published before. They were given to me by Bagnold in connection to his being awarded the Honorary Doctorate by Aarhus University.

The first piece is the manuscript for the opening lecture of a colloquium on Aeolian Sand Transport on Earth, Mars and Venus, held at NASA, U.S.A., January 1977. The second tells of an episode in the story of the Long Range Desert Group.

These documents testifies to Bagnold's eminence as a writer. They are included here in agreement with him.

## [1] Wind and Sand Interactions
**by R. A. Bagnold**

I have been asked to outline my original work on the movement of blown sand and how it was initiated. At the same time, I will take the opportunity to update the ideas where necessary.

In 1926, as an officer in the British Army I was by pure chance posted to Egypt, to a unit stationed in Cairo. There, again by pure chance, I fell in with a small group of young officers whose hobby was to pioneer the cross-country performance of their privately owned Model T Ford cars. In those days motor transport in the Middle East was still in its infancy. There were no roads. Cars followed ancient camel tracks as best they would. It took a day to get a car the 50 miles from Cairo to Suez.

Potentially, Cairo was an ideal center for the sightseeing of past history. Initially our main object was to get to interesting places. To the east lay Sinai and all the biblical lands, then almost inaccessible to the tourist. To the west there stretched out the immensity of the Egypto-Libyan Desert; half a million square miles of rainless, lifeless and still mysterious country, untrodden since prehistoric times. This desert had never yet been crossed from east to west, and the first crossing from north to south had been achieved only three years previously. It was still a land of legends – of lost oases and ancient armies overwhelmed by sand.

Our hobby led us to discover the possibility of travelling entirely self-contained in fuel, water, food and spare parts on cross-country journeys exceeding 1500 miles. We devised a system of navigation by sun-compass and theodolite which changed the very real fear of getting lost into complete self-reliance. Moreover, we devised a quick and sure method of extracting our cars when stuck in soft sand.

Within 300 miles westward of the Nile lie a group of inhabited oases – Khanga, Dakhla, Farafra and Bahariya – depressions wind-scoured down to the artesian water table which is fed by underground seepage from far away in Central Africa. Between the oases the only real obstacles were isolated dune chains. Like everyone else we accepted these as uncrossable. Beyond the oases there was known to lie a continuous barrier of giant dunes, range beyond range as far as the eye could see. The map of Egypt ended in the west with the word "Great Sand Sea; limits of dunes unknown". No one had attempted to penetrate this mysterious region (now known to cover some 50.000 square miles) since the near-disastrous Rolfs expedition of fifty years before.

Here indeed was a challenge. Against everyone's advice we decided to have a try. Ford had by then produced the greatly improved Model A car, and we had gained a great deal of experience. To our astonishment we succeeded. Having once taken the measure of the first great dune range a kilometer wide and some hundreds of feet high, our three cars were able to cross each succeeding range with little difficulty. We found that every 5 to 10 miles along the length of each chain the wall of towering slipfaces was broken by a half-hidden lane, very steep and but a few yards wide, of surprisingly firm sand. By charging the dune here at top speed it was possible to surmount the first unclimbably steep hundred feet or so. There enormous things were all identical. It was this discovery, and the resulting experience of travelling for days high above solid ground without seeing any fixed landmark, nothing but endless chains of smoking dune summits from horizon to horizon in all directions that first gave me the urge to investigate the physics underlying the strange behavior of the sand. You see, we felt the dunes to be alive. Tolerant and even friendly in calm weather, but furious and intent on destroying us during days of storm. On two occasions they nearly did.

In this desert local rain falls but once in half a century or so, and in consequence no vestige of vegetation exists to interfere with and confuse the simple interaction of wind and sand. The evident high degree of organization here demands answers to many questions. Why for instance does sand collect into discrete dunes instead of scattering uniformly over the desert? Why, when a storm wind blows, scouring away the sand from beneath one's feet, does the dense cloud of flying grains confine itself to a few feet above ground leaving heads and shoulders projecting against a clear blue sky? Why, again, do the great dune chains run so straight for a hundred miles and more and precisely parallel to one another? Why, far to the south where the lanes between the ranges are of exposed stony desert, are the margins of the dunes so clearly defined – as if, in the words of walrus “forty maids with forty mops had swept for half a year”? Why, again, in certain areas do the almost ubiquitous longitudinal dune chains give place to columns of discrete barchans?

I retired from the army in 1935 after 20 years service, which, incidentally had started in the trenches early during World War I, and decided to devote much of my time, by way of a private hobby, to an experimental study of sand movement. The field was wide open. No scientific study had been made. Some argued that the grains of the low-flying sand clouds were suspended by atmospheric turbulence. Others maintained that the upward dispersion must be due to the electrical repulsion of highly charged grains. I was sure from experience that neither mode of transport could be true. The clear-cut ceiling to the sand cloud ruled out random turbulence, and none of us had ever got an electric shock by touching a car body during a sand storm. The remaining explanation was saltation, but could a quartz grain rise to shoulder height by mere bouncing off the ground?

This was easily tested. I made a turntable of plywood and set pieces of glass at a chosen angle round the edge. The turntable was spun at the peripheral speed of a typical storm wind, and a succession of sand grains was let fall vertically from above. The envelope of the resulting trajectories was traced by catching the grains on sticky fly-papers. The grains did indeed rise to the observed heights. Moreover, I found no evidence that the violent impacts had shattered any of the grains. The impacts seemed almost perfectly elastic.

Turning from experiment to theory, the equations of motion of a sand grain rising into a wind are mutually interlinked because the fluid drag acts always in the varying direction of the relative velocity vector. Furthermore, those equations contain a dimensionless group representing the ‘Penetration’ i.e., the number of diameters a free grain travels relative to the fluid while its relative velocity is halved, or doubled, by fluid drag. This parameter may be useful when considering grain movement on Mars. Assuming a given wind velocity gradient and given initial conditions it is possible to trace a grain’s approximate trajectory by making a succession of small arbitrary step-by-step decrements in the relative velocity. The method can be computerized with considerable increase in accuracy. Note that the scale of the grain trajectories is set by wind and grain conditions; so it is not at all safe to place any reliance on small scale wind tunnel models purporting to imitate dune morphology. Moreover, the morphology of dunes in a necessarily uni-directional wind is quite different from that of dunes formed by far more common multi-directional wind regimes.

Nevertheless, measurements in my home-made wind tunnel of the relation between the transport rate of sand and the strength of the transporting wind threw an immediate light on why sand collects into dunes. I covered the tunnel floor with a layer of uniform sand. The saltation over the surface was much lower than in the previous experiment. The evident reasons was that instead of elastic bouncing off rigid surfaces the saltation grains now splashed into a yielding grain bed. The higher the saltation the faster the surrounding wind, hence, presumably, the greater the transport rate. Noting that a sand grain only twice as big has eight times the mass and so will act much like an anvil, I scattered somewhat larger grains over the surface of uniform sand. Sure enough the measured transport rate was nearly doubled. Here then was the simple explanation of why sand tends to deposit on existing surfaces of like sand rather than elsewhere. For a local decrease in the transport rate is synonymous with a local deposition, and vice-versa. The Bedouin understands this. When an advancing dune threatens his water-hole he scatters pebbles over it. The sand shrinks away because the pebbles increase the local transport rate. The dune tends to re-form downwind.

Turning again to theory, it has long been known that the available power from a windmill increases as the cube of the wind speed. Mechanical power is measured as an applied force times the velocity with which it acts. Regarding the surface wind as a machine for moving sand against the granular friction of the surface, the available transporting power must similarly be measured as the wind’s applied shear stress times its effective velocity. The shear stress can be expressed by $\rho v_*^2$ where $v_*$ is the so-called friction velocity. We do not know at what height to measure the effective wind velocity but we know it should be proportional to $v_*$. Hence, we should expect the transport rate of sand to be proportional to $\rho v_*^3$. From the then quite recent work of Prandtl, Von Karman and others on the structure of shear turbulence, the value of $v_*$ could be got directly form measurement of the wind velocity gradient against log distance from the bed surface. The wind tunnel experiments left no doubt of the truth of this cubic relation.

Thinking again of the wind as a transporting machine it is evident that it cannot do work in moving sand without itself losing some of its power. Hence one would expect its velocity as measured over a fixed surface to be reduced appreciably by the act of transporting sand. Wind measurements disclosed a very marked change in the velocity pattern. As the wind strength was increased, the effective surface roughness, as measured by the intercept on the ordinate of zero velocity, became larger. Thus, as the wind gradient was increased the velocity profiles began to cross one another so that they came to an approximate focus at a nearly constant velocity. It now seems probable that this constant velocity represents the relative or slip velocity corresponding to the mean wind force on the average saltating grain.

Before leaving the subject of wind tunnel measurements I'd like to point out that my book described an amateur's venture into scientific research. The notation I used was regrettably inconsistent. In the more generally acceptable notation I have adopted subsequently the fluid velocity in the direction x of flow is denoted by little u, big U being reserved for the grain velocity. This leaves the V's to denote velocity components in the perpendicular y direction. Thus V becomes u. These later symbols are appropriate alike for wind-blown and water-driven grains.

The subject of sand movement by wind is inseparable from that of meteorology. Wind tunnel experiments cannot take account of the effects of natural variations of wind direction. In a varying wind regime the cubic relation often causes the mean annual direction of sand movement to differ appreciably form that of the conventional 'prevailing wind'. For the direction of sand movement is usually dominated by that of a few short periods of storm. If continuous meteorological records covering a period of years are available, the resultant sand movement direction can be computed from the original daily sheets. These alone give details of both strength and direction. A simple method is to divide the range of wind speeds into say six or eight bands, each defined by its mid-band speed. Subtracting the threshold speed of about 4 meters per sec. from each of these speeds and cubing the remainder one gets a set of factors, each appropriate to a particular speed band. On previously prepared record sheets one then goes through the daily records sheet by sheet, making a mark in the appropriate place. Then, for each compass bearing in turn the number of marks is multiplied by the relevant factor. The total of each column is proportional to the component sand transport vector for that direction. The annual resultant can be got by drawing the usual vector diagram. I have found that on average it takes 1 ½ hours to analyse a year's records. The annual direction may vary somewhat from year to year according to the number of storms. So it is as well to examine a period of at least 5 years. A knowledge of this mean sand-driving vector is of great importance to the oil wells.

I should be borne in mind that while a change of climatic wind regime may take a whole millennium to make any noticeable change in the direction of the great dune chains whose cross-sections may exceed 20.000 square meters, the shape and orientation of a little dune or drift may change in a few weeks or even days. So the study of dune morphology is rather abortive in the absence of continuous wind records. It is unfortunate that established meteorological stations are seldom if ever found in the near neighbourhood of dune fields. The local wind regime may be very different from that at the nearest meteorological station perhaps a hundred miles away. In fact the lack of reliable wind data has prevented our getting any but the scantiest understanding of detailed dune forms. From direct observation however two things are certain. Dunes living under a multi-directional wind regime take the form of longitudinal chains of continuously connected summits. They grow like tape-worms by adding segments or summits. Dunes, on the other hand, that live under a nearly uni-directional wind regime take the very different form of individual barchans. The limiting degree of directional variation is unknown for lack of precise wind data. Again, even the scantiest of vegetation can have a profound effect on dune morphology.

Incidentally I was once able to watch a barchan in the act of breeding. The baby was born as a little mound about a meter across in the densest sand stream – that just downwind of an outstretched horn. It soon grew its essential little slip-face, some 15 cm high and thereby became fully fledged. The rate of advance of a barchan being inversely proportional to the slipface height, the baby began running away ahead of its mother, gathering nourishment and growing as it ran.

To account for the remarkable straightness, parallelism and more or less uniform spacing of the dune chains the suggestions I make in 1952 has not been seriously challenged as far as I know. I suggested that the largescale, alternately-rotating vortices, due to thermal instability, which manifest themselves at their tops as the long parallel cloud formations in the sky in temperate regions, may in a hot desert manifest their rotation below by influencing the direction of the local ground wind. On a model scale pairs of corkscrew vortices are known to be produced by heating a gentle current of air form below. Since the saltation of sand grains provides a very effective heat exchange during hot months, the pattern of the instability is likely to be linked with that of a dune system. Thus a pair of oppositely rotating vortices situated in a lane between two dune chains, down in the middle and up at the sides, would not only provide the curious 'forty maids with forty mops' effect, but would determine the spacing of the dune chains and tend to perpetuate their direction. On the model scale it appears that the overall width of a vortex pair is some six times the height of instability. Thus a dune chain spacing of 2 to 6 kilometers would mean a thermal instability between 300 and 1000 meters high.

The link between meteorology and dune direction on this planet is evident on a global scale, from the striking similarity of the orientation patterns of all four of the great dune systems – in Australia, South Arabia, Egypt-Libya and Algeria. While the general drift is in each case towards the Equator, the orientation makes the same 90° wheel, roughly in the neighbourhood of the Tropic, forming an eastward to a westward bearing. The pattern, interestingly, seems independent of the Continental geography.

There is another aeolean problem, to my mind almost as intriguing as dune behavior, in a way the reverse of it. Text books write glibly of peneplanes. But what physical conditions could possibly create the great sand sheet of southern Egypt? More than 20.000 square miles of absolutely flat, level and featureless rippled sand? Wind erodes differentially, creating isolated depressions, escarpments etc. and leaving plateau and isolated hills. Water erodes to form valleys large or small. It is not at all easy to imagine a primitive surface of such a size, sedimentary or otherwise, of such absolute uniformity that erosion has left no features whatever. Professor Haynes has seen this region. Perhaps he will touch on this problem later. I think it may well be relevant in the present context.

In conclusion I might mention the strange sequel to our old desert expeditions. We had made these for our own private satisfaction at our own expenses, for the fun of it. None of us had ever dreamed that war could possibly come to the dead interior of N.E. Africa. But hardly was my book finished and in the hands of the publisher than I was recalled to the army for Word War II and posted to East Africa. By the pure accident of a troopship collision in the Mediterranean I was back in Egypt. I found myself the only man on the spot who knew that the Great Sand Sea could be crossed, and who had the special knowledge and experience to grasp the implications. With the relatively enormous range of self-contained action I knew was possible the whole interior of Libya could be used as a sort of $4^{th}$ dimension in which a small specially equipped force could travel anywhere in secret.

Italy declared war. The Mediterranean was closed to us. The large Italian army in Libya nearly ½ million strong, was about to invade Egypt along the coast. A few incomplete formations were all that were available for its defence. The small voice of a retired major having been ridiculed by the ‘usual channels’. I went directly to General Wavell, the C in C of all our available land forces from India to West Africa. Within less than half an hour I came away with that remarkable man’s order to raise, equip and train within six weeks a special force of my own design. Foreseeing difficulties and misunderstandings with his formidable GHQ staff, he gave me an extraordinary talisman. “To all heads of departments and branches”, it read “I wish every demand made by Major Bagnold in person to be met immediately and without question”. He also gave me carte blanche to operate anywhere I chose in the enemy hinterland. We were ready on time and the embryo Long Range Desert Group disappeared secretly from Cairo.

On the same day that the Italian Army began to cross the frontier eastward into Egypt, along the coast, three little troops, each of 10 armed trucks and 40 volunteers, mostly New Zealand sheep farmers, crossed the same frontier, westward, far to the south, to operate independently within enemy territory. Small supply columns vanished without trace. Aircraft and supplies were burnt on unmanned refueling grounds. Isolated outposts were attacked by unidentified raiders on the same day several hundred miles apart. Uncertainty is the bane of the high command in war. Graziani, who was already having logistic difficulties in maintaining an army of 10 divisions at the end of the single 1200 mile coast road, began to disbelieve his own intelligence reports. Wavell’s bluff paid off. The whole Italian army halted for three vital months. Meanwhile reinforcements arrived, having had to be diverted round Africa. With these, in a quick and brilliant campaign Wavell put the Italian army in the bad.

To end with a little cautionary tale. Increasing distances made it difficult for us to keep in touch with the general situation, radio being too dangerous for us. So we bought two single-engined light aircraft form an Egyptian pasha. With a young French army pilot I had borrowed from Colonel Leclerc away in French Equatoria we were flying westward over the Sand Sea in the intense summer heat. The little engine began to overheat. We had to land immediately. I saw continuous dunes everywhere below. The dunes got larger and more formidable as we descended. The end would come in a few seconds. We must inevitably crash. There was a slight bump. Young made perfect landing on the dead flat surface of the Calansho Serir. We had left the real dunes several miles behind. There they were along the skyline. What we had seen, and could still plainly see on the ground were the ghosts of former dunes that had migrated away long ago, leaving their outlines realistically mosaicked on the flat pebble surface. The moral is, I think, that desert landscapes viewed from above do not always represent what previous experience suggests.

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## [2] An episode in the story of the Long Range Desert Group

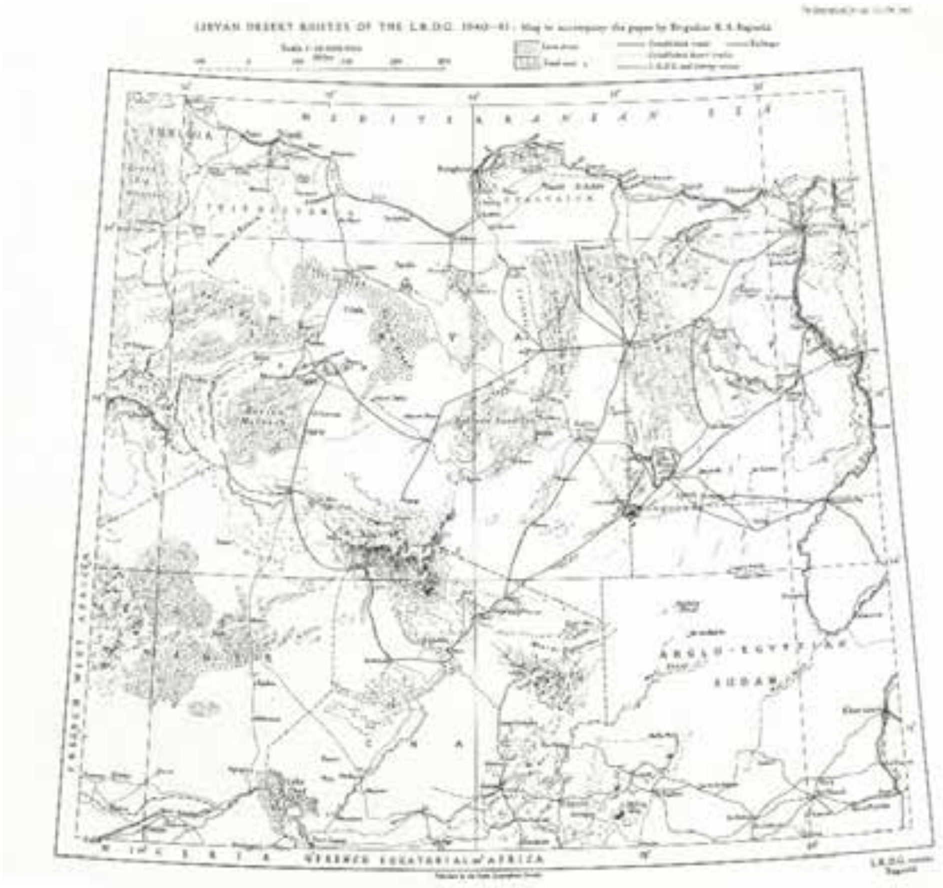
The following episode in the LRDG story has never been published; for it involved the French war-time dilemma, best forgotten – loyalty to one’s own established national government, though a mere German-dominated puppet, or loyalty to a single almost unknown rebel, de Gaulle, with risk to family and future.

No up-to-date and comprehensive small-scale map of the whole interior, from the Nile to the Algerian border, existed. Pat Clayton, professional map-maker, had compiled one for our own use, from many scattered sources and from our own recently gained knowledge.

Towards the end of 1940 Clayton, Shaw and I, with this map on the table, were considering what further raids would alarm the enemy most. ‘Do you see what I see?’ said Clayton. ‘Murzuk?’ I said ‘right in the far southwest corner of Libya. The back of beyond. Garrison secure and probably asleep. The going looks practicable. The lie of the dunes should be just right. It’s far beyond our range, though. At least 2500 miles there and back. I wonder. Suppose we could get supplies from the French in Equatoria. But GHQ here has no dealings with them. We don’t even know which side they’re on. Probably high politics’.

‘Why not ask Douglas’ said Shaw. ‘He’s bound to know much more about that than the army people here.’

**Pat Clayton's map**




The next day I flew the thousand miles south to Khartoum, and spent the evening with Douglas Newbold, an old friend of ours, now Civil Secretary, head of the Sudan government.

'Yes' said Douglas ,I know the Governor of Chad Province. He's my next-door neighbour so to speak. He's a little negro from Martinique named Eboué. Very shrewd. I think he trusts me, and I think he'd like to come in on our side. But there's dissention. He has to carry his people with him. They fear being attacked as rebels by Niger Province which is definitely pro-Vichy. And his own military commander is also pro-Vichy or is sitting on the fence. All Eboué can do at the moment is to allow us the use of Fort Lamy airport for RAF re-fuelling. He can explain that as force majeure. My own hands are tied. Relations with the French overseas dependencies are top-most-level Churchill-de Gaulle stuff. Very tricky. But look here. You have no political status. Why don't you go to Chad and see them yourself? I'll arrange a flight for you tomorrow morning. It's an interesting country. Very big, like mine, and even more primitive. Desert, great mountains, tropical jungle, rivers swarming with crocodiles, and wild pagan tribes controlled by a very tough army.'

So I flew on westward to Fort Lamy, half-way across Africa, in a BOAC airliner all to myself. There I suddenly collapsed on the tarmac, of a violent fever I used to get occasionally, probably the long aftermath of malaria.

I came to in a tastefully furnished bedroom, nursed by a charming little negro lady I discovered was Mme Eboué.

A few hours later there followed a strange bed-side conference in French. The governor came in with a fair-haired officer in full-dress camel-corps uniform who towered above him. 'This is Colonel d'Ornano, second-in-command of the army. A Corsican' he added to explain the name. d'Ornano peered round 'He's not here?' 'No' said the governor 'and he won't be!' 'Good. Then I'll speak for the army'. The governor nodded.

d'Ornano turned to me. 'Were those your people who came and scared my outpost at Tekro a month or so ago? And they drove all the way from Cairo and back without any supplies en route? Tiens!

But you haven't come here for nothing. What do you want?'

I told them quite frankly about the planned raid on Murzuk, and handed d'Ornano a short list of what we should need. Petrol, water, and some French army rations.

They looked at one another. d'Ornano said to the governor 'This is it. We've got to decide now. Now – thumping the table – this is our chance. My younger officers are getting restive, I can't hold them inactive much longer. You agree?' Eboué nodded again.

'Then' said d'Ornano to me 'I'll do all you ask; but on one condition. You take me with you to Murzuk, me, and one of my officers[6] and one NCO. And we fly the French flag alongside yours.'

They were ready to burn their boats, ignoring their military commander. The unexpected participation of the French would greatly increase the psychological effect on the enemy. I agreed at once. We fixed a date and a rendezvous north of the Tibesti mountains. d'Ornano there and then wrote out a formal contract between the French army of Chad and Major Bagnold, and we both signed it.

d'Ornano's death during Clayton's successful raid on Murzuk was a tragedy. He was the first Free French casualty in action against the Axis Powers.

My unorthodox journey to Chad, made with Newbold's discrete connivance, had two immediate consequences, apart from making the Murzuk raid possible. Chad Province came in openly on the Allied side, the only French dependency to do so voluntarily. This allowed de Gaulle, then fighting to establish Free French rule further south in Gabon, to send Colonel Leclerc[7] to Chad as military commander, to improvise a mobile force for his daring and successful attack on Kufra.

Leclerc and I became friends. We were able to help each other in various small ways.

In order to bolster de Gaulle's movement it was given out officially from London that the Murzuk operation had been a purely French enterprise.

As all the other principals – Eboué, d'Ornano, Newbold, Leclerc and de Gaulle – are now dead, it may be as well to record the truth about even so relatively minor an episode of the war.

R.A.B 1971

## Postscript by Ralph Bagnold

I returned to Chad early in 1941 to meet the raiding party on its return form Murzuk. Leclerc had arrived with a young British liaison officer named George Mercer Nairne[1]. Having a day to spare at Fort Lamy, Mercer Nairne and I borrowed a motor boat and cruised up one of the rivers flowing into the Lake Chad swamps. The river was in flood. The banks were yellow. But the yellow squirmed slowly down into the water as we passed by. Crocodiles as densely packed as holiday makers on Brighton beach. We passed villages where children were bathing in the river, right among them. The things were evidently gorged with fish. It was tropically hot and the river very tempting. We both stripped, dived in and had a long swim. The crocodiles ignored us.

**Murzuk raid 1941. Village notables of Traghen coming out to surrender keys.(RAB)**

**Murzuk raid 1941. Rendez-vous with French north of Tibesti. From the right Massu, Clayton, d´Ornano. (RAB)**

## 5.5 SAND ORGANS

This Section contains a letter to me from Ralph Bagnold and two related letters, from Michael Welland and from David Kendall, both letters commenting on the phenomenon of 'sand organs' or 'singing sands' as it is also called.

**From a letter to O.E. Barndorff-Nielsen from R.A. Bagnold:**
Rickwoods, Mark Beech, Edenbridge, Kent, 13 March 83

Dear Ole,

...

Have you, in your Fascination of Sand, said anything about C.E.S. Phillips' 'sand organ'. He was a neighbour of ours when I was a boy. He had inherited a big private lab and dabbled in science. I saw his sand organ. He played it to me. It was a set of vertical glass tubes, varying in diameter and in wall thickness. Each was full of dry sand. On opening a valve below, the whole sand column was allowed to descend slowly, emitting a musical note. The stick-slip motion starts a circumferential oscillation in the glass of the tube, so that each expansion allows the whole sand column to fall a little. He wrote a short paper I think in Proc.Roy. Soc.Edin. around 1912. His difficulty, if I remember right, was that there was an uncertain delay in starting the vibration.

I have seen so many booming sands in action in various places that I think I know quite a lot about the physical mechanism – in outline. (Did I give you a copy of my paper Proc.Roy.Soc. A 295 of 1966?). But the fascinating mystery remains. How to distinguish a dune sand that will boom from the majority that will not. Booming sands may be dirty or clean, of barchans or seif dunes, desert-dry or coastal-salty. There seems to be nothing peculiar about the size distribution. In common with nearly all sands, the grains are Quartz. What can the answer be? I have pointed out somewhere that sand dunes can have all the essential attributes of life – in their own mineral way. They absorb nourishment and grow; they maintain a definite shape and repair damage to it; they move from place to place; and they can breed, giving birth to baby dunes that start their own life. To all this one can add that they can communicate with one another at a distance, in the sense that they can respond to a distant signal. I enclose a note describing a startling event, as I remember it. ...

The note referred to is reproduced below:

A REMOTE DUNE EVENT
In March 1938 Ronald Peel and I were getting up at dawn from a night in the shelter of a barchan dune, about 1 km west of the great cliff of the Gilf Kebir plateau in S.W.Egypt. There was no wind and no other living thing within 200 km. Other isolated dunes lay scattered some ¾ km distant. Abruptly out of the silence a booming roar burst upon us. It came out of the foot of our dune less than two metres away. The ground we stood on vibrated. A sand avalanche was pouring steadily down the dune's slip-face, but, as on other occasions, the sound came

not from the avalanche itself but from immediately below it where the static sand was being thrust down the slope by the increasing weight of the avalanche and was being sheared within itself. The booming was so loud we had to shout to be heard. It went on for perhaps a minute.

In the sudden silence which followed we heard two answering calls from far away. They were not mere echoes, for their two distinct frequencies combined to a slow beat about once a second. Whether transmitted by air or ground wave, the vibration must have started avalanches down at least two distant dunes. The disturbance faded to a confused murmur as more distant dunes became involved.

Activated dynamically, as we are chemically, by the atmosphere, these great granular forms seem capable of an odd imitation of life. Undoubtedly the dunes of this colony were communicating with one another and responding.

There is a casual account of the above in Geographical Journal XCIII April 1939. There are many accounts of booming dunes but I know of no record of a distant dune response.

R.A.Bagnold 1983

**From: Michael Welland**
**Sent: 08 November 2006 11:33**
**To: Ole E. Barndorff-Nielsen**
**Subject: video**

Dear Ole,
Your package has arrived safely – many thanks! The video is the first time I have had an opportunity to see and hear Bagnold and was really enjoyable. I have also passed on a copy to Stephen Bagnold with whom I went yesterday to the British Film Institute and viewed film of the 1930 and 1932 Western Desert expeditions – quite extraordinary. There were several lingering and beautiful shots of dunes, blowing sand and general desert topography - perhaps a sign of his growing interest in how it all worked.

Thank you also for the publications, which I have much enjoyed. I noted that, in his letter to you in 1983, Bagnold refers to C.E.S. Phillips' "sand organ" and comments that he thought that Phillips had written a note about it in the Proceedings of the Royal Society of Edinburgh. The latter have done a search for me, but came up with nothing. Do you have any clues as to where this might have been published?

I look forward to our continuing communication, and, I hope, meeting one day.

Best wishes
Michael Welland

UNIVERSITY OF CAMBRIDGE

DEPARTMENT OF PURE MATHEMATICS AND MATHEMATICAL STATISTICS

STATISTICAL LABORATORY

Telephone
Cambridge 65621

16 Mill Lane
Cambridge CB2 1SB

Professor D. G. Kendall, FRS

Prof. O. Barndorff-Nielsen
Afdeling for Teoretisk Statistik
Matematisk Institut
Aarhus Universitet
DK - 8000 AARHUS C
Denmark

23 February 1983

Dear Ole,

You wrote to me about the reference to singing sands in Marco Polo. I have looked this up for you and enclose some xeroxes. The only copy at my immediate disposal is the English edition published as number 306 in Everyman's Library in 1908 (reprinted 1939). My impression is that it is rather a poor edition. When you read it you keep finding editorial notes attached to various places in central Asia saying "this has not been identified". You then open almost any modern atlas and there the place is, a thriving town. So I think you ought not to refer to this edition, but to some more recent and more scholarly edition, and possibly even by preference to an Italian edition. However, you will be able to find the reference by noticing that it occurs in Chapter 36. After finding this, I remembered that I had a copy of the very interesting book by Mildred Cable and Francesca French called The Gobi Desert, published Hodder and Stoughton in London in 1942 and reprinted many times since. These were two Christian missionaries who spent most of their lives in the Gobi desert and then ultimately they wrote this very extraordinary book about the people and places that they had seen. From this I will xerox for you pages 64 and 65, where they not only refer to the singing sands of the Gobi desert, but also to Marco Polo's own observation seven hundred years ago. I hope that these references will be useful to you in producing the article on "The Fascination of Sand" which I look forward very much to seeing. When you next are in touch with Bagnold, do please greet him for me, although we have never met; the excuse for this must simply be that for forty years now he has been one of my number one heroes, and I should of course love to meet him if ever there were a possibility. With best regards. Yours ever,

David

# 6 THE BAGNOLDS

## 6.1 BLACKHEATH — AARHUS — LONDON

This first letter from Ralph Bagnold was the beginning of a delightful collaboration and friendship. I had the privilege to visit Ralph Bagnold and his wife, lovingly known as "Plankie", several times at their mansion house in Blackheath, thus getting a fascinating insight into Bagnold's military and scientific career. Also, there I had the privilege of meeting Ralph Bagnold's sister Enid Bagnold, as described later, and his son Stephen.

**Photo of Ralph Bagnold taken in his garden in Blackheath, by the author**

And my wife and I had the delightful pleasure of having Ralph and 'Plankie' visit Aarhus for several days, on the occasion of him being awarded the Dr. Honoris Causa at the Aarhus University. This was but one of a number of visits by Ralph Bagnold to Denmark, including a scientific excursion with colleagues in the 'Sand Gang' (see later) to the Northern and Western part of Jutland.

## 6.2 ENID BAGNOLD

Ralph Bagnold's sister Enid Algerine Bagnold, Lady Jones, CBE, (1889-1981), was a very productive novelist and playwright, well known at the time. She attended the Walter Sickert art school in London, and shortly after finishing that she started working for Frank Harris, author of the notorious, sexually explicit book *My Life and Loves*. She became the mistress of Harris, the first of a long list of male attachments. She too was military active, participating in the First World War at first as a nurse, from which position she was dismissed after writing critically about the hospital administration, and later as a driver, describing her driving experiences in the novel *A Happy Foreigner*.

In 1920, in spite of the remarkable independence of her soul, after a very short period of courting by a quite recent acquaintance George Roderick Jones, until then a sworn bachelor, the two of them were married and she lived with him till his death in 1962.

Her husband, later Sir Roderick Jones (1902-1962), was Chairman of Reuters (the press agency). Much of their life together was spent in the leading circles – in politics, the arts and the nobility, but throughout she kept a strict regime of independence for her creative work.

In Roderick Jones's autobiography *A Life in Reuters* there is, in Chapter XXIII: "A Fateful Meeting: Cherchez La Femme", a vivid description of their life together and of how they were introduced to each other and where he describes her as "vibrant with spirit and imagination, impulsive, generous and untamable". They had three sons and one daughter.

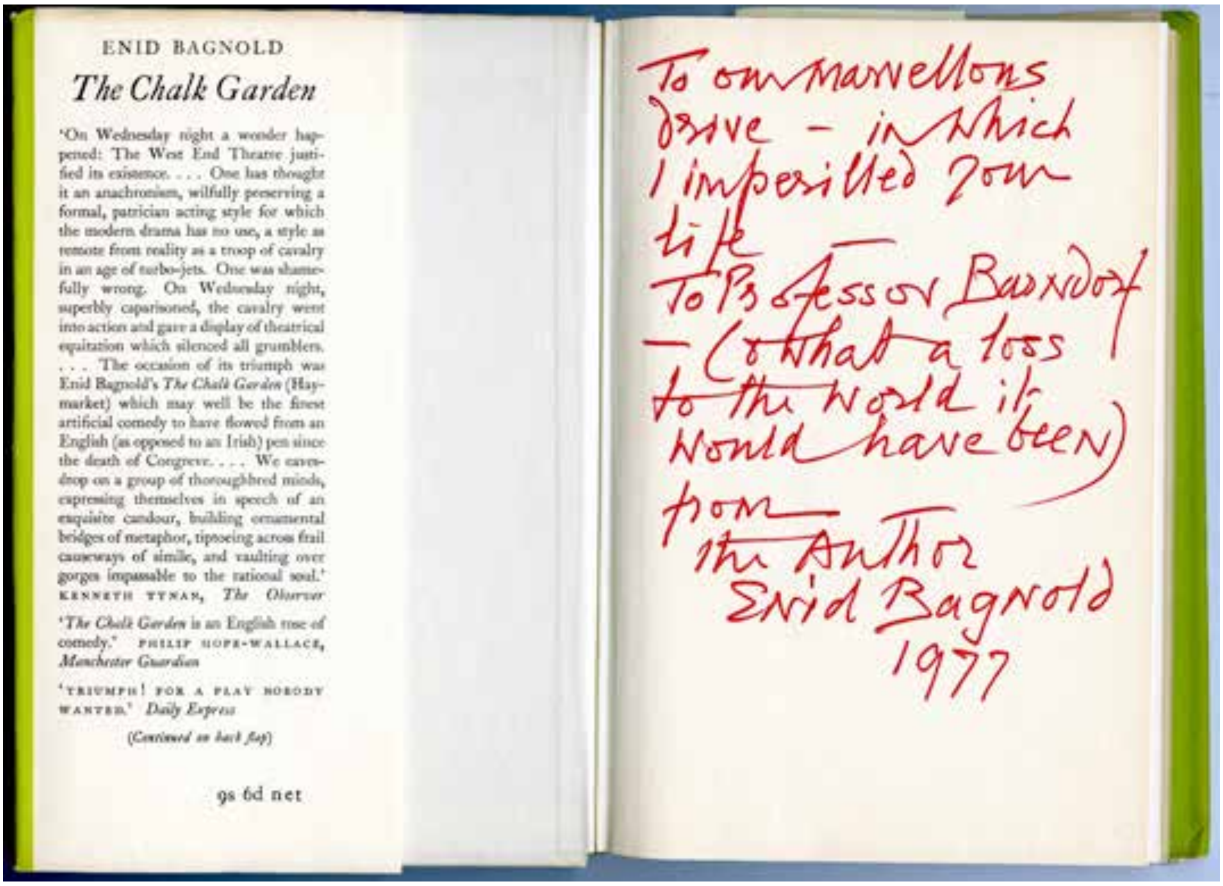
Enid Bagnold's life is described both in her autobiography *Enid Bagnold's Autobiography* and in *Enid Bagnold, the Authorised Biography* by Anne Sebba. The latter contains photographs of her and her brother's parents and of the family home on Shooter's Hill.

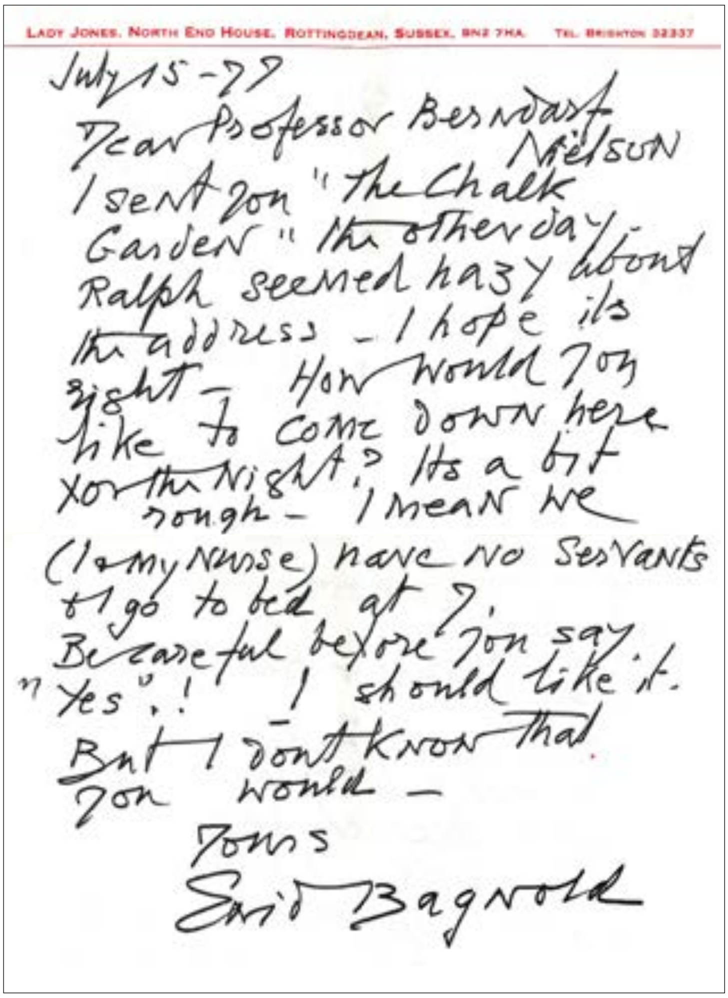
This brings me to my own encounter with her, when she was about ninety years old. At the time I was staying with the Bagnolds at their house in Black Heath and on the day I was to leave Enid Bagnold came to visit. We had a very pleasant family lunch and interesting conversations, and despite her age she was not past flirting. Then in the late afternoon she dosed off on the sofa. After a while I said that it was time for me to return to London, by train as usual, but at that moment she woke up and said 'I can drive professor Barndorff-Nielsen back to London". Considering her age and what we had drunk for lunch both Bagnold, his wife and I were somewhat wary of this proposal but I resolved that politeness demanded my saying thank you, and we set off towards London.

Shortly after beginning our journey along the small winding country roads Enid told me "My eyesight is not too good, so while I concentrate on the steering wheel you better look ahead and warn about oncoming traffic". All went well till we came to a roundabout near the motor road to London where we drove straight into and half way up the earth mound in the middle of the roundabout. Enid Bagnold took it lighthearted just exclaiming "Oh, nearly two people killed". But we had a delightful and for me very fascinating conversation on the way. And shortly after I had returned home to Denmark she sent me a copy of her play "The Chalk Garden" with the nice inscription shown next to here. With my statistical leanings I was surprised to find, in the middle of the play the following words in a conversation between two main characters of the play:

> *Maitland: What rings true then?*
> *Madrigal: The probability. The likelihood.*
> *They work together to make things fit.*

This splendidly encapsulates the essence of mathematical statistics, though – I am pretty certain – the author had no inkling of this.





LADY JONES. NORTH END HOUSE. ROTTINGDEAN, SUSSEX. BN2 7HA. TEL. BRIGHTON 32337

July 15-77

Dear Professor Barndorff Nielsen

I sent you "The Chalk Garden" the other day. Ralph seemed hazy about the address – I hope its right – How would you like to come down here for the Night? Its a bit rough – I mean we (& my Nurse) have no servants & I go to bed at 7.

Be careful before you say "Yes"! I should like it. But I dont know that you would –

Yours

Enid Bagnold

Accessing Google under the name Bagnold will lead to numerous entries on the lives and works of both Ralph and Enid Bagnold.

As an example, an article in the Telegraph from April 2012, written by Michael Thornton, a much younger, friend of Enid Bagnold reports on an exhibition celebrating the life of Enid Bagnold, under the title “the fascinating first lady of a gilded society”. It tells of their friendship and of Enid’s adventurous life, in a vivid and illuminating way.

The following citation from Ralph Bagnolds book *Sand, Wind and War* (p. 146-147) throws additional light on his relation to his sister and on her exceptional character:

> Since my return to England I found myself closer than before to my sister Enid. I had always been welcome at her house, but the six-and-a-half-year age difference had long kept us apart. I had always been “my little brother” to her. We had different interests. Hers were literary and social. She had made a name for herself as an authoress and playwright. She had married Sir Roderick Jones, the owner and autocratic chairman of Reuters, the worldwide news agency. She had a full life bringing up her four children, writing, and managing three big houses with a commuting indoor staff of ten or more. One house was in London, at 29 Hyde Park Gate, another was Northend House at Rottingdean in Sussex, and the third was Kipling’s former estate. In the early days, Kipling was still living there and would sometimes drop in for tea. I remember him as a thin old man with heavy, projecting black eyebrows.
>
> Enid an I had two things in common – fond memories of our early childhood days in Jamaica and a disregard for convention which in our different ways amounted to a practical inventiveness. If something seemed useful to be done, we did it, even though it had never been done or thought of before. As an example, Enid wanted fresh milk for her children, something difficult to get in wartime London. She took her Guernsey cow, Daisy, from Rottingdean to Hyde Park Gate and stabled her in one of the garages. Daisy grazed happiliy in the royal parkland just across the Piccadilly-Kinsington thoroughfare. Every morning and evening the police

held up the traffic for Daisy to cross: "make way for Lady Jone's cow." Much the same order was given when Churchill drove out from No. 28 next door. The police were merely amused. One after another, the various authorities sent inspectors but could find nothing to forbid. And no one objected to Daisy's modest little pats.

## 6.3 SOME CORRESPONDENCE WITH RALPH A. BAGNOLD AND STEPHEN BAGNOLD

My collaboration and friendship with Ralph Bagnold led to an extensive correspondence between us, some of which is shown here. This was followed later by a correspondence with Ralph Bagnold's son Stephen.

**Ralph Bagnold**
**March, 1983**

Ricksoods, Mark Beech, Edenbridge, Kent 13 March 83

Dear Ole

Many thanks for sending me David Kendall's letter. I greatly appreciate his remarks about me at the end. As my father used to say "More people know Tom Fool than Tom Fool knows".

I read that remarkable book by Mildred Cable and her partner some years ago. One can just imagine those two old dears having fun tobogganing down that dune and making it boom. I have also read Owen Lattimore's book "The Desert Road to Turkestan" I got to know Lattimore in Peking about 1933. I asked him why none of the place-names in his book had any connection with those on the published map. He said "Previous travellers didnt realise that its highly disrespectful to the Spirit of the place to speak His name in His presence. They will invent some name to satisfy you. But if you wait till the place is left some miles behind, and ask casually what the name was, they will tell you without hesitation" This might well explain line 8 of Kendall's letter. "This place has not been identified". It wouldnt be if it was fictitious.

Have you, in your Fascination of Sand, said anything about CES Philips'sand organ? He was a neighbour of ours when I was a boy. He had inherited a big private lab and dabbled in science. I saw his sand organ. He played it to me. It was a set of vertical glass tubes, varying in diam and in wall thickness. Each was full of dry sand. On opening a valve below, the whole sand column was allowed to descend slowly, emitting a musical note. The stick-slip motion starts a circumferential oscillation in the glass of the tube, so that each expansion allows the whole sand column to fall a little. He wrote a short paper I think in Proc.Roy.Soc.Edin. around 1912. His difficulty, if I remember right, was that there was an uncertain delay in starting the vibration.

I have seen so many booming sands in action in various places that I think I know quite a lot about the physical mechanism - in outline. (Did I give you a copy of my paper Proc.Roy.Soc. A 295 of 1966 ? ). But the fascinating mystery remains. How to distinguish a dune sand that will boom from the majority that will not. Booming sands may be dirty or clean, of barchans or seif dunes, desert-dry or coastal-salty. There seems to be nothing peculiar about the size distribution. In common with nearly all sands, the grains are Quartz. What can the answer be ?

I have pointed out somewhere that sand dunes can have all the essential attributes of life - in their own mineral way. They absorb nourishment and grow; they maintain a definite shape and repair damage to it; they move from place to place; and they can breed, giving birth to baby dunes that start their own life. To all this one can add that they can communicate with one another at a distance, in the sense that they can respond to a distant signal. I enclose a note describing a startling event, as I remember it.

16 March. I have just got your letter of the 9th. Thanks for the Laplace material. I will send this letter care of Prof Cox as I doubt if it will reach you at Aarhus before you leave. Looking forward to hearing from you by phone next week. We are both beginning to recover from a rather bad go of 'flu, unfortunately.

With our best wishes

Ralph

**Ralph Bagnold**
**October, 1983**

Rickwoods, Mark Beech, Edenbridge, Kent
5 October 83

Dear Ole

Many thanks for your letter of 29 September. There has been a hitch in the sale of Rickwoods, so now we are unlikely to be moving before the end of this month. We shall therefore be delighted to see you here during the week-end 29/30 October.

I enclose the two more offprints you want, and I have already posted to you a packet of old photos in two envelopes, one of our private pre-war expeditions and the other of the war-time LRDG. I am sorry that only a few of the photos are labeled but that can be remedied if you bring the packet along when you come.

I have written to Lord Rothschild briefly saying that both the distributions he gave fail to fit the new facts disclosed by the log-frequency plots. Neither give the required asymptotic tails. I think that is what he suspected.

I shall be very interested to see your model of the sand sorting mechanism.

As follow-up to the letter-per-word distribution I have run through the gazetteer of my Oxford World Atlas 1952. From only a slim sample - less than 1000 names - the distribution of place names by number of letters is clearly log-hyperbolic. The mode ln $q_x$ falls in the right place. The mean chance $\bar{C}$ = 0.24 approx. and the intersection does not, as I suspected it would, occur at ln $\phi$ = 0 like the dictionary distribution. This was a little puzzling until I realised that human volition very probably intrudes. The cartographer alone must decide what place to insert and what to omit, and a cartographer unconsciously dislikes long names because they mess up his map. Hence the distribution should be unnaturally narrow - $\bar{C}$ considerably smaller than 0.5. What I am looking for is some other distribution in which the variate, like that of letters per word, consists of a number y of absolute units One indivisible entity, a letter or a person etc. But it must be untainted by any human volition. Can you think of such a distribution ? I have a hunch that it would have a mean chance $\bar{C}$ close to 0.5. ( I cant think that a lexicographer could have any similar objection to long words )

Possibly Professor Hammerich might be interested to do the same analysis of some other dictionaries, say Classical Latin and Greek, modern Russian etc. I note that in my Table 1, page 288, the synthetic oriental languages Arabic and Sanskrit have a narrower distribution $\bar{C}$ = 0.3 and 0.35, whereas the non-synthetic European languages are wider. This seems reasonable because in Arabic and Sanskrit dictionaries the many declensions and conjugations are omitted.

Yours
Ralph

**From the author**
**February, 1989**

afdeling for teoretisk statistik
matematisk institut · aarhus universitet · dk - 8000 aarhus c · telefon (06) 12 71 88

27 February 1989
OEBN/lja

Brigadier R.A. Bagnold, F.R.S., O.B.E.
7 Manor Way
Blackheath
London SE3 9EF
England

Dear Ralph and Plankie,

The itinerary for my visit to United Kingdom in May has now been largely fixed. I shall be participating in a meeting on Coastal Sands arranged by the Royal Society of Edinburgh, in Edinburgh, 17-19 May. From there I am going to Oxford to stay and work with Sir David Cox 20-24 May, leaving for London early on the 24th and going back from Gatwick in the afternoon of the 25th. Might I come and see you either on the 24th or the 25th, for instance in the afternoon of the 24th? (In all likelihood I shall be occupied for the evening of the 24th.)

Enclosed please find a card from Jakob and a small note of mine which may interest Ralph.

Looking forward to hearing from you and, hopefully, seeing you in May.

Many kind regards, also from Bente and Jakob.

As ever

During one of my visits to Ralph Bagnold and his wife 'Plankie' at their home in Mark Beech I had the good fortune to meet their son Stephen and his family. Our contact continued after the death of Plankie as reflected in the below copies of some of our correspondence.

7 Manor Way
Blackheath
London SE3 9EF

Professor O E Barndorff-Nielsen
Afdeling for Teoretisk Statistik
Matematisk Institut
Dk - 8000 Aarhus
Denmark

21 August 1990

Dear Ole

Thank you for your kind letter of 6 July, and my apologies for not having replied sooner. But it has been a busy time for us.

Mummy died in June last year. She had been growing increasingly frail for some while until it was difficult for her to leave her bed. Then she had a severe stroke, from which it was clear she wouldn't recover. Despite all her infirmities, however, she remained alert and caring for those around her right until the end.

Daddy took it all very stoically, although he missed her greatly. It was fortunate indeed that we managed to find two excellent and very caring housekeepers (both Irish) for him, one after the other. And of course with my family and I just round the corner there was someone in to see him each day. Britt managed the shopping and I helped him with his correspondence and so forth. Then this May he caught pneumonia and died, peacefully in his sleep, within 24 hours.

As to my father's various papers, I am currently in the process of sorting through his files and wish myself, first, to have a clear idea of their compass. As you may know, the most immediate several previous generations of Bagnolds were long-lived and late-marrying, with the consequence that there have been preserved a great many interesting family documents going back more than 200 years. These have been specifically left to me in my father's will, and I have not yet had an opportunity to study them in depth or to consider how they may relate to my father's own papers. Some of these latter are personal, some scientific, and some military. I will certainly consider the Churchill Archives as a home for the latter two categories in due course, providing ownership can remain with the family. It will however be a few months before I will be in a position to make a decision. In the meantime, I would be grateful if you would kindly thank Professor Kendall for his offer and let him know that if he would like to get in touch we can take things further in due course (I should perhaps mention we are away on holiday during the first half of September).

I hope you are well and that you will come and see us if you are over in the UK. We are in the process of moving in to Number 7, and with one son about to go up to university and another quite possibly joining the Army shortly, there will be plenty of room!

With best wishes,

Stephen Bagnold

**Stephen Bagnold**
**August, 1990**

**From: Stephen Bagnold**
**Date: Monday 18 July 2016 13:07**
**To: Ole E. Barndorff-Nielsen**
**Subject: RE: Bagnold on Mars**

Dear Ole
Thanks for this. I was unaware of your connection with Imperial College and David Cox. Small world!

Please do feel entirely free to use any of our correspondence in your autobiography. I would of course love to read it once it's published.

One extra little bit re RAB, sparked off by interest here in the Battle of the Somme, which as you no doubt know started 100 years ago this June and lasted, with almost unimaginable slaughter, until November. He fought in it and recorded its eve in his diary war diary. I see I already sent you the relevant excerpt, but some sections of it are included again here in the attachment for easy reference – together with an account of the eve of another battle, Agincourt, fought in Europe 600 years ago last year, as described by Shakespeare in the Prologue to Act 4 of Henry V. One could make too much of the comparisons I guess. But still fascinating nonetheless, particularly as to my almost certain knowledge Daddy only discovered Shakespeare in his seventies.

All good wishes
Stephen

## ENDNOTES

1 https://www.youtube.com/watch?v=u73mtDZXZV8

2 The latter example served to demonstrate that a commonly held view that the observed shape of distributions of sand sediments came about through a mixture of three normal distributions, one for the larger grains that move by creeping along the surface, one for saltating grains and one for grains in suspension, was invalid.

3 Sir Francis Galton (1822-1911), FRS, was a Victorian multi-genius: Geographer, Meteorologist, pioneer in Eugenics, Statistician…

4 Fellow of the Royal Society, Order of the British Empire

5 [Shaw (1945)]

6 Capitaine Massu, later General Massu, army commander-in-chief in Algeria during the upheaval of its independence.

7 Phillippe de Hauteclocque.
Professional soldier. Escaped from France June 1940 and joined de Gaulle under the assumed name of Leclerc which he chose to retain. Later General Leclerc, commander of the French division which liberated Paris. Killed soon after the war in a rather mysterious air crash. An inspiring leader and brilliant organizer.

8 Ronald F. Peel, later Professor of Geography, University of Bristol

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# HYPERBOLIC LAWS

# 7 SOME APPLICATIONS OF HYPERBOLIC DISTRIBUTIONS

While the origin of the hyperbolic distribution lay in the size distributions of windblown sand the distribution has found numerous applications in a wide range of areas. Beyond sedimentology, [Bagnold, BN (1980)], [BN, Jensen, Sørensen (1983)], its uses in the study of droplets in sprays, and on wind shear affecting aircrafts during landing are briefly indicated below. Another, more exotic application has been to the remnant magnetisation of Icelandic lava flows [BN, Blæsild, Jensen, Sørensen (1985)].

*Proc. R. Soc. Lond.* A **417**, 335–352 (1988)
*Printed in Great Britain*

## Erosion, deposition and size distributions of sand

By O. E. Barndorff-Nielsen[1] and C. Christiansen[2]
[1] *Institute of Mathematics, University of Aarhus, Ny Munkegade, DK-8000 Aarhus C, Denmark*
[2] *Institute of Geology, University of Aarhus, Ny Munkegade, DK-8000 Aarhus C, Denmark*

(*Communicated by P. R. Owen, F.R.S. – Received* 13 *July* 1987 – *Revised* 26 *November* 1987)

A mathematical–physical model for erosion and deposition of sand is formulated and related to the logarithmic hyperbolic distributional form of mass–size distributions. The location-scale invariant parameters $\chi$ and $\xi$ of the hyperbolic distribution express, respectively, the skewness and the kurtosis of that distribution, and the triangular domain of variation of these two parameters is referred to as the hyperbolic shape triangle. The erosion–deposition model implies that erosion will tend to move the $(\chi, \xi)$-position of a given sediment to the right-hand part of the shape triangle and that deposition will move the $(\chi, \xi)$-position towards the left part of the triangle, along specified curves. This is confirmed by sediments from a variety of natural environments. An empirically determined curve bisecting the shape triangle is found to separate the samples from predominantly depositional environments as compared with the samples from predominantly erosional environments. The hyperbolic shape triangle is also found to discriminate well between samples of different but closely related origins.

## 7.1 DROPLETS IN SPRAYS

Sometime around 1980 I was invited to give a talk on the hyperbolic distribution at the Danish Technical University. One of the people attending was Professor Franz Durst, LSTM-Erlangen (Chair of Fluid Mechanics) at University of Erlangen-Nürnberg. At first, he expressed great skepticism in regard to the validity of what I presented. However, he soon became convinced and this led to a series of active contacts between his group at University of Erlangen-Nürnberg and the Aarhus ‘Sand Gang’.

Franz Durst initiated a study of the applicability of the hyperbolic law to questions concerning particle and drop size distributions. This resulted in a series of papers by Durst and coauthors. In [Durst and Marcagno (1986)] it is demonstrated that the hyperbolic distribution provides satisfactory descriptions of particle size distributions from a great variety of fields, including glacio-fluval sediments, aerosols, sprays and molecular weights. Previous studies in these fields had separately attempted to fit a number of known distributions – log-normal, Rosin-Rammler-Sperling, etc. with limited success and this had led to the impression that ’each field is governed by its ‘own law of particle physics’.

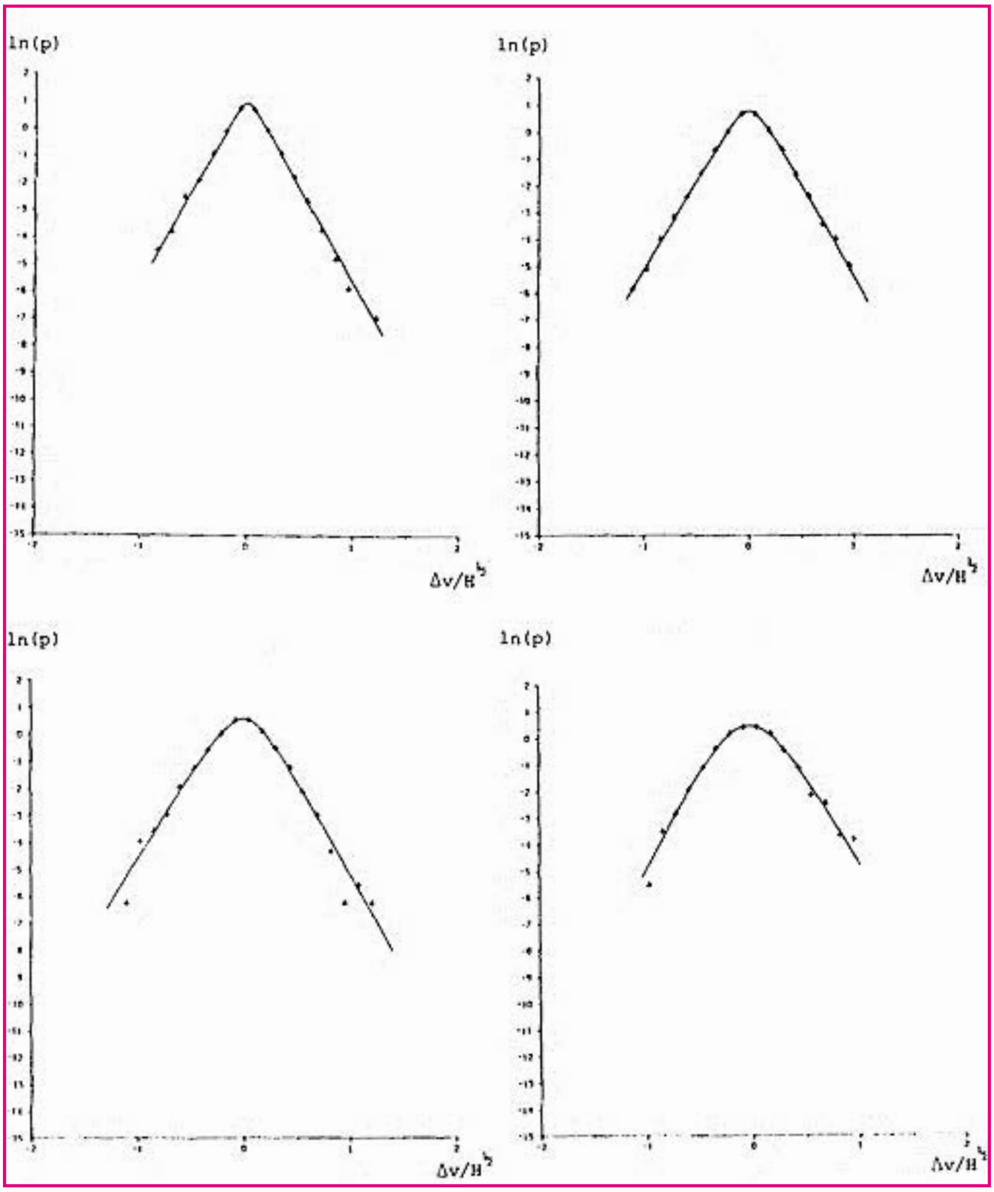
## 7.2 WIND SHEAR AND AIRCRAFTS

Major wind gusts meeting aircraft during landing and take-off constitute a major problem for navigation and may lead to fatal accidents. Numerous studies devoted to this type of problems have been published.

The adduced Figure (p. 81) shows examples of the fit of hyperbolic distribution to recordings of wind shear affecting air crafts during landing, according to various types of landing conditions. The data and an account of their experimental context were published in a paper by Kanji (1985) and the hyperbolic analysis was presented in [BN, Jensen, Sørensen (1989]).

# 8 HYPERBOLIC UNIVERSE

It was recounted above how the hyperbolic distribution arose through mathematical formulation of an idea, due to Ralph Bagnold and related to his studies of wind blown sands. The distribution has found application to a number of other areas, and among its multidimensional versions the three-dimensional type occurs in the theory of special relativity.



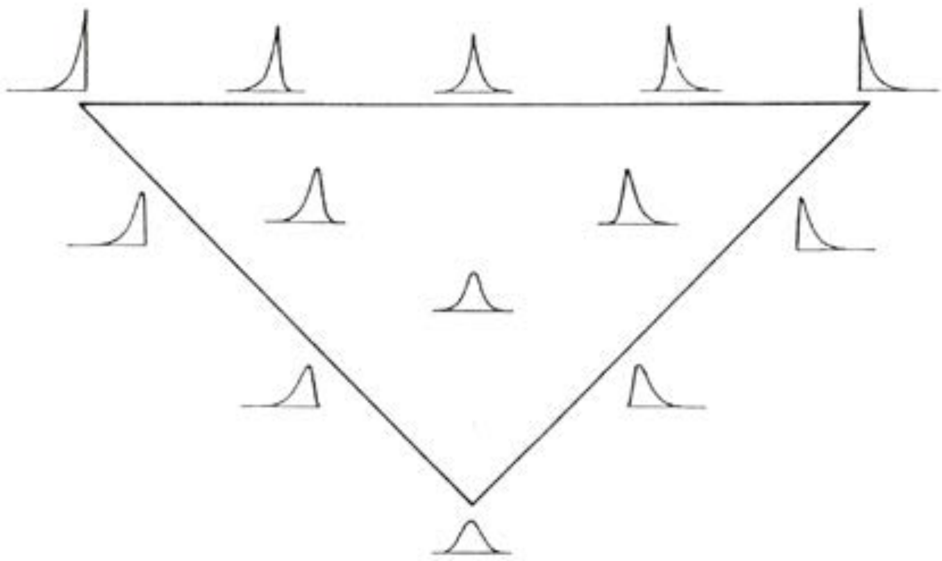
## 8.1 ONE-DIMENSIONAL SETTING

Up to a change in location the probability density of the hyperbolic law is given by

$$p(x) = \frac{\gamma}{2\delta\sqrt{\gamma^2+\beta^2}K_1(\delta\gamma)} \exp\left(-\sqrt{\gamma^2+\beta^2}\sqrt{\delta^2+x^2}+\beta x\right) \qquad (*)$$

where $\delta$, $\gamma$ and $\beta$ are parameters and where $K_1$ is known as a Bessel function.

The character of the hyperbolic distributions can be conveniently illustrated by what is called the hyperbolic shape triangle. The adjacent Figure shows the hyperbolic triangle with a number of instances of the $p(x)$ functions plotted in the Figure, corresponding to various values of the parameters $\gamma$ and $\beta$.[1]



The curves on the borders of the triangle all correspond to probability laws that were known in advance. In particular, the standard Gaussian distribution is represented by the bottom point of the triangle while the distributions on the upper borderline are the skew and symmetric Laplace distributions. Plotting the estimated values of the parameters from a range of experiments provides a depiction of the relationships of the distributions involved, often indicating a dynamic connection.

## 8.2 MULTIVARIATE VERSION

The representation of (∗) as the mixture of the Gaussian distributions defined by endowing the variance of the normal by one of the generalised inverse Gaussian laws leads naturally to a $d$-dimensional hyperbolic distribution, the density of which is, up to a change in location and a linear transformation, given by

$$p(x) = \frac{(\gamma/\delta)^{(d+1)/2}}{(2\pi)^{(d-1)/2} 2\sqrt{\gamma^2+\beta\cdot\beta} K_{(d+1)/2}(\delta\gamma)} \cdot \exp(-\sqrt{\gamma^2+\beta\cdot\beta}\sqrt{\delta^2+x\cdot x}+\beta\cdot x) \qquad (**)$$

where the $K_{(d+1)/2}$ are Bessel functions, $\gamma > 0$ and $\beta = (\beta_1, \dots, \beta_d)$ are parameters, and where $\beta\cdot\beta = \beta_1^2 + \dots + \beta_d^2$, $x \cdot x = x_1^2 + \dots + x_d^2$ and $\beta \cdot x = \beta_1 x_1 + \dots + \beta_d x_d$.

## 8.3 RELATION TO RELATIVISTIC PHYSICS

Remarkably, the three-dimensional hyperbolic distribution, i.e. where $d = 3$, has the character of being the relativistic counterpart to the three-dimensional Gaussian, a fact that I became aware of some time after introducing the multidimensional hyperbolic laws, in [BN (1982)].

It is a classical result, due to the Scottish mathematical physicist James Maxwell (1831–1879)[2], that the distribution of the three-dimensional velocity vector $v$ or, equivalently, the momentum $\bar{v} = mv$, of a single particle in an ideal gas, follows a three-dimensional Gaussian law. The derivation is based on statisticalmechanics according to which the probability density of $\bar{v}$ is given by

$$ae^{-\lambda E(\bar{v}))}$$

where a and $\lambda$ are constants and $E(\bar{v})$ denotes the kinetic energy of the particle, which is given by

$$E(\bar{v}) = \tfrac{1}{2}(\bar{v}\cdot\bar{v})/m.$$

In the early days of relativity theory it was proved by Ferencz Jüttner [Jüttner (1911)], who was a student of Max Planck, that the relativistic analogue of this derivation leads to $\bar{v}$ having probability density

$$p(\bar{v}) = a\exp(-\lambda c\sqrt{m^2c^2+\bar{v}\cdot\bar{v}})$$

for some constant $\lambda$ and where $m$ is the rest mass of the particle and $c$ denotes the speed of light.

This result, which is known as Jüttner's (or Maxwell-Jüttner's) law, is seen to be of the form (∗∗) with $(\delta, \gamma, \beta)$ satisfying

$$\lambda c = \sqrt{\gamma^2 + \beta^2}$$

for some constant $\lambda$ not dependent on $c$; moreover,

$$\delta = mc$$

and

$$\beta = 0.$$

The norming constant a is determined from the condition that the integral of $p(\bar{v})$ has to integrate to 1 in order for $p$ to be a probability density.

However, this was not a full relativistic treatment. Such a treatment requires consideration of the setting where the inertial frame of the observer of the gas moves at a constant speed in relation to the gas.

To include this it is necessary to invoke the Lorentz transformation which relates the coordinates of an event in space-time expressed in each of the two inertial frames. Denoting the two inertial frames respectively by $S$ and $S'$, with $S'$ being the moving frame, the coordinates are, respectively, $(x_1, x_2, x_3, ct)$ and $(x_1', x_2', x_3', ct')$.

Assuming that the frame $S'$ moves along the $x_1$ axis at speed $u$ the Lorentz transformation is

$$\begin{aligned} t' &= \psi\left(t - \frac{u}{c^2}x_1\right) \\ x_1' &= \psi(x_1 - ut) \\ x_2' &= x_2 \\ x_3' &= x_3 \end{aligned}$$

where

$$\psi = \frac{1}{\sqrt{1 - u^2/c^2}}.$$

The character of the special theory of relativity appears clearly from these equations. If the speed u of the moving system is negligible in comparison to that of light then these equations turn into the classical, i.e. there is no difference coordinatewise between the two systems.

Jüttner, who studied the question at the suggestion of Max Planck, assumed initially that the two systems were stationary relative to each other; but in a slightly later paper he considered the general case. However, as discussed in a delightful account by Ingo Müller of the History of Thermodynamics [Müller (2007)], his argument was based on an erroneous opinion of Planck's.

The general case was firmly established much later by ChristianMøller[3] [Møller (1968)] and the formula for $p(\bar{v})$ that he established is again of the hyperbolic type but now with

$$\frac{\lambda c}{\sqrt{1-u^2/c^2}} = \sqrt{\gamma^2 + \beta \cdot \beta}$$

$$\delta = mc$$

and

$$\beta = \frac{\sqrt{\gamma^2 + \beta \cdot \beta}}{\sqrt{1-u^2/c^2}}(u/c, 0, 0).$$

As reported by Müller, Jüttner himself was rather despondent about the possible observability and applicability of his results, as the difference to Maxwell's formula is negligible except under extremely high temperatures. But, in fact, it has turned out that the formula is relevant in connection to the study of White Dwarfs, cf. chapter 10 of [Müler (2007)].

In essence, the derivation of the Jüttner-Møller derivation is based on the key formula of the theory of special relativity which specifies the squared length of the 'spacetime interval' between two events as

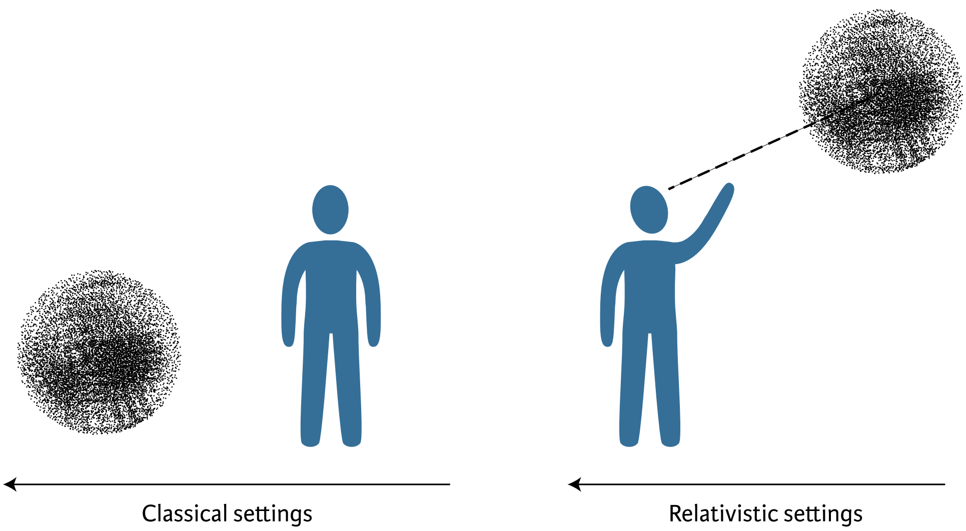
$$\Delta^2 s = c^2 \Delta^2 t - \Delta^2 x_1 - \Delta^2 x_2 - \Delta^2 x_3.$$

This exhibits how calculation of length in space-time must involve not only the squared classical length $\Delta^2 x_1 + \Delta^2 x_2 + \Delta^2 x_3$ but also time. The hyperbolic aspect of space-time becomes clearer by reexpressing the formula as

$$c^2 \Delta^2 t = \Delta^2 s + \Delta^2 x_1 + \Delta^2 x_2 + \Delta^2 x_3$$

and recalling that in elementary mathematics the equation for a hyperbel is

$$y = \sqrt{a^2 + x^2}.$$



For another aspect of the hyperbolic character of the Universe, suppose that a rocket is lifting up from Earth with an acceleration that is constant as measured from a clock aboard the rocket. Denoting this acceleration by $\alpha$, the relation between time $\tau$ shown by the rocket's clock and time $t$ measured at the launching pad is given by

$$\tau = \frac{c}{\alpha} \ln\left( \frac{\alpha t}{c} + \sqrt{\left(\frac{\alpha t}{c}\right)^2 + 1} \right).$$

## ENDNOTES

1 For the purpose of presenting the triangle, the pair of parameters $\gamma$ and $\beta$ were transformed into $\xi = (1 + \gamma^2)^{-1/2}$ and $\chi = \beta(\gamma^2 + \beta^2)^{-1/2}\xi$. Here $\xi$ varies between 0 and 1, expressing the steepness of the flanks of the curves, while $\chi$, which varies between –1 and 1, represents the possible skewness.

2 James Clerk Maxwell, F.R.S., F.R.S.E., is known particularly for formulating the theory of electromagnetic radiation, an achievement referred to as the 'second great unification of physics', the first being that of Isaac Newton.

3 Christian Møller (1904–1980) was a Danish theoretical physicist, known for his contributions to relativity theory, gravitation and quantum mechanics, in particular for his theory of collisions of electrons, the 'Møller-scattering'. He was Professor at the Niels Bohr Institute.

# THE SAND GANG

For a number of years following the mathematical formulation of Bagnold's 'hyperbolic hypothesis' a great deal of my time was spent as participant in a research group that grew up in connection with the establishment of a wind tunnel, built in the mould of the wind tunnel that Bagnold constructed and based at the Institute of Physics and Astronomy. The group consisted of sedimentologists, physicists and mathematicians and was colloquially referred to as 'The Sand Gang'.

# 9 WIND-DRIVEN SAND AND SEDIMENTOLOGY

The group developed contacts and collaborations with a number of the internationally leading researchers in the field of Sedimentology, in particular Ron Greely, Jim Iversen, Paul Robert Owen and Brian Willetts.

Ron Greely (1939-2011) was Regents Professor of Planetary Geology at Arizona State University; together with Jim Iversen (1933-), Professor at Department of Aerospace Engineering, Iowa State University, he undertook extensive research in planetary studies, with focus on wind regimes on Mars, Venus and Titan and their impact on surface topology, particularly as regards dune fields and shapes. Greely was deeply involved in nearly every major space probe mission flown in the solar system in the early stages of planetary exploration. In Sections 44 and 12 I return to the relations to Jim Iversen and to Paul Owen.

Several manifestations of these collaborations took the form of international Workshops. The first was an 'International Workshop on the Physics of Blown Sand' held 1985 in Aarhus and documented in three volumes as Memoirs no. 8 from the Department of Theoretical Statistics, with contributions, among others, by Ralph Bagnold, James C.R Hunt, Jim D. Iversen, Brian B. Willetts, Bruce R. White, Paul B. Owen, Dale Gillett, Haim Tsoar, Kenneth Pye and Monique Manguet, all leading researchers in the topics of the Workshop. Another Workshop was held 1991 in Edinburgh, the Proceedings of which were published in Acta Mechanica, cf. Section 11.

And part of the collaborations took place in the framework of a project 'Modern Mathematical Statistical Methods in Aeolian Processes' funded by the NATO Scientific Affairs Division.

# 10 FIELD STUDIES

The Aarhus 'Sand Gang' consisted of researchers from the Departments of Geology, Mathematics and Physics at Aarhus University and included Christian Christiansen, Kristian Dalsgaard, Jens Tyge Møller, and Keld Rømer Rasmussen (geology), Preben Blæsild, Christian Halgreen, Jens Ledet Jensen and Michael Sørensen (mathematics) and Henry Loft Nielsen (physics). Another group member was Hans Kuhlman (geologist, Copenhagen University).

A number of field studies and experiments were carried out, mostly at the West Coast of Denmark, where miniature forms of some of the type of sand dunes found in the Libyan Desert are common. Samples of sand grains were carefully scraped up from the very uppermost layers of the dunes and brought back to the laboratory for determination of the size distributions.

**Hans Kuhlman sampling from surface layer**

Of course, the scraping technique was rather crude and one of our international colleagues, Haim Tsoar, Professor at the Ben-Gurion University of the Negev and one of the leading scientists in the area of sand drift and sand dunes, suggested as a possible improvement to acquire a really cheap and sticky ladies' hair spray in order to fix the uppermost layer. This in fact works well as the fixed sample can easily be cut loose by a knife and the spray removed in the laboratory.

**Young versions of two members of the Sand Gang: Michael Sørensen and Jens Ledet Jensen**

**At Raabjerg Mile, with a freezing Ralph Bagnold**

Another favoured study area was Raabjerg Mile, a very large sand dune in the middle of Jutland. This dune is slowly wandering from West to East across Jutland and is a Nature Reserve which will be allowed to continue its course although in time it will wipe out some settlements. Bagnold accompanied us at one of the excursions to the Mile and the adjacent photo show a rather cold Ralph Bagnold on the top of the Mile, together with several members of the Sand Gang.

**Excursion to the Danish West Coast. In the middle Ralph Bagnold and to the right, Jens Tyge Møller and the author.**

There, in particular, a study of the variation in size distribution across a barchan type dune was carried out, the results of which are reported in a paper entitled 'The Fascination of Sand' [BN, Blæsild, Jensen, Sørensen (1985)].

Also, at a location on the West Coast of Denmark, sand particles were trapped at various heights to study the influence of the various modes of transport of sand (creep, saltation and suspension) by the wind.

**Billedtekst?**

# 11 WIND TUNNELS

Very early on (around 1980) it was decided to build a wind tunnel at Aarhus University to study the dynamical aspects of sand drift. Funds for this were scarce but a seed grant from the Faculty of Science and good help from the technical staffs at the Departments of Geology and of Physics enabled construction of a suitable tunnel which was and is located in the basement of the Department of Physics. Also helpful was the inspired idea of Jens Tyge Møller to try to obtain a motor for driving the wind tunnel fan from the Danish Air Force. He enquired with the Air Force whether they might have a used airplane jet engine that could be applied for this purpose. It turned out that they had a stack of Rolls Royce engines that had served their time. They were pleased that one of those could be of use and delivered it for free to the Physics Institute.

**Early stage of the Aarhus wind tunnel**

One of the focal areas of investigation in the period was the studying of the physics of blown sand in relation to surface conditions on Mars, this being triggered among other things by the photos of the Martian surface, send back to Earth by the Viking 1 capsule after its landing on Mars, which showed a striking similarity to sand structures on Earth. The adjacent photo, given to me by Bagnold, includes one of the photos from Mars mentioned.

The adjacent picture shows one such photo taken by the Viking Lander and given to me by Ralph Bagnold who in turn had been given it by Carl Sagan, scientist and author of the famous book 'Cosmos'. The picture also shows part of a photo of Kolmogorov given to me by Albert Shiryaev. Together with Carl Sagan, Bagnold participated as consultant in NASA's work on space craft missions to Mars, and he and Sagan wrote a joint paper on ''Fluid transport on Earth and aeolian transport on Mars'' (Icarus 26 (1975), 209-218).

A rather extreme aspect of this interest was an attempt to obtain knowledge on sand particle transport under near zero-gravity conditions.

This involved a young American geologist Bruce White flying in a NASA plane used to train astronauts, but this time the purpose was to watch what happened in a small wind tunnel in the very short period where zero-gravity was approximately obtained by having the airplane fly at great speed to a high altitude and then turn quickly around to a steep dive, the observations of the wind tunnel being undertaking at the few seconds at the top of the flight. This implied Bruce White being heavily drugged to keeping him from nausea or loosing conscience.

Other types of investigations of the properties of sand under low gravity conditions have been undertaken by conducting experiments on granular material in space, cf. the following quote from the book (page 59) by Michael Welland, entitled *Sand. The never ending story.*[1]

This book is a delightful read, setting out the multitude of roles of sand in life on Earth in regard to physics, geology, engineering, biology, archeology, medicine, literature and the arts.

"Because liquefaction is a triumph of gravity over friction and surface tension, considerable effort has been put into eliminating the effects of gravity in order to concentrate on what's going on between the grains and the water.

How? By conducting experiments on granular materials in space. By testing the behavior, wet or dry, in the microgravity aboard a space shuttle, not only can we prepare for building on the Moon or Mars, but we can better understand the quirks of granular materials here on Earth, including the potential for better construction engineering in earth quake areas. Such experiments have been an ongoing feature of shuttle missions."[2]

In 2000 a film on the sand project was made on the initiative of the then President of the Bernoulli Society, Victor Perez Abreu. The title of the film is *Stochastics in Science. The Aarhus Sand Project.* It is available under YouTube on the internet.

As indicated earlier, around the time when the Aarhus wind tunnel was built there was internationally a major revival of interest in particle transport in wind and water. Much of the work carried out in the period 1980 to 1990 was reported in a two volume Supplement to Acta Mechanica on 'Aeolian Grain Transport', the first volume, on 'Mechanics', dedicated to the memory of Ralph Alger Bagnold, the second on 'The Erosional Environment' to the memory of Paul Robert Owen, one of the leading scientists and pioneers in the study of sand and the erosion of desert areas (and a range of other fields). For more on Paul Owen see Section 12.

A glimpse of the state of affairs at that time is given by following quote from the Preface to the above mentioned two volumes:

A successful workshop on 'The Physics of Blown Sand', held in Aarhus in 1985, took a decisive step in forming a research community with interests spanning geomorphology and grain/wind process mechanics. The encompass and cohesion of that community was reinforced by the Binghampton Symposium on Aeolian Geomorphology held in 1986 and has been fruitful in the development of a number of international collaborations. The objectives of the present workshop, which was supported by a grant from the NATO Scientific Affairs Division, were to take stock of the progress in the five years to 1990 and to extend the scope of the community to include soil deterioration (and dust release) and those beach processes which link with aeolian activity on the coast.

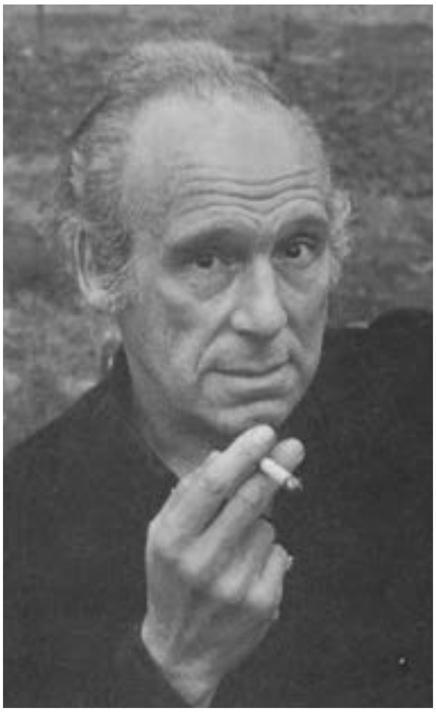
The meeting satisfied these objectives and proved most stimulating. Presentations described both completed studies and work in progress, drawn from many countries and several disciplines. They provided stimulating discussions, the fruit of which will become evident during the next few years. The standard of presentations was generally high and most of them are represented in this collection of papers. Authors have had opportunity to incorporate the outcome of discussion where this was appropriate. So much good material was produced that the papers extend to two volumes.

The 1990 Workshop was preceded by a Symposium on 'Coastal Sand Dunes' organized by the Royal Society of Edinburgh and held in May 1989. The papers presented at the Symposium were published in the Proceedings, Series B, of the Society.

This area of research is still very much of forefront interest. For instance, a three months research program was held in 2013 at the Kavli Institute for Theoretical Physics in Santa Barbara, announced as

> **Fluid-Mediated Particle Transport in Geophysical Flows**
> **Coordinators: James Jenkins, Eckart Meiburg, Alexandre Valance**
>
> Recent progress in the study of granular materials, improvements in field instrumentation, and the increased capabilities of large-scale computer simulations have led to a better quantitative understanding of particle transport by a turbulent fluid, the interaction between transported grains and the bed, the development of surface features on the bed, and their subsequent motion and interaction. Further progress will require advances in our understanding of particle-fluid interactions and the modification of particle-particle interactions in the presence of a fluid. This program is intended to accelerate these advances by bringing together physicists and experts on geological processes and morphology to address the questions that remain.

# 12 PAUL ROBERT OWEN



It was through David Cox that I came in contact with Paul Owen who, like David Cox, was Professor at Imperial College. This led to a strong involvement by Paul Owen in the activities of the Sand Gang.

Paul Owen, engineer and applied mathematician, had a delightful, multifaceted and inspiring personality, as vividly described in the Royal Society Obituary written by A.D. Young and Sir James Lighthill. (Biographical Memoirs of Fellows of the Royal Society, Vol. 38, pp. 269-285). The Biographical Memoir explains how, in the area of particle transport in fluids, Paul Owen came to be '*recognised as a natural successor of R.A. Bagnold in his profound knowledge of how the atmospheric boundary layer interacts with sand particles when wind blows over the desert.*' It also refers to two unpublished works written by him as a help in improving and developing the Aarhus Wind Tunnel.

The Royal Society Memoir opens with the three paragraphs shown here.

PAUL ROBERT OWEN

24 January 1920—11 November 1990

Elected F.R.S. 1971

BY A.D. YOUNG, F.R.S., AND SIR JAMES LIGHTHILL, F.R.S.

1. INTRODUCTION

PAUL ROBERT OWEN was a scientist and engineer of great distinction and versatility. His work ranged widely over the fields of aerodynamics, flight mechanics and fluid dynamics and their applications to aeronautics, environmental and safety problems, building design, the movement of solid particles in an air flow, the ventilation of mines and wind erosion of desert areas. He also had a passionate interest in the arts, particularly literature and music. He wrote and spoke with clarity, style and wit, so that his papers were a pleasure to read, his company was always eagerly sought and letters from him were most welcome for their acute insight coupled with a delightful ironic, often self-deprecating humour. He was the quintessential humanist with wide interests and sympathies and an aversion to dogma; in his later years he became particularly concerned with the problems of the Third World and how technology can best be applied to help solve those problems.

He enjoyed and enlivened good company and conversation but he had an inner core of reserve which his friends were aware of and respected. He was known to some as Robert and to others as Paul; one can only guess at what distinguished the two groups, but it seems he decided the appropriate name to be used early on in the development of a friendship. He had a deep musical speaking voice and a pleasant bass-baritone singing voice, and in his youth he was a talented amateur actor. Woodwork was a favourite form of relaxation and in his later years he became a skilled restorer of old furniture.

It was characteristic of him that he left no written record of his life, he probably thought it was of no great interest, and available details of his early life are very sketchy. He was born of Jewish parents in north London, his father was a business man of comfortable means, and he had a younger brother who became an accountant. There is nothing obvious in his background to explain his outstanding scientific ability except to note that he was very attached to his mother, who died relatively young, and that she gave him much encouragement to develop his academic potential. Margaret, his wife, recalled that he sometimes spoke warmly of an uncle with a red beard who taught him to fly kites!

**Source: Paul Robert Owen. 24 January 1920-11 November 1990**
**Authors: A. D. Young and James Lighthill.**
**In: Biographical Memoirs of Fellows of the Royal Society, Vol. 38 (Nov. 1992), pp. 268-285**
**Published by: Royal Society**

Having read in this that he left no written record of his life I decided to try and contact his family. By the help of David Cox and a former colleague of his, Professor Peter Bearman, I obtained the email address of his wife Margaret. The ensuing correspondence with her and their children Dan and Natasha is reproduced below, after two examples from my extensive correspondence with Paul Owen.

Boulder, Colorado, USA.
Monday, 7 April 1986.

Dear Ole,

Dale Gillette passed your message to me but, by the time I could secure a telephone from which one could make overseas calls, I had lost the piece of paper with your number on it. At any rate, I understand that the postponed date of my arrival is not as distressing to you as I had feared, and I now look forward very much to seeing you around 20 May. On my return to England about 10 days from now, I shall book my flights and write again, giving you the details.

By coincidence, the day you phoned I was supposed to have had the pleasure of a visit from Kathleen Gerety, but a sudden, happily short, change in the weather, which brought an overnight snowfall of 20 cm or more, temporarily closed the airport at Denver and she was unable to get here from the East Coast.

Incidentally, have you given any thought to the problem I mentioned when we met in London: that is, given a sample of sand from near the surface of a deposit, from which the distribution of mass with respect to particle diameter is determined — say, by sieving and weighing — what is the expected shape of the surface? The answer, I believe, is crucial to the problem of saltation and may help to reconcile the many differing measurements of mass flux.

I hope that my London house removal does not leave me in too neurotic a state and spoil the pleasure I am taking in contemplating my visit to Aarhus.

With warmest wishes and many thanks for your phone call

Yours sincerely,
Paul

Flat 1,
Stanley Lodge,
25, Stanley Crescent,
London W11 2NA,
England.

16 May 1990.

Dear Ole,

Your telegram arrived Monday, 14th. I now hasten to thank you very warmly for your kindness in sending it, and for the good wishes it expressed: and to tell you that it gave me immeasurable pleasure to read your words and thus to learn that I had not been forgotten by my friends in Denmark. More than that: it helped to expunge the awful feeling of guilt that possessed me ever since I was forced to break my promise to attend the Workshop. Until two weeks ago, when I phoned Brian Willetts in a state of despair, I had been determined to make the journey and, in disregard of the rules, to present a paper which I hoped could be left with you. But, alas, it was not to be. When I got down to the preparation of Viewgraphs, I realised that the state of my hands made holding a pen, let alone drawing, impossible.

Allow me to go back a little in time. The past fifteen months or so had been a nightmare, which I have no wish either to recall or to expose you to the burden of its clinical detail. All I need say is that I am now, technically, fully-cured. Indeed, my doctors proclaimed that news at the end of last year, thereby encouraging the belief that I could confidently commit myself to participating in the Workshop. What they did not disclose, but which I had, perforce, begun to suspect, was that a side-effect of the treatment was a partial loss of feeling in my hands and feet. My physicians now assert that my recovery will be complete "in time": yet, with customary medical discretion, no scale is attached to that prediction. I can only hope that it is not geological! However, I must credit them with a certain robustness in applauding my agnostic refusal to play the part of an invalid; for, a few weeks ago, I abandoned my walking stick (except on public transport, where its exaggerated use secures me a seat) and made a resolute effort to type, which I do slowly, with great discomfort and more than usual inaccuracy.

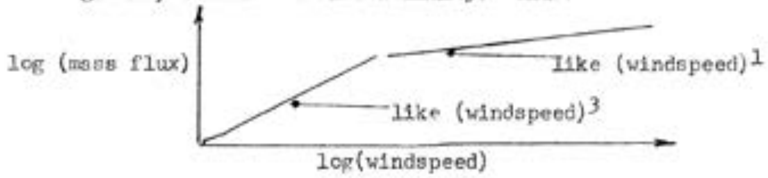
Incidentally, I discoveredthat Psychiatry plays a more dominant part than hitherto in British medical training; due, no doubt, to it being cheaper than clinical or laboratory research. Hence, the approval of my defiant attitude to infirmity. I can also relate that, towards the end of a prolonged and traumatic confinement to hospital, my interest in the World outside suddenly returned, and I began again to read newspapers. It was a period coincident with the overt initial signs of unrest in eastern Europe. Feeling, like Rip van Winkle, that I had been asleep for twenty years, I unguardedly voiced the opinion that "Gorbachev evidently meant what he said about reducing Soviet armament", whereupon, the immediate consequence was to provoke insistent attention from the psychiatrists who implied the suspicion that the virus with which I was infected* had reached my brain! Tiring of a somewhat malicious game I played with them, I put an abrupt end to it by declaring that I had been a friend for twenty five years of the late Will Sargant, a prominent figure on the psychiatric stage in Britain, and had engaged in innumerable contentious arguments with him about the subject.

The object of this garrulous rambling is simply to provide a background to

---

* The advice you were given four or five years ago, as I recall it, by your biologist colleagues, not to visit a certain Country, turned out to have been well-founded. It took nearly two months of my doctors' time, countless X-ray scans, and a couple of operations to identify the cause of the disease, contracted overseas.

my predicament: a frustration at being unable formally to pursue a lot of ideas and enthusiasms, owing to a lack of manual dexterity. I had never fully appreciated the gift of tactile sensation, and the casual reliance one places upon it, until it was denied me – temporarily, I trust. But as soon as I feel confident again in the use of a pen, I shall be importunate in seeking another invitation to Aarhus, so that I may describe to a responsive audience some of the work I have done on the problem of what Dale Gillette calls 'Non-erodible Elements' In addition, I should like to tell you about my explanation for an intriguing feature that may be deduced from Paul Stockton's measurements: the subject I intended to talk about at the Workshop. He, at any rate, appeared to be convinced by it when he visited me in London last year. What it claims to elucidate is the inference that, when plotted against windspeed, the slope of the curve of mass flux, as recorded by a sensor at a fixed distance from the ground, exhibits a discontinuity. Thus:




THE FIRST STRAIGHT LINES I HAVE BEEN ABLE TO DRAW FOR OVER A YEAR!!!

This peculiarity, it is my belief, vindicates a long-standing criticism I have offered of experimental data that fail to distinguish between different modes of sand transport.

Unfortunately, at this stage, it is not possible to estimate when I shall be fit to travel again: exasperatingly, even to the land described by Dante as "il bel paese lá dove soná il si". The sole residual obstacle lies in the difficulty experienced in walking, standing for long periods characteristic of the luxury afforded by airports, and handling the tiresome bits of paper essential to modern transport: tickets, passports and so on, which I am inclined inadvertently to drop, hence exposing myself to the unmitigated insolence of Immigration officials. Onthe other hand, you may have plans to visit London, in which case, it would give me delight to meet you. Since I rarely visit Imperial College, for reasons I shall divulge wh en we see each other, we could talk in my apartment, not far from the College. One of the (heretical?) ideas I should like to put to you is the more vigorous application of your wind tunnel to practical problems. In suggesting this, I do not mean to imply an echo of the current hunting cry of "die Technik über alles"; on the contrary, it stems from the belief that, by relaxing the cosy assumption of a wind that blows steadily and uniformly, one might thereby reveal phenomena, perhaps vaguely suspected but hitherto unexplored in detail. In effect, it amounts to a plea for a severance from the hypothesis of a 'flat Earth'; to be replaced by an acknowledgement of the fact that most natural phenomena are three-dimensional and, in the case of airflow, are strongly influenced by embedded vertical structures. I could go on to cite many examples, but to do so in a letter would place an agonising strain on my fingers. I shall therefore restrict myself to reminding you of the suggestion I made during a previous visit to Aarhus, to investigate the behaviour of of saltation in an accelerating or decelerating flow: to which one could add the pattern of saltation generated by the flow over a wavy wall. The remainder can best be discussed over a bottle of claret.

Perhaps, in straying so widely from my original intention in writing to you, I am, after all, tacitly conceding a substance to the psychiatrists' fantasies. I shall, therefore, return forthwith to that intention and express my deep gratitude for your telegram , for your good wishes and those of all my friends at the Workshop. Please convey my thanks to them, as well as my deep regret at having been compelled to absent myself from what I am confident has turned out to be a stimulating and successful meeting.

In particular, may I beg you to give warm wishes to your Aarhus colleagues,

-3-

together with Dale Gillette and Paul Stockton and others from the US, to Mohamed Hassan and Haim Tsoar: and to embrace ,on my behalf, Monique Mainguet. Tell her that Daniel Bernoulli (or, Bernouilli: see 'Encyclopaedia Britannica') of $\frac{1}{2}\rho U^2$ fame, along with Father Jacques, Uncle Jean and Brother Nicolas, were Swiss. However, not wishing to protract our correspondence on the subject, I have made no attempt to trace the origin of the prolific Bernoulli family, but suspect that, living in the 18th Century, they may well have had Niçois connexions (like Garibaldi) when that town was ruled by Sardinia, before 1860. To this day, one encounters many French surnames that sound Italian, but that may be due to genuine migration, before Chicago and New York achieved a competitive popularity.

Is there any possibility of my acquiring a copy of the papers presented at the Workshop?

With kindest regards; and, once again, thanks,

Very sincerely,

Paul

## 12.1 CORRESPONDENCE WITH MARGARET, DAN AND NATASHA OWEN

**On 29 Jan 2014, at 17:48, Ole E. Barndorff-Nielsen wrote:**

**Dear Margaret Owen**
Please apologise me for contacting you out of the blue, as it were. The reason is that many years ago I had a close contact and collaboration, most warmly remembered, with your late husband, and I am now writing some memorabilia from my life in science and in particular in relation to the study of the physics of sand. The link below leads to a manuscript 'Sand Traces' where, among other things, at the end I recall the participation of your husband in the Aarhus Sand Project.

I am sending this in the hope that you will find it agreeable, and if you have any comments or suggestions to this I should be very grateful if you would let me know.

I have obtained your email address through my friend and colleague Sir David R. Cox, who was formerly Professor at Imperial College and who instigated my contact to your husband.

With best regards
Ole E. Barndorff-Nielsen

**From: Margie O**
**To: Ole E. Barndorff-Nielsen**
**Cc: Dan Owen, Charlotte Owen, Natasha Owen**

Oh oh oh how absolutely lovely to get such a message recalling my very beloved and still much missed (over 23 years) husband P R Owen. How marvelous to hear from you.

I am thanking you even before I open, with anticipation, your attachment.
I remember well the Aarhus Sand Project.
So nice to be contacted thus. You sound as if you are Danish? I might contact you again if you do not mind, once I have read the text!

With best wishes Margaret Owen

**On 1/30/14 1:13 AM, Margaret Owen wrote:**

Two replies keep disappearing. Text hopeless on I pad. His letters so amazing, his dear handwriting, have made me weep. I will explain when more another time

Dedication in my book world of widows is "to dearest Robert without whose death this book would never have been written"

Www. Widowsforpeace.org. Am just back from Syria. Rojava. And Iraq

He would be 93. Are you same? Would love to talk to you. Wrote you long reply and it vanished

He must have regarded you as a close friend to write so feelingly and honestly about his failing health. Of course I have cried. Thank you

Margaret Owen

**From: Dan Owen**
**Date: Wednesday, February 5, 2014 2:18 PM**
**To: Ole Eiler Barndorff-Nielsen**
**Subject: Fw: Sand Traces**

Dear Professor,

Just a quick word to thank you for sending on the Sand Traces manuscript. It has been illuminating and deeply touching to read through your correspondence with my father, especially the letters he wrote just a few months before he died.

Were you connected to the work he was doing with the University of Khartoum? A few years ago, I ended up working with a Sudanese colleague on a W. Africa project who turned out to be the niece of the university rector or chancellor who fondly remembered the desert research work from the '80s....

best wishes,
Dan Owen

**On 3/3/14 2:31 PM, Natasha Owen wrote:**

Dear Ole

I am one of Paul Robert Owen's daughters and my mother forwarded your email several weeks ago. I think we've all been incredibly moved by the correspondence between you and our father, charting not only his part in the story of sand but also his state of mind, private preoccupations, and physical decline in what were to be his final years. He must have valued your friendship deeply to have shared to that degree alongside your professional collaboration.

After reading and re-reading the extract with his letters, I went back and read your pages from the beginning. The story is beautifully put together, both your own writing and Bagnold's, and the story of sand is fascinating. I always remember my father explaining the origin of the word 'saltation', coming from the Latin 'to dance', suggesting it also had this playful quality... But I wasn't aware until reading your memoirs of all the other 'living' qualities of sand – the architecture, singing, movement, reproducing. That must be part of the fascination for scientists who choose to study it, I suppose!

I didn't inherit my father's scientific brain so I have to admit the physics is beyond me, and that's why it's taken this long to digest. Perhaps I should try harder! I'd read a little of Bagnold's writing because my partner is a desert traveller and enthusiast, and was close to the British explorer Wilfred Thesiger in Thesiger's later life.

Apologies for the slow reply and thank you for this gift. I wish you well wherever you are publishing it. Is it intended as a book?  I also hope this finds you and your family well. Perhaps we'll have the opportunity to meet one day. Warm wishes,

Natasha Owen

**On 3/5/14 9:56 AM, Ole E. Barndorff-Nielsen wrote:**

Dear Natasha

Many thanks for your kind words.

Your father was wonderful human being in all respects, and I felt a deep rapport to him from the very beginning of our acquaintance.

The responses from your mother, brother and yourself are heartwarming, both in the direct sense but also since in writing reminiscences down one wonders whether this will be of any interest to others. I am encouraged to go on and am thinking of putting together a booklet describing my life in the developments of the field of stochastics in science, a field that offers an incredible variety of interesting and stimulating aspects concerning phenomena of randomness - how to discover them as important features and how then to formulate them in mathematical terms.

If you wish to pursue the topic of sand I can strongly recommend having a look at a new book by Michael Welland that I refer to briefly in my manuscript. It is a delightful, nontechnical read that describes the fascination and importance of sand studies and how the properties of sand have a role to play in an astonishing number of walks of life.

With warm regards
Ole

## ENDNOTES

1 Published 2009 by University of California Press

2 http://marslab.au.dk/windtunnel-facilities/wind-tunnel/

## REFERENCES

Barndorff-Nielsen, O.E., Jensen, J. L. and Sorensen, M. (1985): *The fascination of sand.* In A.C. Atkinson and S.E. Feinberg (Eds): *A Celebration of Statistics* The ISI Centenary Volume. New York: Springer. Pp. 57-87.

# MAPHYSTO AND THIELE

To emphasise that the Institute had broadened its original scope as one of pure mathematics its name was changed to Institute of Mathematical Sciences. At the same time a grant was given to set up a small Mathematical Centre for Mathematics at Aarhus University (MCAA) of which I served as Scientific Director 1n the period 1993-1995.

This in particular created a platform for an application to the Danish National Science Foundation for a research centre, to be called MaPhySto, the main aim of which was to develop research in the Mathematics of Stochastics and Physics. The application was granted and MaPhySto was inaugurated on 16 September 1998.

# 13 MAPHYSTO

**Mephistofeles and Faust, Drawing by Eugène Delacroix**

MaPhySto
Centre for Mathematical Physics and Stochastics
Department of Mathematical Sciences,
University of Aarhus

The group of Danish researchers that participated in MaPhySto as principal investigators came not only from the University of Aarhus but also from **University of Copenhagen**, **Odense University** and **Aalborg University**. MaPhySto organized workshops, conferences, concentrated advanced courses, short and long term visits and had postdoc positions, at all four Universities.

The main fields of activity of MaPhySto were Mathematical Physics (e.g. quantum mechanics, statistical mechanics, quantum field theory) and Stochastics (e.g. stochastic analysis, interactive particle systems, stochastic matrices, free probability), with some particular emphasis on the interplay between these two fields. Aspects of Stochastic Computation, Inverse Problems and Analytic Number Theory were also part of the ambit of the Centre.

With some of my main international collaborators and friends. From left to right: Michael Sørensen (University of Copenhagen); Victor Perez Abreu (CIMAT, Mexico); Claudia Klüppelberg (Technical University Munich); Jean Bertoin (Universität Zürich); Jean Jacod (Université VI, Paris); Philip E. Protter (Columbia University, New York).

**Dean Karl Pedersen's welcome at MaPhySto's inauguration. Sitting from right to left: Peder Olesen Larsen, director of the Danish National Research Foundation, Henrik Tvarnø, Rector Southern Danish University, Henning Lehmann, Rector Aarhus Universitet and Ole E.Barndorff-Nielsen.**

The MaPhySto grant was for five years, with the possibility of a subsequent five years. At the end of the original period I decided to step down as Director in order to concentrate on my own research. The Centre was continued as a Network but this was terminated two years later.

It was then decided to establish a centre at the Institute of Mathematical Sciences to further foster research in Stochastics, in areas of particular interest to the probabilists and mathematical statisticians at the Institute and in the spirit of MaPhySto and drawing on the collaborative links to researchers elsewhere around the World established under MaPhySto. The name chosen for this was the T. N. Thiele Centre for Applied Mathematics in the Sciences, and its activities began in 2014.

For me personally, an enormous benefit of the existence of these Centres was that it led to close collaborations and friendships with a large number of preeminent researchers in the field of Stochastics (in the sense of the theories of Statistics and Probability and their applications).

Centre staff and most of the Principal Investigators in MaPhySto. From left to right (back row): Oddbjørg Wethelund (centre secretary), Jan Philip Solovej, Eva B. Vedel Jensen, Arne Jensen, Bergfinnur Durhuus, Steffen L. Lauritzen, Michael Sørensen, Ole E. Barndorff-Nielsen, Martin Jacobsen, Erik Skibsted and Erik Balslev. Front row: Jesper Møller, Søren Have Hansen (project manager), Jens Ledet Jensen, Goran Peskir and Uffe Haagerup. Absent are Søren Asmussen, Jan Ambjørn, Jørgen Ellegaard Andersen, Svend Erik Graversen and Jørgen Hoffmann-Jørgensen.

Close friends and colleagues: The statistics-probability clan of the Department of Mathematics. From left to right: Preben Blæsild, Jens Ledet Jensen, Eva Bjørn Jensen, Jan Pedersen, Jørgen Granfeldt, Jørgen Hoffmann Jørgensen, Anders Holst Andersen, Svend Erik Graversen. At a Workshop held in connection with 65 birthday.

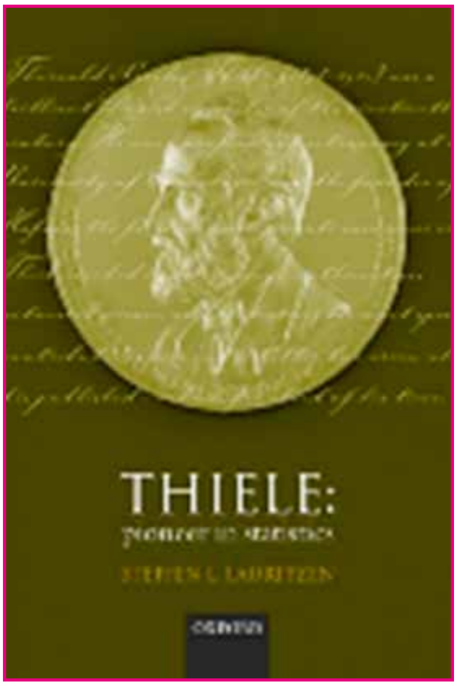
## 13.1 ERCOM

Around the time when MaPhySto was established there grew up in Europe a number of similar mathematical research centres, in addition to the, relatively few, 'classical' Centres such as the Isaac Newton Institute for the Mathematical Sciences, the Institut Henri Poincare, the Instituto Nazionale Alta Mathematica Francesco Severi and the Max Planck Institute for Mathematics. The then President of the European Mathematical Society (EMS) Jean-Pierre Bourguinon therefore took the initiative to set up a committee under EMS with the name of ERCOM (European Research Centres on Mathematics) and he asked me to be the first Chair of this, in which capacity I served 1997-2002.

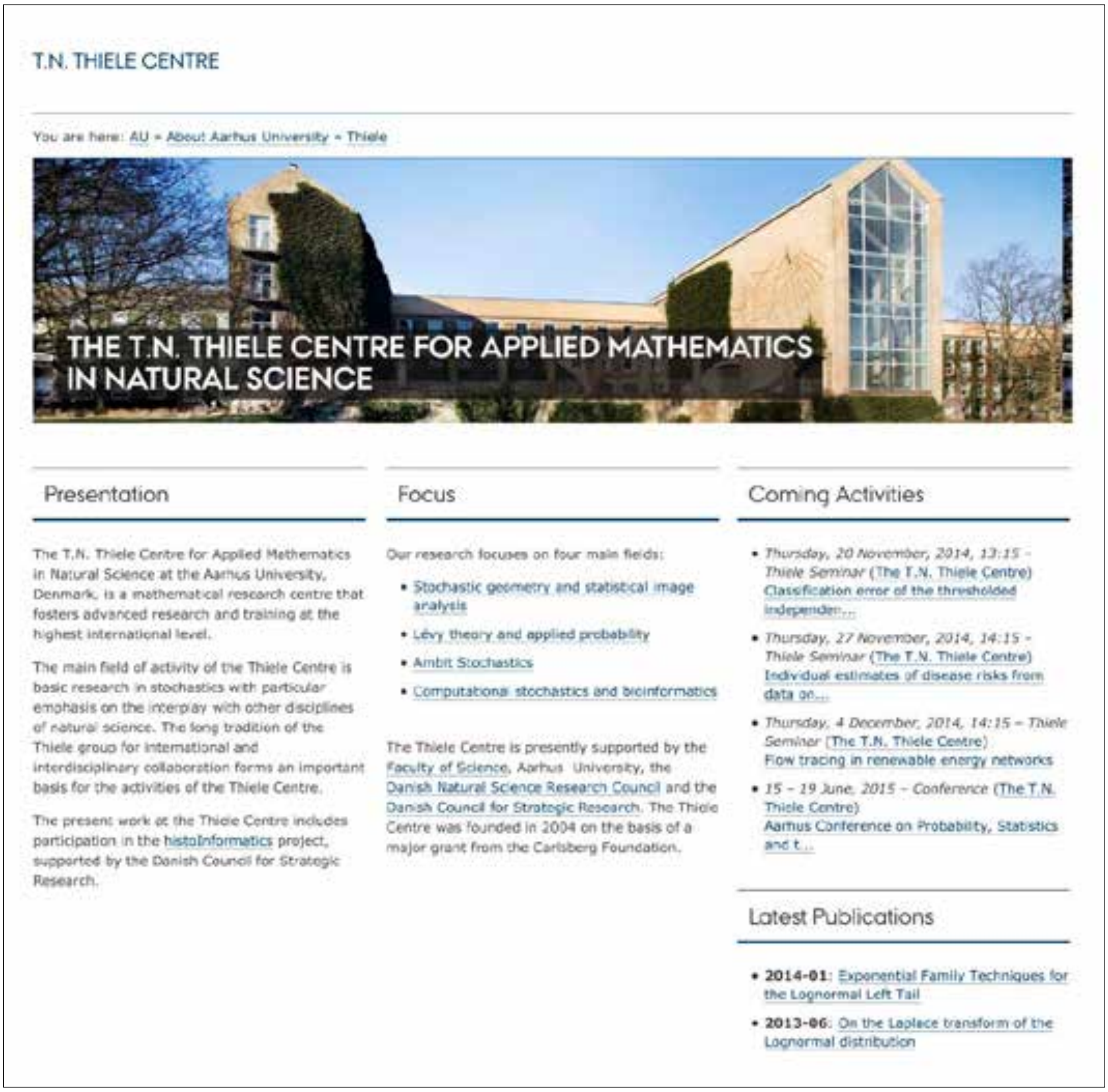
Jean-Pierre Bourguinon

# 14 THIELE CENTRE

The following material is an extract from 2014 from the home page of the Thiele Centre. Besides indicating the mission of the Centre and its main foci of research at the time it also presents a brief biography of Thorvald Nicolai Thiele, with a reference to Steffen L. Lauritzen's book on Thiele's life.




A fascinating account in book form of Thiele's life and work has been written by Steffen Lilholt Lauritzen.



T.N. THIELE CENTRE

You are here: AU » About Aarhus University » Thiele

THE T.N. THIELE CENTRE FOR APPLIED MATHEMATICS IN NATURAL SCIENCE

## Presentation

The T.N. Thiele Centre for Applied Mathematics in Natural Science at the Aarhus University, Denmark, is a mathematical research centre that fosters advanced research and training at the highest international level.

The main field of activity of the Thiele Centre is basic research in stochastics with particular emphasis on the interplay with other disciplines of natural science. The long tradition of the Thiele group for international and interdisciplinary collaboration forms an important basis for the activities of the Thiele Centre.

The present work at the Thiele Centre includes participation in the histoInformatics project, supported by the Danish Council for Strategic Research.

## Focus

Our research focuses on four main fields:

- Stochastic geometry and statistical image analysis
- Lévy theory and applied probability
- Ambit Stochastics
- Computational stochastics and bioinformatics

The Thiele Centre is presently supported by the Faculty of Science, Aarhus University, the Danish Natural Science Research Council and the Danish Council for Strategic Research. The Thiele Centre was founded in 2004 on the basis of a major grant from the Carlsberg Foundation.

## Coming Activities

- *Thursday, 20 November, 2014, 13:15 - Thiele Seminar* (The T.N. Thiele Centre) Classification error of the thresholded independen...
- *Thursday, 27 November, 2014, 14:15 - Thiele Seminar* (The T.N. Thiele Centre) Individual estimates of disease risks from data on...
- *Thursday, 4 December, 2014, 14:15 – Thiele Seminar* (The T.N. Thiele Centre) Flow tracing in renewable energy networks
- *15 – 19 June, 2015 – Conference* (The T.N. Thiele Centre) Aarhus Conference on Probability, Statistics and t...

## Latest Publications

- **2014-01**: Exponential Family Techniques for the Lognormal Left Tail
- **2013-06**: On the Laplace transform of the Lognormal distribution

# PROJECT GROUPS

- Stochastic geometry and statistical image analysis
- Lévy theory and applied probability
- Ambit Stochastics
- Computational stochastics and bioinformatics

## Stochastic geometry and statistical image analysis

Principal investigators

Eva B. Vedel Jensen, Ute Hahn and Markus Kiderlen

Project members

Jérémy Auneau, Linda V. Hansen, Elisabeth Pedersen

Focus Points

Stochastic geometry, digital stereology, Lévy driven Cox processes, modelling of new image modalitites in neuroscience

## Lévy theory and applied probability

Principal investigators

Søren Asmussen, Steen Thorbjørnsen

Project members

Lars N. Andersen, Ole E. Barndorf-Nielsen,  Anders Rønn-Nielsen

Focus Points

General Upsilon transformations and subordination of free Lévy processes, Lévy processes with two reflecting barriers, statistical inference for Lévy based models, heavy tails computer reliability.

## Stochastic processes and spatio-temporal modelling

Principal investigators

Ole E. Barndorff-Nielsen, Jürgen Schmiegel

Project members

Søren Asmussen, Andreas Basse, Eva B. Vedel Jensen, Steen Thorbjørnsen

Focus Points

Ambit processes, multipower variation, turbulence

## Computational stochastics and bioinformatics


Principal investigators

Jens Ledet Jensen, Asger Hobolth

Focus Points

Analysis of high-dimensional data, inference in evolutionary models

## THORVALD NICOLAI THIELE

Thorvald Nicolai Thiele was a brilliant researcher and worked as an actuary, astronomer, mathematician, and statistician.

Thiele was born in Copenhagen on Christmas Eve, 24 December 1838, and grew up in a prominent family and a culturally and intellectually stimulating environment. His father, Just Mathias Thiele (1795-1874), was private librarian to King Christian VIII of Denmark and director of the Royal College of Prints.

Thiele obtained his master's degree in astronomy from the University of Copenhagen in 1860 and his doctoral degree (Sc.D.) in 1866, based on a thesis about the orbits of double stars. In 1875 he became professor of astronomy and director of the Astronomical Observatory at the University of Copenhagen, positions he kept until his retirement in 1907. He was the founder and Mathematical Director of the Danish insurance company Hafnia from 1872 until his death in Copenhagen on 26 September 1910.

An important facet of Thiele's personality is his unique abilities as initiator. The Danish Mathematical Society was founded in 1873 on his initiative in cooperation with H.G. Zeuthen and J.P.C. Petersen. The Danish Actuarial Society was also founded in 1901 on his initiative.

Et Møde i Videnskabernes Selskab by P.S. Krøyer

The painting is the work of the well-known Danish painter P.S. Krøyer. T.N. Thiele is sitting at the fore-front. The painting is courtesy of the Royal Danish Academy of Sciences and Letters.

A short biography with emphasis on Thiele's scientific achievements can be found at "Thorvald Nicolai Thiele" in the Mac Tutor History of Mathematics.

An excellent account on Thiele's life and work is Steffen L. Lauritzen's book "Thiele: pioneer in statistics", Oxford, 2002.

A. Hald
Furesøvej 87 A
2830 Virum

15. november 2000

Professor Ole E. Barndorff-Nielsen
Afdeling for teoretisk statistik
Matematisk institut
Ny Munkegade
8000 Aarhus C

Kære Ole.

Jeg ser i det sidste nummer af "Meddelelser", at der skal afholdes et symposium til ære for dig den 16.-18 november. Jeg ville gerne have deltaget i hyldesten, men er desværre forhindret; derfor dette brev.

Siden din disputats i 1973 har du mere end nogen anden bidraget til højnelsen af dansk teoretisk statistik med artikler og bøger af en karat, som minder mig om Thiele's lignende indsats 100 år før; han skrev sin første statistiske artikel i 1873.

Hvis der havde eksisteret en Thiele-medalje, og jeg skulle have uddelt den, ville jeg have givet den til dig som den første.

Med venlig hilsen

Anders.

## REFERENCE

Lauritzen, S.L. (2002): Thiele: *Pioneer in Statistics.* Oxford University Press.

## WEBSOURCES

"Mephistopheles and Faust" by Delacroix http://www.ngv.vic.gov.au/explore/collection/work/25928/

http://www.maphysto.dk/

http://www.maphysto.dk/publications/MPS-NL/1998/1.pdf, side 2

http://www.maphysto.dk/publications/MPS-NL/1999/2.pdf, side 2

Jean-Pierre Bourguignon https://en.wikipedia.org/wiki/Jean-Pierre_Bourguignon#/media/File:Jean-Pierre_Bourguignon_-_2017_(cropped).jpg

https://www.google.com/imgres?imgurl=http://t3.gstatic.com/images?q%3Dtbn:ANd9GcR9lIT4B-S9eCPey8qwX7QwPq7Uq3EkCP8H7oOm-wWogFaFzpJXeR&imgrefurl=https://books.google.com/books/about/Thiele.html?id%3DnuPnCwAAQBAJ%26source%3Dkp_cover&h=1080&w=717&tbnid=x5l6bcqBqAzeeM:&q=Thiele+lauritzen&tbnh=160&tbnw=106&usg=__VYogWBenTC-0GHx8Bn-02ot9Mkxw%3D&vet=10ahUKEwjGsZq3t_jbAhWO16QKHbgPDG8Q_BoIgwEwCg..i&docid=ijPUv5ZWe2iysM&itg=1&client=firefox-b&sa=X&ved=0ahUKEwjGsZq3t_jbAhWO16QKHbgPDG8Q_BoIgwEwCg http://thiele.au.dk/

http://thiele.au.dk/research/project-groups/

http://thiele.au.dk/about-us/t-n-thiele/

# A FAIRY TALE

In 1983 I announced the position as Executive Secretary of the International Statistical Review, the main journal of the International Statistical Institute, of which I was Editor 1980-1987.

By extraordinary serendipity [1] among the applicants was one Oddbjørg Wethelund, of Faroe Islands extraction, whose excellent professional qualifications were a par with a sedate and immediately appealing personality that made my choice among the applicants easy.

In 1986 she became moreover Departmental Secretary to the Stochastics group at the Department of Mathematics (at the time Institute of Mathematical Sciences, a name much preferable to the present).

Then, when I became Editor in Chief of Bernoulli for the first years of its publication (1994-2000), Oddbjørg took up the position as Executive Secretary for Bernoulli, a major and highly significant responsibility. Here her work comprised a multitude of responsibilities: cataloguing submissions and reviews of manuscripts, correspondence with authors, Editors, Copy Editor and Desk Editor, and preparation of manuscripts for Copy Editor.

And following that she had the highly significant responsibility of serving as Secretary for MaPhySto, the Research Centre described in Part V. This involved planning and assistance at Conferences and caretaking of international guests, accounting, and coediting MaPhySto News.

My involvement in a number of international research collaborations have also much benefitted from Odd's assistance.

Apart from her secretarial skills, her general human qualities have been much called for in connection to the many international Conferences and Workshops organised by the Stochastics group. In this regard she has been quite unique in the helping and caring for the guests, as widely praised by them.

1 This somewhat unusual word goes back to 1754 and was coined by Horace Walpole, inspired by *The Three Princes of Serendip*, the title of a fairy tale in which the heroes 'were always making discoveries, by accident and sagacity, of things they were not in quest of'.

# EARLY MIDDLE YEARS

My academic life has involved intensive international travelling, for scientific collaborations on a personal basis and for participation in conferences and workshops.

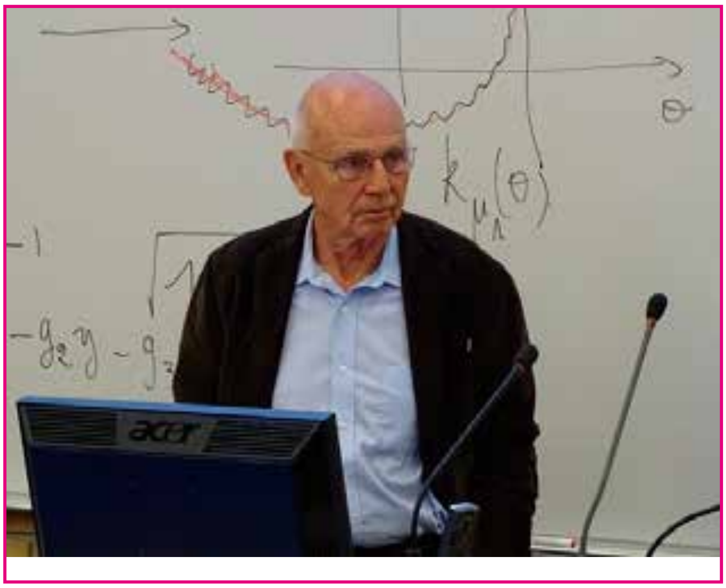
Among many research groups with which I have had close contacts are those of the probabilists and statisticians at Université Paul Sabatier, Toulouse, and at Katholieke Universiteit of Leuven. Those groups were led by Professor Gerard Letac and Professor Jozef Teugels, respectively.

# 15 TOULOUSE — AARHUS

## 15.1 GERARD LETAC

At the time where Gerard Letac and I first met and throughout the most intense phases of the Toulouse-Aarhus collaborations he was Professor at Institut de Mathematiques, at Université Paul Sabatier. The beginning and some of the developments of the collaborations are nicely described in the following quotation from the paper presented by him at the 2015 Aarhus Conference and published in the book *The Fascination of Probability, Statistics and Their Application*, mentioned in Section XVII:

*Ole and I met for the first time in the Summer School of Saint Flour in 1986. Having been converted to statistics by V. Seshadri 2 years before, I had learnt about exponential families through Ole's book (1978) and I had fought with cuts and steepness. Marianne Mora had just completed her thesis in Toulouse and was one of the Saint Flour participants: she was the first from Toulouse to make the pilgrimage to Aarhus the year after, followed by many other researchers from Toulouse: Evelyne Bernadac, Célestin Kokonendji, Dhafer Malouche, Muriel Casalis, Abdelhamid Hassairi, Angelo Koudou and myself. Over the years all of us were in contact with the everflowing ideas of Ole. During these Aarhus days (and Ole's visits to Toulouse) we gained a better understanding of the Lévy processes, of generalized inverse Gaussian distributions and their matrix versions, of differential geometry applied to statistics.*



**Gerard Letac giving his Lecture at the Aarhus Conference 2015.**

## 15.2 FRENCH-DANISH COLLABORATIONS MORE BROADLY

Through the 1990-ies there were lively collaborations between the research group in Stochastics at the Institute of Mathematical Sciences, Aarhus, and various research groups in France, in particular the group around Gerard Letac at l'Université Paul Sabatier, Toulouse, as already described in the previous Section, and that around Professor Emmanuel Jolivet at INRA (Institut National de la Recherche Agronomique).

## 15.3 DR. HONORIS CAUSA, UNIVERSITÉ PAUL SABATIER TOULOUSE JUNE 1993

In June 1993 I was awarded an Honorary Doctorate at l'Université Paul Sabatier, Toulouse.[1]

**Award Ceremony**

Université Paul Sabatier

Laboratoire de Statistique et Probabilités
Unité de Recherche Associée au C.N.R.S. 745

Gérard LETAC, Professeur

Monsieur le Recteur, Monsieur le Président,
Chers Collègues, Mesdames, Messieurs,

C'est une grande joie pour moi de vous présenter Ole Barndorff-Nielsen, Professeur de Statistique à l'Université d'Aarhus au Danemark. Ce petit pays de 5 millions d'habitants, à la géographie si plate que les visiteurs d'Ole Barndorff-Nielsen disent affectueusement que le point culminant du Danemark, c'est lui, ce petit pays donc, a toujours fait preuve, et jusque dans l'histoire récente, d'une grande personnalité et s'est monté une pépinière d'hommes remarquables : Kierkegaard, Andersen, Tycho Brahé bien sûr, mais aussi les frères Böhr, qui s'illustrèrent comme champions olympiques avant de le faire en physique et en mathématiques.

Ole Barndorff-Nielsen est l'un de ces hommes remarquables. Arrivé de Copenhague à Aarhus en 1960, il y a rencontré Sven Bundgaard mathématicien qui fit sortir de terre entre 1950 et 1960 une magnifique université, dont la bibliothèque, et les installations pour accueillir les visiteurs, sont connus des mathématiciens du monde entier, en particulier d'une dizaine de membres du Laboratoire de Statistique et Probabilités de l'université Paul Sabatier. En effet O. Barndorff-Nielsen a créé là un laboratoire de statistique, s'y est entouré d'une douzaine de chercheurs de talent et se montre d'une énergie infatigable pour organiser des rencontres, s'occuper des jeunes, frapper à toutes les portes pour obtenir les financements nécessaires, animer diverses revues (y compris les Annales de Toulouse) et sociétés savantes, et accueillir ses visiteurs d'Amérique, du Japon, d'Europe.

bât. 1R1 — 118, route de Narbonne — 31062 Toulouse cedex — France
Tél. : 61 55 67 72 — Fax : 61 55 60 89 — Télex : UNIPSAB 521880 F

Je viens de parler de l'administrateur : mais on a vu à quel point cette activité est imbriquée dans l'activité de l'enseignement, dans le souci, l'obsession de former des jeunes, de ne pas laisser perdre et gaspiller le savoir et les années d'expérience, en cherchant à ramasser toute cette connaissance convenablement décantée en des livres, des polycopiés, des séminaires.

Après l'administration et l'enseignement, le troisième pied de l'activité du professeur d'université, c'est la recherche. J'ai dénombré 152 entrées dans les publications d'Ole Barndorff-Nielsen ; nous sommes très fiers que certaines d'entre elles aient été faites avec des Toulousains. La plus connue est certainement un livre écrit en 1978, universellement connu et cité par les statisticiens du monde entier, riche en idées que quinze ans plus tard on n'a pas fini d'exploiter. Son oeuvre est celle d'un mathématicien complet : il sait se tourner vers les applications : savez-vous qu'il a étudié le mouvement des dunes de sables après avoir rencontré un extraordinaire vieux monsieur, R.A. Bagnold, et créé une petite soufflerie à Aarhus pour les nécessaires simulations ? C'est aussi un théoricien, capable d'abord de sortir l'essence d'une situation concrète, la modéliser, ensuite d'axiomatiser, de développer une théorie au point où elle devient capable d'en féconder une autre, enfin de résoudre des problèmes, tout simplement, et d'en donner une solution élégante.

Pourquoi ai-je mentionné plus haut l'activité olympique de Niels et Harald Böhr ? Parce qu'il y a dans toute l'activité de O.B.N. quelque chose qui rappelle cet idéal harmonieux : je l'ai vu se dépenser à l'animation de l'Opéra d'Aarhus, j'ai éprouvé les foudres de son service et de ses incontournables montées au filet au tennis. Je l'ai vu accueillir régulièrement nos chercheurs. Il nous a rendu visite une dizaine de jours par an pratiquement chaque année depuis 1987. Ces séjours, ces conférences qui les accompagnaient ont toujours été pour nous un encouragement, un enrichissement, une stimulation. Monsieur le Recteur, Monsieur le Président, chers Collègues, Mesdames, Messieurs, il me semble qu'Ole Barndorff-Nielsen nous a apporté beaucoup, et que l'attribution d'un Doctorat Honoris Causa est un moyen paticulièrement idoine de le remercier.

### My speech of thanks to l'Université Paul Sabatier[2]

Monsieur Le President et Recteur, chèrs Collégues, Mesdames et Messieurs

Les liens culturels et scientifiques entre la France et le Danemark sont très anciens.

Je n'insisterai pas sur la période Viking, si ce n'est pour mentionner qu'on raconte que le peintre francais Fernand Legér affirmait qu'il devait être d'origine Viking, arguant du fait qu'au cours des dicsusisons il avait tendance à garder les bras le long du corps, au lieu de gesticuler à la manière gauloise.

Bon nombre de peintres Danois de talent ont étudié avec Legér, y compris Asger Jorn, et il y en ce moment une exposition des travaux de Legér à Copenhague, intitulée "Legér et le Nord".
Parmi les nombreux autres liens Franco-Danois dans le domaine artistique, on pourrait citer par exemple la visite que Hans Christian Andersen fit à Madame de Stäel, et la période pendant laquelle le chorégraphe August Bournonville étudia à l'Opéra de Paris au milieu du dix-neuvième siècle. (Bournonville était Danois comme son nom ne l'indique pas). On peut aussi citer à titre d'exemple la forte influence de la culture francaise dans l'oeuvre de Karen Blixen, comme le montre l'évidence le film "Le Festin de Babette" de Gabriel Axel, metteur en scène Danois qui travaille en France. Un autre metteur en scène de cinéma ayant de forts liens avec la France est Carl Th. Dreyer don't le film le plus connu "Jeanne D'Arc" a été tourné dans ce pays. Enfin, qu'on me permettre de citer la "Grande Arche" de Johan Otto von Spreckelsen comme un exemple remarquable, aussi bien conceptuellement que dans sa réalisation, ainsi que la gigantesque fresque de Dewansne qui y est peinte.

Dans le domaine des sciences un lien très ancien fut un college pour étudiants Danois qui a existé à Paris de 1275 à 1429.

La connection la plus celébre est sans aucun doute celle qui a ammené l'astronome Danois Ole Rømer à la découverte de la finitude de la vitesse de la lumière. J'ai le plaisir de vous dire que Ole Rømer est né à Aarhus, en 1644, et il est mort à Copenhague en 1710. En 1671-1672 l'astronome Francais Aldei Jean Picard vint au Danemark, envoyé par l'Académie des Sciences de Paris, pour determiner les coordonnées exactes de l'Observatoire Uranienborg de Tycho Brahe sur l'ile de Hven. Le roi du Danemark Christian V demanda au professeur d'astonomie à l'Université de Copenhague de designer un jeune étudiant brilliant qui pourrait aider Picard dans sa tâche, et cet étudiant fut Ole Rømer. Picard le rammena avec lui à Paris en 1672 où il étudia jusqu'en 1681, et, le 7 décembre 1676 il publia son résultat célebre sur la vitesse da la lumière, qui s'appuyait sur des observations faites à Paris. Rømer revint au Danemark en 1681 et devint professeur d'Astronomie á l'Université de Copenhague. Cependant, comme il avait de nombreaux talents, il fut aussi nommé chef de la police, et plus tard maire de la ville. Un de ces premiers travaux comme chef de la police fut d'enlever, sur l'ordre du Roi, une énorme quantité de crottin de cheval nauséabond qui s'était accumulée devant le Palais Royal.

Un autre scientifique Danois de renom, Niels Steensen (Steno), bien connu pour ses travaux en Anatomie, en Physiologie, en Paléontologie et en Géologie, est lui aussi venu en visite à Paris à cette époque. En 1665, il donna à Paris une conference celébre sur la nature et le rôle du cerveau, rejetant en particulier le modèle de Descartes sur le fonctionnement du cerveau.

Plus avant, j'ai brèvement parlé de Tycho Brahe. Comme vous le savez peut-être, il avait perdu une partie de son nez dans un duel à Hambourg, et portait à la place une prothèse en or. La cause du duel était de savoir, qui de Brahe et de son adversaire, était le meilleur mathématicien. Heureusement, de tells querelles sont aujourd'hui démodés, mais cette anecdote me donne l'occarsion de saluer avec gratitude les relations cordials que j'ai toujours entretenues avec les collègues et les étudiants de cette Université, depuis notre premier contact en 1986.

Les seuls duels que le professeur Gérard Letac a jamais envisagés se déroulent sur le court de tennis et n'ont aucune relation directe avec les mathématiques.

Permettre mois de mentionner enfin une dernière remarque plus anecdotique. En 1919, l'Académie Royale des Lettres et Sciences du Danemark à

change ses statuts pour pouvoir accueillir des membres féminins, ceci dans l'idée d'élire Marie Curie qui devint effectivement membre en 1920.
Après avoir cité ces noms illustres, qu'on me permettre d'ajouter, sans aucune comparaison, que je suis enchanté d'avoir eu la chance d'être un lien, même ténu, dans cette noble tradition.

Je me sens très honoré par le titre qu'on me confère aujourd'hui, que j'apprécie chaleureusement, et j'espère pour l'avenir une collaboration très fructueuse avec mes collegues de l'Université Paul Sabatier.

Je vous remercie.

## 15.4 CONRAD MALTE-BRUN

Had I, at the time of the Honoris Causa event, known about Malthe Conrad Bruun, internationally renowned under the name Conrad Malte-Brun, I would certainly have mentioned him in my speech of thanks.

Conrad Malte-Brun was a remarkable and highly versatile human being – poet, publicist, journalist, and geographer.

He was born in a small Danish town Thisted in 1775 and died 1826 in Paris.

After a tumultuous youth in Copenhagen where he was very active as poet and at publishing political pamphlets, criticising the political system and advocating the Rousseausian ideas of equality and brotherhood, he was forced to leave the country and in 1799 he moved to Paris, where he stayed till his death.

In Paris he participated first in the publication of Journal d'Empire, pro-Napoleonic journal, and later, having lost his enthusiasm for Napoleon, in the publication of Quotidienne, as well as contributing to other journals and pamphlets of political observance.

In Copenhagen, Malte-Brun had made brief excursions into studying both theology and law, but in Paris he begun, without any professional background, to write about geography. Amazingly, this led eventually to him being internationally renowned and he is generally considered to be the founder of geography as a science.

As geographer he authored ai prodigious number of works, particularly noteworthy are *Géographie mathématique, physique et politique de toutes les parties du monde* (16 Volumes, 1803-1807, jointly with others), *Annales des voyages* (24 Volumes, 1808-1815), *Nouvelles annales des voyages* (30 Volumes, 1819-1826), *Précis de la géographie universelle* (8 Volumes, 1810-1829), *Tableau historique et physique de la Pologne* (1807).

Malte-Brun was on collegial and friendly terms with Alexander von Humboldt, and he was a driving force in the creation of La Société de Geographie in Paris. The founding of this Scientific Society took place in Hotel de Ville in Paris and the meeting was attended by numerous explorers and leading scientist, among which were Alexander von Humboldt, the chemist and physicist Joseph Gay-Lussac and the mathematician Pierre de Laplace.

Pierre-Simon de Laplace was elected President of the Société and Malte-Brun became its first General Secretary.

Malte Brun's life is described in great detail in the biography *Manden der ville vise Verden* (The man who wanted to show the World) by Bjørn Birnir, 2011, Copenhagen: Gyldendal.

## 15.5 PREBEN BLÆSILD

**Preben Blæsild**

On the Aarhus side Preben Blæsild was one of the most active participants in the Toulouse-Aarhus endeavours.

I first met Preben while he was still a student. Since then we have grown closer and closer – as colleagues and friends (including numerous doubles tennis matches, a sport where, unlike me, Preben is very skilled).

Scientifically we have collaborated in a wide range of fields, often with colleagues from abroad: Exponential Distributions; Statistical Inference; Hyperbolic and Inverse Gaussian distributions; Statistical Transformation Models; Differential Geometry and Statistics. All in all, together we wrote some twenty papers and Lecture Notes.

And we have jointly participated in numerous scientific visits and Conferences: France 8x, Denmark 7x, England 5x, Japan 4x, Italy 3x, Norway 2x, Finland 2x, Spain 2x, Brazil 1x, Bulgaria 1x, Canada 1x, Germany 1x, Poland 1x, Sweden 2x, The Netherlands 1x, Tunisia 1x, Turkey 1x and USA 1x.

# 16 LEUVEN-ETH-AARHUS

## 16.1 JOZEF TEUGELS

It was through my participation in the activities of the International Statistical Institute and its Committee for Statistics in the Physical Sciences, started in 1969 by the initiative of Jerzy Neyman, that I got to know Jozef Teugels, colloquially known as Jeff. We also shared interests more broadly in the applications of probability theory and, indeed, in Opera.
Jozef Teugels pioneered the development of research and studies in Probability and Insurance Mathematics as the first Professor of Probability Theory at the Katholieke Universiteit of Leuven. He was a leading figure in the creation of the Bernoulli Society which arose out of the Committee for Statistics in the Physical Sciences, primarily through the initiative and efforts of David Kendall. Jozef served of President of the Bernoulli Society from 1995 to 1997 and as President of the International Statistical Institute 2009-2011.

At the time where our acquaintance began Jeff had a PhD student, Paul Embrechts, with whom I also struck a friendship and to whom I have later made a number of longer visits after he moved to Zurich as Professor at ETH.

Both Jozef and Paul have had Insurance Mathematics as a main field of research. In recent years Paul's main works have been in Financial Finance.

## 16.2 DR. HONORIS CAUSA, KATHOLIEKE UNIVERSITEIT LEUVEN APRIL 1999

It was at the instigation of Jozef Teugels that I was awarded an Honorary Doctorate at Katholieke Universitaet Leuven..

### Programma Academische Zitting

18 u Verwelkoming door prof. L. Vanquickenborne, decaan Faculteit Wetenschappen

18u10 Laudatio door prof. J.L. Teugels, promotor

18u20 Uitreiking van het eredoctoraat aan prof. dr. O.E. Barndorff-Nielsen door rector A. Oosterlinck

18u25 Muzikaal intermezzo
Triosonate in Fa voor twee Blokfluiten en Cello van Georg Friedrich Händel:
Adagio, Allegro

18u35 Dankwoord en lezing door prof. dr. O.E. Barndorff-Nielsen, University of Aarhus, Denmark
**Stochastic Aspects of Laser Cooling and Trapping**

19u10 Muzikaal intermezzo
Triosonate in Fa voor twee Blokfluiten en Cello van Georg Philip Teleman:
Vivace, Largo, Allegro

19u20 Receptie in de Jubileumzaal

De muzikale intermezzi worden verzorgd door Nele Joosen en Pieter Campo, blokfluiten en Ellen Geubbelmans, cello.

## Laudatio

Professor Ole Eiler Barndorff-Nielsen, die vandaag door de Faculteit der Wetenschappen wordt voorgedragen voor een eredoctoraat, werd in 1935 in Kopenhagen geboren. Na het beëindigen van zijn middelbare studies in 1954, lijkt het er even op dat hij carriere zal maken in de biostatistiek want voor een periode van vier jaar blijft hij werkzaam aan het Departement Biostatistiek van het Deense Serum Instituut. Toch vinden we hem in 1958 terug als assistent aan de Universiteit van Kopenhagen. In 1960 behaalt hij de graad van Magister in de Wetenschappen in wiskunde en wiskundige statistiek aan de Aarhus Universiteit van zijn geboorteland.

From July 1960, Barndorff-Nielsen has been associated with the University of Aarhus where he smoothly passes through all the ranks of a professional career. Right from the beginning we notice a returning theme in his activities: wherever he goes, whatever he does, everything under his responsibility gets a remarkable upgrade. His first appointment in 1960 was at the Institute of Mathematics; the institute soon became the Department of Mathematical Sciences. After receiving his doctorate from the University of Copenhagen in 1973, he was elected professor at the Department of Theoretical Statistics in Aarhus, which was quickly rebaptized into the Department of Theoretical Statistics and Operations Research.

The first time that I was informed about the existence of Professor Barndorff-Nielsen was in 1965 when I was following the Doctor of Philosophy Programme at Purdue University in the United States of America. In one of his lectures, Professor Glenn Baxter made a nice compliment on a Danish scientist who had been working at Stanford University, somebody with a - for Americans - impossibly complicated name. If I recall correctly, Baxter was referring to an unexpected and refreshing generalization of the Borel-Cantelli lemma, a well-established basic result in probability theory.

We physically met for the first time in Poland in 1975 on the occasion of the 40th Session of the International Statistical Institute. At that time, Warsaw saw the creation of the Bernoulli Society for Mathematical Statistics and Probability, a scientific society that since then has been a recurrent theme in the life of Barndorff-Nielsen. Right from the start, he became one of the key people in this brandnew scientific organisation, in particular as chairman of its European branch. His enthusiasm has been so convincing that many people abusively think of the Bernoulli Society as a European society. The Bernoulli Society actually is the only international association dealing with mathematical statistics and probability, with interests ranging from the profoundest theory to the most unexpected application. In 1993 Professor Barndorff-Nielsen was elected the ninth president of the Bernoulli Society. During his presidency, he succeeded in materializing an old dream, namely the establishment of an international statistical journal that would emphasize the intrinsic unity as well as the extensive diversity of statistics.

Barndorff-Nielsen has played a crucial role in at least three distinct domains of human endeavour: scientific research, university administration and international cooperation. Allow me to dwell a bit on each one of these topics.

- The most recent list of publications contains 185 items, 5 books, 4 edited volumes, papers and contributions in proceedings and even one film on *sand*. Particularly important are his textbooks on asymptotic statistics that have been used worldwide. As one reviewer wrote *" authoritative and comprehensive books which develop complex theoretical results in a coherent and logical manner. Anybody interested in recent developments in parametric likelihood theory should read them."* One of the gratifying aspects of his scientific output lies in a striking balance between his - shall I call it private? - research where we find some 90 items and as many publications in collaboration. That he has written papers with more than 50 scholars clearly proves his capacity to make friends through science. But to keep friends is even more remarkable: over one third of his co-authors have written more than one paper with him. The variety of topics on which Barndorff-Nielsen has published is denumerable and impressive: from theoretical statistics and probability to deep applications in chaos, demography, econometrics, finance. Other titles refer to less usual topics such as wind tunnels, river networks, sand, hydrology, hare populations.

- On the platform of university administration his contributions are not less impressive. In 1973 he created a Department of Theoretical Statistics at the university in Aarhus. Right from the beginning, he surrounds himself with young but highly capable colleagues. In less than no time, the statistics group at Aarhus becomes a point of reference because of its impeccable international reputation. A few years later, Ole Barndorff-Nielsen becomes scientific director of the *Mathematical*

*Research Centre at Aarhus University*. However, the crowning piece is definitely the creation in 1998 of the state supported *Centre for Mathematical Physics and Stochastics* where a slate of internationally renowned scientists share their expertise with one another and with selected youngsters interested in the interplay between quantum mechanics at large and stochastic analysis. Aspects of stochastic computation, interactive particle systems, stochastic matrices and financial models form also part of the ambit of the centre. The call name of the centre is *MaPhySto*. I'm convinced that this acronym has been inspired by a famous figure from Goethe's Faust. This is no accident. Our honorary guest and his wife are fervent opera enthusiasts. We share this passion and so, a couple of years ago, we told them that our National Opera was staging a new production of Moussorgsky's Khovantchina. So, they came by car from Aarhus to Brussels in order not to miss this unique event.

- Finally, Barndorff-Nielsen has an impressive record of services, rendered to the scientific community at large. He has been editor-in-chief of *International Statistical Review* and more importantly, he still is the main editor of *BERNOULLI* and associate editor of half a dozen other statistical journals. He has been visiting professor in Stanford, Minnesota, Toulouse, Cambridge, invited lecturer at universities in Europe, Australia, Brazil, Japan, Mexico, North America and also at international meetings organised by the *International Statistical Institute*, the *American Statistical Association*, the *Institute of Mathematical Statistics*, the *Biometric Society*. Last but not least, he received a honorary doctorate from the *Université Paul Sabatier* at Toulouse in 1993.

## ENDNOTES

1 **Translation of Professor Gérard Letac's presentation of the motivations for the award of the Dr. Honoris Causa**

Mr. Vice-Chancellor, Mr. President, dear colleagues, ladies and gentlemen,

It is a great pleasure for me to introduce you to Ole Barndorff-Nielsen, Professor of Statistics at Aarhus University in Denmark. This small country with 5 million inhabitants is so flat that Ole Barndorff-Nielsen's visitors affectionately say that he is the highest point in Denmark. This small country has, however, fostered several remarkable personalities: Kirkegaard, Andersen, Tycho Brahe of course, but also the brothers Bohr who shine like Olympic champions.

Ole Barndorff-Nielsen is one of these remarkable persons. He came to Aarhus from Copenhagen in 1960 and met Svend Bundgaard, mathematician, who between 1950 and 1960, established a splendid university, its library and the guest department being known by mathematicians from all over the world, and, in particular, known by several members of the Laboratoire de Statistique et Probabilités de l'Université Paul Sabatier. In fact, Ole Barndorff-Nielsen established in Aarhus the Department of Statistics together with a dozen of talented researchers, and with an untiring energy organized research meetings, taking care of young researchers, knocked on all doors in order to achieve the necessary funding, active on several journals, and welcoming visitors from the United States, Japan, and European countries.

I have mentioned the administrator, but we have seen to what extent this activity is associated with education, with concern, eagerness to form the young people, not to lose and waste knowledge and years of experience, by striving to pick up all this knowledge conveniently gathered in books, collections, seminars.

After the administration and the education, the third activity of a university professor is research. I have counted 152 publications on Ole Barndorff-Nielsen's list of publications; we are very proud that a number of these have been written in cooperation with researchers from Toulouse. The best known is certainly a book written in 1978, known and cited by statisticians worldwide, containing ideas that fifteen years later have not been fully exploited. His work is that of a real mathematician: he is able to turn to applications. Did you know that he has studied the movement of sand dunes after having met the grand old man, R.A. Bagnold, and created a small wind tunnel in Aarhus for the necessary simulations? He is also a theoretician, capable of first grasping the essence of a concrete situation, to model it, then to axiomatise it, to develop a theory, to solve the problems, and to present an elegant solution.

Why did I emphasize the olympic activity of Niels and Harald Bohr? Because in all activity of OBN there is something that resembles this harmonic ideal: I have seen his enthusiasm at the Opera of Aarhus, I have experienced his temper at the tennis court. I have seen him welcome our researchers on several occasions. He has visited us a couple of weeks practically every year since 1987. These visits and conferences have always been an encouragement, an enrichment, a stimulation for us. Mr. Vice-Chancellor, Mr. President, dear colleagues, ladies and gentlemen, I think that Ole Barndorff-Nielsen has contributed a great deal to us, and the award of the Doctorat Honoris Causa is a proper way of thanking him.

2 **Translation of my speech of thanks to l'Université Paul Sabatier**

Mr. President and Vice-Chancellor, dear Colleagues, Ladies and Gentlemen,

The cultural and scientific ties between France and Denmark have a long history.

I would not mention the Viking period if it were not for the story of the French painter Fernand Legér who claimed that he was of Viking origin because, during discussions, he had a tendency to keep his arms along his body instead of gesticulating in the Gaelic way.

A number of talented Danish painters have studied with Legér, among others Asger Jorn, and during that time an exhibition of works by Legér entitled "Legér and the Nordic Countries" took place in Copenhagen.

Among the numerous French-Danish liaisons in the cultural area one could for instance mention

Hans Christian Andersen's visit to Madame de Stäel, and the period during which the choreographer August Bournonville studied at the Opera of Paris during the middle of the Nineteenth Century. (Bournonville was Danish although his name does not indicate so). One could also, for instance, mention the strong influence of the French culture on Karen Blixen's works, as is evident in the film "Babette's Feast" by Gabriel Axel, a Danish director who was working in France. Another film director who had a strong connection to France was Carl Th. Dreyer whose best known film "Jeanne D'Arc" was recorded in this country. Finally, allow me to mention Johan Otto von Spreckelsen's "Grande Arche" as a remarkable example, both conceptually as well as in its realisation; furthermore, it should be mentioned that Dewansne's gigantic fresco is painted on the arc.

In the area of science there is an ancient link to a college for Danish students, existing in Paris from 1275 to 1429.

The most renowned connection is undoubtedly the one that led the Danish astronomer Ole Rømer to discover the speed of light. I have the pleasure to inform you that Ole Rømer was born in Aarhus in 1644, and he died in Copenhagen in 1710. In 1670-1672 the French astronomer Aldei Jean Picard came to Denmark, sent by the Academy of Sciences in Paris, to determine the exact coordinates at the observatory Uranienborg of Tycho Brahe at the island of Hven. The King of Denmark, Christian V, asked the Professor of Astronomy at the University of Copenhagen to appoint a young talented student to help Picard with his work, and this student was Ole Rømer. Picard brought him to Paris in 1672 where he studied until 1681, and on the 7th December 1676 Rømer published his renowned result on the speed of light, that he had established during his observations made in Paris. Rømer went back to Denmark in 1681 and was appointed Professor of Astronomy at the University of Copenhagen. However, although he had numerous talents he was also appointed Head of Police, and later Mayor of Copenhagen. One of his first tasks as Head of Police was, on the King's order, to remove a great amount of horse dropping that had accumulated in front of the Royal Palace.

Another renowned Danish scientist, Niels Steensen (Steno), well-known for work in Anatomy, Physiology, Palaeontology, and Geology, also visited Paris at the same time. In 1665 he organized a renowned conference on Nature and the role of the brain, rejecting in particular Descartes' model of the function of the brain.

Earlier I shortly mentioned Tycho Brahe. As you may know he had lost part of his nose in a duel in Hamburg and carried a gold prosthesis. The cause of the duel was to determine whether Brahe or his opponent was the best mathematician. Fortunately, such feuds are outdated nowadays.

But this anecdote gives med the occasion to express my gratitude for the warm-hearted relations with the colleagues and the students of this university that have existed since our first contact in 1986. The only duels that Professor Gérard Letac has ever considered have taken place at the tennis court, with no relations to mathematics.

Allow me finally to make a more anecdotal remark. In 1919 the Royal Danish Academy of Sciences and Letters changed their rules in order to be able to admit female members, with the aim to elect Marie Curie, who actually became a member in 1920. After having mentioned these famous names, allow me to add, sans aucune comparison, that I am delighted to have had a chance to be a part of this noble tradition of French-Danish ties.

I feel deeply honoured by the title that is conferred on me today, it is warmly appreciated, and I hope for fruitful collaboration in the future with my colleagues at Université Paul Sabatier.

Thank you.

# SOME AREAS OF RESEARCH

This Chapter describes some of the research areas to which I have contributed. Other areas, such as turbulence or finance, are considered in more detail elsewhere in this book.

# 17 CLASSES OF PROBABILITY DISTRIBUTIONS

## 17.1 PROBABILITY LAWS AND INFINITE DIVISIBILITY

By the law of an observable *X* – say a temperature *T*, the velocity *V* of a particle, the price *S* of a stock, ... – is meant its probability distribution *P* where, for instance, *P* (a<*X*≤b) is the probability that the value of *X* belongs to the interval (a,b]. Often it is possible to represent the probability as the integral over (a,b] of a positive function *p* called the probability density of *X*.

The property of infinite divisibility of a probability law is of key importance for modelling by stochastic processes. Here infinite divisibility means that the distribution may arise as the result of (infinitely) many small and independent random effects. In probability theory two events are said to be independent if the probability that they both take place is equal to the product of the individual probabilities of their occurrence. This concept extends directly to any number of events.

In addition to its theoretical interest, in the building of stochastic models of dynamic phenomena there is often considerable gain in terms of tractability and interpretability by using infinitely divisible elements in the construction. Such elements may be single or multiple observables or, more generally whole processes and fields.

The concept of infinite divisibility was introduced and studied in the 1930-s by Lévy, Khintchine and Kolmogorov. They established a fully specified representation of the characteristic function (Fourier transform) of any such distribution, known as the Lévy-Khintchine formula. In case the characteristic function is known the question of infinite divisibility can then in principle be decided by means of the Lévy-Khintchine formula. A stochastic version of this formula is due to Ito and is known as the Lévy-Ito representation.

## 17.2 THE ROLE OF PARAMETRIC FAMILIES OF DISTRIBUTIONS

For applications of probability theory an elementary prerequisite is to have available a catalogue of specific instances of flexible families of distributions indexed by a – possibly multidimensional – parameter.

The classical example of such a family is the class of Gaussian or normal distributions, which is indexed by the mean value $\mu$ and the standard deviation $\sigma$. Other classical examples are the Poisson laws and the gamma laws.

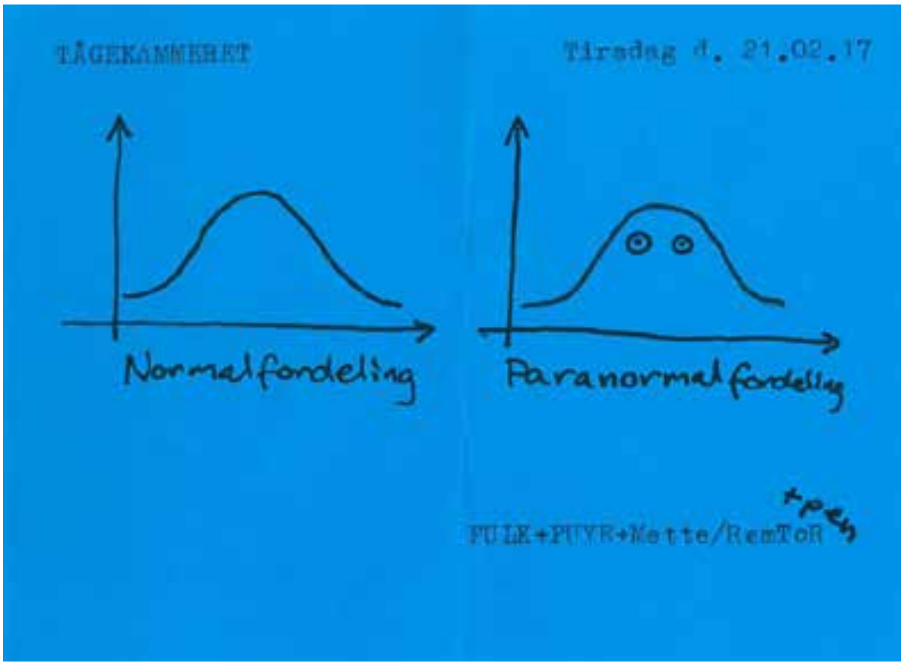
As stochastic analysis and its applications have developed the question of whether a specified family possesses deeper probabilistic properties has become of considerable interest. Without such properties the use of the family remains of limited interest. In particular, the property of infinite divisibility is of key importance for modelling by stochastic processes.

Initially few examples of distributions having explicitly given probability density were known to be infinitely divisible. The list gradually grew over a long period but the progress was slow as each single case seemed to call for special, often quite deep, arguments to establish the infinite divisibility.

In many instances an interesting class of distributions have initially arisen not from very theoretical mathematical considerations but through finding a mathematical formula that fitted a range of observed empirical distributions well and therefore served to provide a condensed and interpretable description of key traits of the phenomenon under study. Subsequently one may then ask for nontrivial and useful probabilistic properties of that class of distributions. Examples of such developments are discussed in Sections 17.3 and 17.5.

Families of distributions, having few parameters, are useful if they are capable of describing empirical distributions from a great variety of fields. Not only do they provide a simple characterisation of any given distribution of the kind in question, but they open the way for a parsimonious representation, by the parameters, of classes of experiments, often allowing incisive tests of hypotheses about the laws of nature behind the phenomena studied.

In a multitude of situations one encounters empirical distributions where the lower and upper tails are considerably heavier than for a normal distribution and where the distributions may exhibit a degree of asymmetry. This calls for distributions that have two further parameters in addition to position and standard deviation. The hyperbolic laws are of this kind, with applications in particular to particle size distributions from windblown sand and to liquid sprays, cf. Part ##. Another important such class are the normal inverse Gaussian laws, the NIG laws, which will be considered in #### and which also have a multitude of applications, in particular to turbulence and to financial econometrics.




**The theory of laws of probability distributions has always been a popular theme in the stochastics group at the Aarhus Institute of Mathematics. The manifestation above prepared by the student organization Taagekammeret (the Fog chamber), bears witness to this.**

## 17.3 THE GH CLASS

The hyperbolic and the normal inverse Gaussian classes belong to a general family of probability laws called the generalized hyperbolic laws, or *GH* laws. This was briefly introduced in my original paper on the hyperbolic distribution [ref]. It turned out that the *GH* class encompasses not only the *H* and *NIG* laws but also a number of classically known distributions, such as the normal and the Cauchy[1] distributions, in addition to several new types with their own domains of application.

Table **** gives an overview of the mathematical formulae for the most important subfamilies of the *GH* class.

The *GH* law arises from the normal distribution by thinking of the standard deviation $\sigma$ as being random, but following a special type of positive distributions, the generalised inverse Gaussian laws, or *GIG* laws. The *NIG* distributions correspond to the member of *GIG* known as the inverse Gaussian law *IG* while the hyperbolic arises from the reciprocal inverse Gaussian (or *RIG*), as indicated in Table ###. The *GIG* laws are considered in the next Section.

| Distribution | $\sigma^2$ | Density function |
|---|---|---|
| $\mathrm{GH}(\nu,\alpha,\beta,0,\delta)$ | $\mathrm{GIG}(\nu,\delta,\gamma)$ | $\dfrac{(\gamma/\delta)^{\nu}}{\sqrt{2\pi}\alpha^{(\nu-1/2)}K_\nu(\delta\gamma)}\{\delta^2+y^2\}^{(\nu-1/2)/2} \cdot K_{\nu-1/2}\left(\alpha\sqrt{\delta^2+y^2}\right)e^{\beta y}$ |
| $\mathrm{RH}(\alpha,\beta,\mu,\delta) = \mathrm{GH}(-1,\alpha,\beta,0,\delta)$ | $\mathrm{RPH}(\delta,\gamma)$ | $\dfrac{\alpha}{2\delta\gamma K_1(\delta\gamma)}\{1+\alpha^{-1}(\delta^2+y^2)^{-1/2}\} \cdot \exp\left\{-\alpha\sqrt{\delta^2+y^2}\right\}e^{\beta y}$ |
| $\mathrm{NIG}(\alpha,\beta,0,\delta) = \mathrm{GH}(-\frac{1}{2},\alpha,\beta,0,\delta)$ | $\mathrm{IG}(\delta,\gamma)$ | $\dfrac{\alpha}{\pi}\exp\left(\delta\sqrt{\alpha^2-\beta^2}\right)q\left(\dfrac{y}{\delta}\right)^{-1} \cdot K_1\left\{\delta\alpha q\left(\dfrac{y}{\delta}\right)\right\}e^{\beta y}$ |
| $\mathrm{HA}(\nu,\alpha,\beta,0,\delta) = \mathrm{GH}(0,\alpha,\beta,0,\delta)$ | $\mathrm{GIG}(0,\delta,\gamma)$ | $\dfrac{1}{2\alpha^{-1}K_0(\delta\gamma)}\left(\delta^2+y^2\right)^{-1/2} \cdot \exp\left\{-\alpha\sqrt{\delta^2+y^2}\right\}e^{\beta y}$ |
| $\mathrm{NRIG}(\alpha,\beta,0,\delta) = \mathrm{GH}(\frac{1}{2},\alpha,\beta,0,\delta)$ | $\mathrm{RIG}(\delta,\gamma)$ | $\dfrac{\gamma}{\pi}e^{\delta\gamma}K_0\left(\alpha\sqrt{\delta^2+y^2}\right)e^{\beta y}$ |
| $H(\alpha,\beta,0,\delta) = \mathrm{GH}(1,\alpha,\beta,0,\delta)$ | $\mathrm{PH}(\delta,\gamma)$ | $\dfrac{\sqrt{\alpha^2-\beta^2}}{2\alpha\delta K_1\left(\delta\sqrt{\alpha^2-\beta^2}\right)} \cdot \exp\left(-\alpha\sqrt{\delta^2+y^2}+\beta y\right)$ |
| $\mathrm{N\Gamma}(\nu,\alpha,\beta,0) = \mathrm{GH}(\nu,\alpha,\beta,0,0)$ | $\Gamma\left(\nu,\dfrac{\gamma^2}{2}\right)$ | $\dfrac{\left(\alpha^2-\beta^2\right)^{\nu}\alpha^{1-2\nu}}{\sqrt{2\pi}\,\Gamma(\nu)2^{\nu-1}}\overline{K}_{\nu-1/2}\left(\alpha\lvert y\rvert\right)e^{\beta y}$ |
| $T(\nu,\beta,\delta,0) = \mathrm{GH}(-\nu,\beta,\beta,0,\delta)$ | $\mathrm{R\Gamma}\left(\nu,\dfrac{\delta^2}{2}\right)$ | $\dfrac{1}{\sqrt{2\pi}\delta\,\Gamma(\nu)2^{\nu-1}}q\left(\dfrac{y}{\delta}\right)^{-2\nu-1} \cdot \overline{K}_{\nu+1/2}\left\{\delta\lvert\beta\rvert q\left(\dfrac{y}{\delta}\right)\right\}e^{\beta y}$ |
| $T(\nu,\delta,0) = \mathrm{GH}(-\nu,0,0,0,\delta)$ | $\mathrm{R\Gamma}\left(\nu,\dfrac{\delta^2}{2}\right)$ | $\dfrac{\Gamma(\nu+1/2)}{\sqrt{\pi}\delta\,\Gamma(\nu)}\left(1+\dfrac{y^2}{\delta^2}\right)^{-\nu-1/2}$ |

**Summary of the GH distribution and its special cases with $\mu = 0$.**
**Recorded are the densities, where $\overline{K}_\nu(y) = y^\nu K_\nu(y)$, $q(y) = \sqrt{1+y^2}$ and $\alpha = \sqrt{\beta^2+\gamma^2}$.**
**$K_\nu$ is the modified Bessel function of the third kind.**

## 17.4 GENESIS OF PARAMETRIC FAMILIES

As already indicated, in many instances an interesting class of distributions have initially arisen not from very theoretical mathematical considerations but through finding a mathematical formula that fitted a range of observed empirical distributions well and therefore served to provide a condensed and interpretable description of key traits of the phenomenon under study. Subsequently one may then ask for nontrivial and useful probabilistic properties of that class of distributions.

An illustration of this is provided by the historical note on the origin of the Generalised Inverse Gaussian, or GIG, distributions, shown here.

In many ways Halphen's invention of the Generalised Inverse Gaussian distributions was similar to mine of the Hyperbolic and subsequently the Generalised Hyperbolic Distributions. Like Halphen, I took my queue from empirical findings, in my case those of Bagnold on the distributions of sizes of sand particles, deposited on dunes; findings that he confirmed and extended by wind tunnel experiments, as discussed in Section 5.4. Having introduced the hyperbolic distribution I realised that it could be viewed as a variance-mean mixture of the normal distributions using as mixture law a particular distribution that belongs to the class that was later named the generalised inverse Gaussian.

**A historical note on the Generalised Inverse Gaussian Distributions**

Sometime around 1980, after giving a talk in Paris on the hyperbolic distribution I was approached by a French statistician, Michel Maurin, who told me that the class of Generalised Inverse Gaussian distributions had been considered in the early 1940-es by a French statistician Etienne Halphen, who named them type A laws. Had it not been for this Halphen's work might well have remained in obscurity for, except for applications of these distributions in hydrology, especially in the 1940-s and 1950-s, hardly any notice had been taken of this class. In part at least, this was connected to the sad circumstance that Halphen was Jewish and therefore, because of the war, prevented from publishing his results. However, to make them known it was arranged that the French statistician Daniel Dugué presented some of Halphen's results in a brief note to the French Academy of Science and reported in 1941, in Comptes Rendus de l'Académie des Sciences, but without mentioning Halphen's name.

A further reason that Halphen's work went largely unnoticed may well be that, except for the Inverse Gaussian, no probabilistic properties or interpretations of the GIGs had been established at those times so that the use of these laws might seem mostly a case of curve fitting.

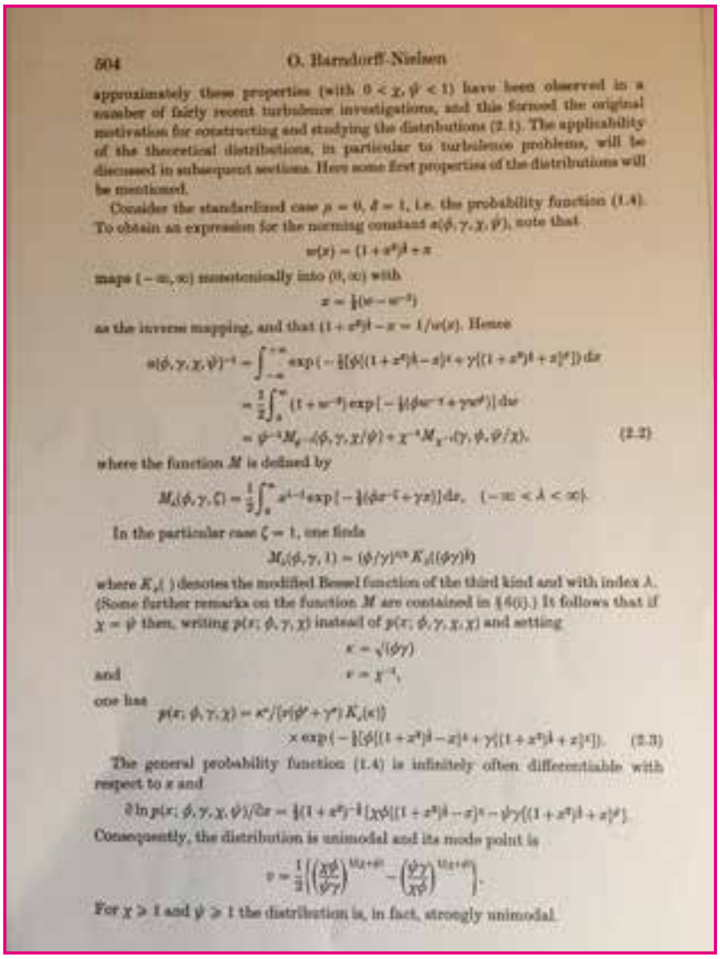
Such properties and interpretations have since been found and are, besides there intrinsic probabilistic interest, of essential value in applications.

A paper, published in Revue des Statistique Appliquée in 1956, one of Halphen's younger colleagues and collaborators, M. Morlat, sets out in considerable detail how Halphen came to define these type A (or GIG) laws, and reports on further developments by which Halphen introduced two other types of distributions, type B and type B-1, again in order to analyse records of flow of water in the French rivers for the purpose, among other things, of predicting future behaviour of the water levels of the rivers and associated basins. These extensions, while of definite value in hydrology, are mathematically less appealing than the GIGs and have found very little use elsewhere.

As described by Morlat a large amount of work, using the best calculating equipment available at the time, went into constructing graphs and tables that would ease the applications of distributions. But the fitting procedures were largely restricted to the method of moments.

The gamma and the inverse Gaussian laws were well known to be infinitely divisible, and it was therefore natural to enquire whether the same holds true of the other members of the generalized inverse Gaussian family and whether such properties were inherited by the generalized hyperbolic distributions. This was found to be indeed the case [BN and Halgreen (1977)].

The Hyperbolic and Generalised Hyperbolic Distributions have immediate extensions to several dimensions, through using the multidimensional Gaussian laws and the mixture representation. In particular, quite unexpectedly it was realised that the three-dimensional version of the Hyperbolic Distribution occurs in Relativity Theory where it provides the law of the momentum of particles in an ideal gas as seen by an observer whose inertial system is moving at a constant speed in relation to the gas. This was in fact discussed already in Section ##.



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approximately these properties (with $0 < \chi, \psi < 1$) have been observed in a number of fairly recent turbulence investigations, and this formed the original motivation for constructing and studying the distributions (2.1). The applicability of the theoretical distributions, in particular to turbulence problems, will be discussed in subsequent sections. Here some first properties of the distributions will be mentioned.

Consider the standardized case $\mu = 0$, $\delta = 1$, i.e. the probability function (1.4). To obtain an expression for the norming constant $a(\phi, \gamma, \chi, \psi)$, note that

$$w(x) = (1+x^2)^{\frac{1}{2}} + x$$

maps $(-\infty, \infty)$ monotonically into $(0, \infty)$ with

$$x = \tfrac{1}{2}(w - w^{-1})$$

as the inverse mapping, and that $(1+x^2)^{\frac{1}{2}} - x = 1/w(x)$. Hence

$$\begin{aligned} a(\phi,\gamma,\chi,\psi)^{-1} &= \int_{-\infty}^{+\infty} \exp\left(-\tfrac{1}{2}[\phi\{(1+x^2)^{\frac{1}{2}}-x\}^{\chi} + \gamma\{(1+x^2)^{\frac{1}{2}}+x\}^{\psi}]\right) dx \\ &= \frac{1}{2}\int_0^{\infty} (1+w^{-2}) \exp[-\tfrac{1}{2}(\phi w^{-\chi} + \gamma w^{\psi})]\, dw \\ &= \psi^{-1} M_{\psi^{-1}}(\phi, \gamma, \chi/\psi) + \chi^{-1} M_{\chi^{-1}}(\gamma, \phi, \psi/\chi), \end{aligned} \qquad (2.2)$$

where the function $M$ is defined by

$$M_\lambda(\phi,\gamma,\zeta) = \frac{1}{2}\int_0^{\infty} x^{\lambda-1} \exp[-\tfrac{1}{2}(\phi x^{-\zeta} + \gamma x)]\, dx, \quad (-\infty < \lambda < \infty).$$

In the particular case $\zeta = 1$, one finds

$$M_\lambda(\phi,\gamma,1) = (\phi/\gamma)^{\lambda/2} K_\lambda((\phi\gamma)^{\frac{1}{2}})$$

where $K_\lambda(\ )$ denotes the modified Bessel function of the third kind and with index $\lambda$. (Some further remarks on the function $M$ are contained in §6(i).) It follows that if $\chi = \psi$ then, writing $p(x; \phi, \gamma, \chi)$ instead of $p(x; \phi, \gamma, \chi, \chi)$ and setting

$$\kappa = \surd(\phi\gamma)$$

and

$$\nu = \chi^{-1},$$

one has

$$p(x;\phi,\gamma,\chi) = \kappa^{\nu}/\{\nu(\phi^{\nu}+\gamma^{\nu}) K_\nu(\kappa)\} \times \exp\left(-\tfrac{1}{2}[\phi\{(1+x^2)^{\frac{1}{2}}-x\}^{\chi} + \gamma\{(1+x^2)^{\frac{1}{2}}+x\}^{\chi}]\right). \qquad (2.3)$$

The general probability function (1.4) is infinitely often differentiable with respect to $x$ and

$$\partial \ln p(x;\phi,\gamma,\chi,\psi)/\partial x = \tfrac{1}{2}(1+x^2)^{-\frac{1}{2}}[\chi\phi\{(1+x^2)^{\frac{1}{2}}-x\}^{\chi} - \psi\gamma\{(1+x^2)^{\frac{1}{2}}+x\}^{\psi}].$$

Consequently, the distribution is unimodal and its mode point is

$$\nu = \frac{1}{2}\left\{\left(\frac{\chi\phi}{\psi\gamma}\right)^{1/(\chi+\psi)} - \left(\frac{\psi\gamma}{\chi\phi}\right)^{1/(\chi+\psi)}\right\}.$$

For $\chi \geq 1$ and $\psi \geq 1$ the distribution is, in fact, strongly unimodal.

## 17.5 THE NIG CLASS

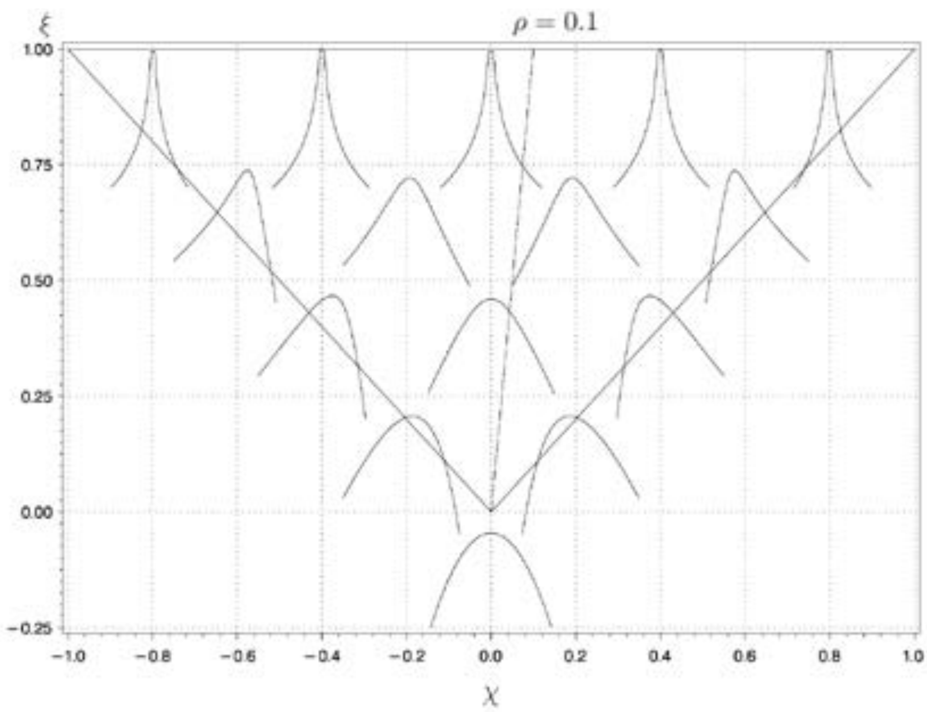
Like the inverse Gaussian is prominent among the generalised inverse Gaussians, the Normal Inverse Gaussian law is the member of leading interest in the class of Generalised Hyperbolic distributions. The Normal Inverse Gaussian law was briefly introduced in [BN (1977)] and studied more closely in [BN (1998)]. It has since been found to have applications in a wide range of fields, especially in Turbulence and in Financial Econometrics, as discussed elsewhere in this book.

**Part of the construction of the generalized hyperbolic laws. From [BN (1979)].**

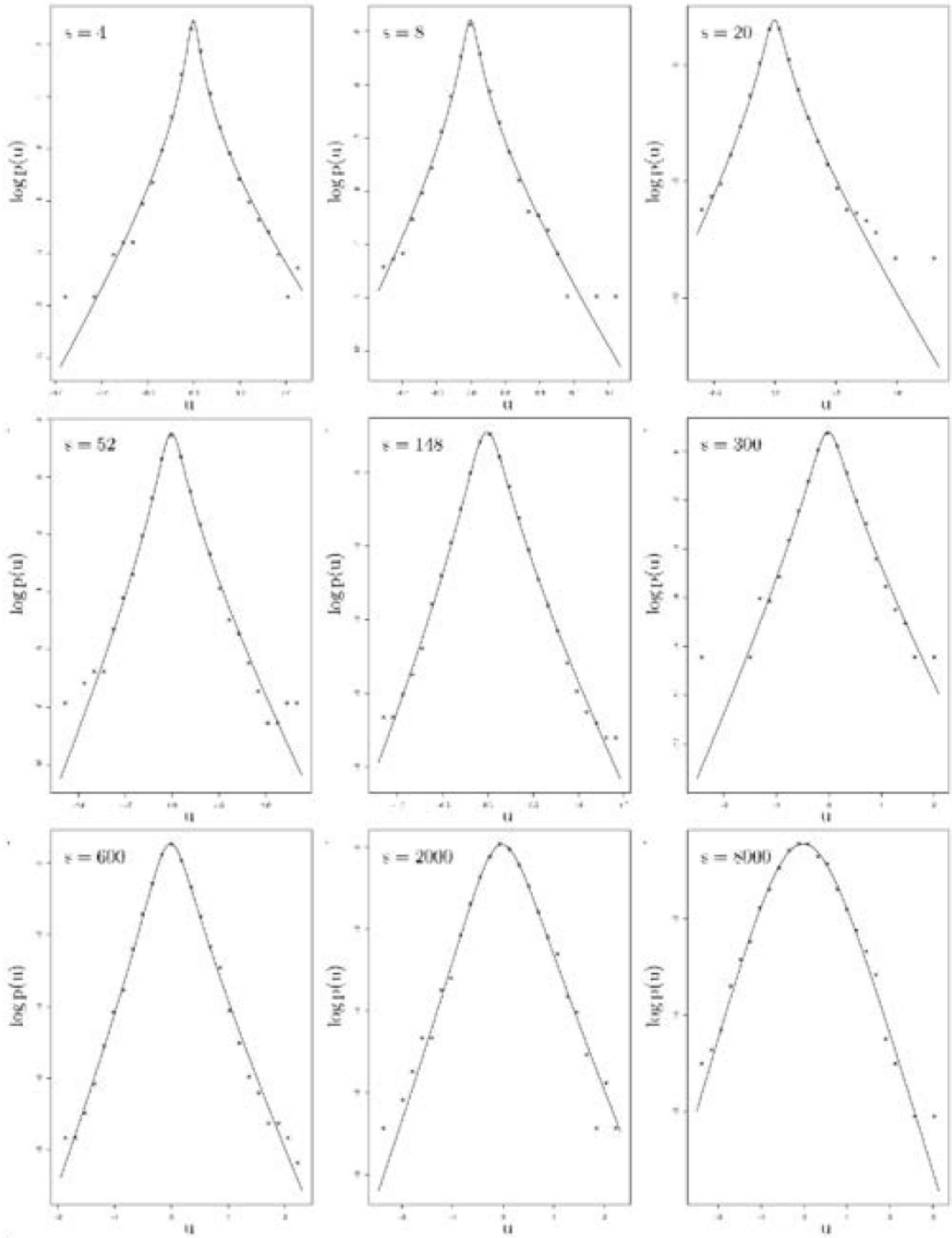
An important recent example is a paper by Michael Sørensen [Sørensen (2016)] that establishes a dynamic stochastic model for the transport and deposition of sand grains by the wind, leading to the NIG as the distribution of the grain sizes in localised samples. This, in a sense, is the ultimate theoretical justification for the applications of the hyperbolic and normal inverse Gaussian laws in sedimentology. In the previous parts of this book the hyperbolic distribution was discussed as the paradigmatic law for the grain size distributions and in fact this was well justified empirically, the fits to actual observational and experimental data being essentially perfect for the type of empirical settings from which the samples came. However, under certain other settings the empirical distributions show relatively minor deviations from the hyperbolic shape while being well fitted by the NIG.

A useful way of illustrating the type of distributions encompassed by the NIG family is the NIG shape triangle. The normal distribution is sitting at the lower tip of the triangle, the Cauchy law at the top about it, while the IG family fills out the right hand edge. The NIG laws themselves fill out the interior of the triangle, each point in the interior corresponding to a particular member of NIG.

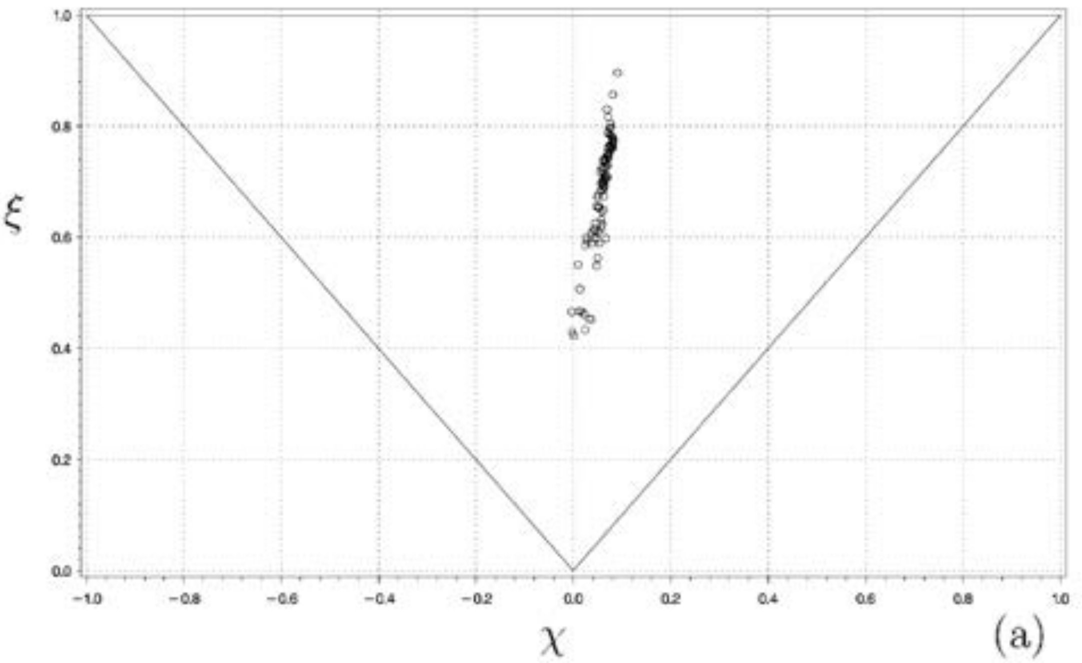
The NIG shape triangle may now, for instance, be used to depict how the distributions of velocity differences in homogeneous turbulence vary with different time lags or Reynolds numbers. The adduced further two figures (from [BN, Blæsild and Schmiegel (2004)]) illustrate this. They are based on a data set from the atmospheric boundary layer. In the second figure sees how with increasing time difference the distributions become more and more symmetric and getting nearer to the normal law.




**The shape triangle of the normal inverse Gaussian distributions with the log density functions, corresponding to various values of the invariant parameters ($\chi$,$\xi$).**




**Approximation of the log pdf of velocity increments by NIG distributions. Data set from the atmospheric boundary layer. For increasing time differences, the distributions look more and more like the Gaussian law.**




**Shape triangle for the evolution of the probability density functions of velocity increments, measured in the atmospheric boundary layer and discussed above.**

# 18 LÉVY PROCESSES AND LÉVY THEORY

## 18.1 FIRST VIEW

Most of the probability laws (probability distributions) that are of interest in Stochastics have the property of being infinitely divisible. This theme is discussed in Section 42.2. From laws of this type flows the elemental idea of Levy processes. Such processes serve as generators of more advanced models that, in addition to their own intrinsic interest, have applications in many areas. Some indications of this are given in Section 42.3.

The Brownian motion may be viewed as a Lévy process. It is the only such process that moves continuously in time. The name Brownian motion refers to work by the English botanist Robert Brown (1773-1858) who in 1827 published a paper discussing his observations of the jittering movements of small particles suspended on the surface of a fluid. This type of phenomena had been observed and described much earlier but Brown was the first to describe the motion in detail, at the same time showing convincingly that the particles concerned were not living organisms but inanimate objects. His results gave fuel to the debate about the existence or not of atoms, a possible explanation for Brown's observations being that the movement of the particles was caused by the impacts of atoms constituting the fluid and moving about in a random fashion. The debate continued unabated till the early

Nineteenth Century when the matter was settle by two papers by Albert Einstein, published in 1905 and 1908, and accompanying experiments by the physicist Jean Baptiste Perrin (1870-1942), who later received the Nobel prize for this work. What Einstein did was to set up a precise mathematical model for the motion that connected surmised physical properties of the atoms to something measurable.

The concept of what is nowadays called Levy processes was introduced and studied in great detail, with many new and deep mathematical results, by Paul Levy (1886-1959), Professor at Ecole Polytechnique in Paris. Levy processes are build up from independent random variables and the probabilistic character of such variables is embedded in the Levy-Khintchine formula, discovered about the same time by Levy and by the Russian mathematician Alexander Y. Khintchine (1894-1959), Professor at Moscow University, who was one of the most significant researchers in the early days of the Soviet School of Probability.

More details on Levy's life can be found as a Foreword to Volumes in the book series 'Levy Matters', which was initiated by Jean Bertoin, Jean Jacod, Claudia Klüppelberg and the author, as a depository for developments in Levy theory that may go beyound what is usually published in scientific journals.

Starting around 1990 there was surge of interest internationally in the theory of Lévy processes, to a large extent rooted in the noble French tradition for research in Probability, with illustrious names from the past such as Pierre de Fermat (1601-1665) and Pierre-Simon Laplace (1749-1827).

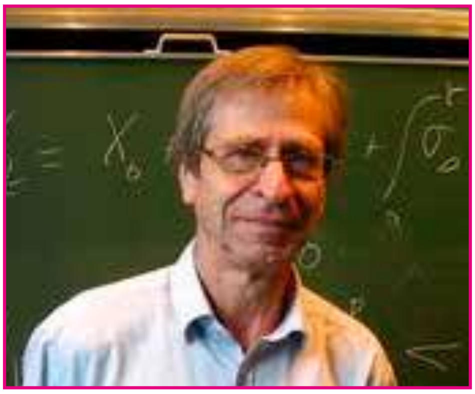
**Pierre de Format** [2]

**Pierre-Simon Laplace** [3]

In the mid 19-th Century Paul Lévy (1886-1971), Professor at Ecole Polytechnique, was a major figure in the field of Probability Theory with many basic contributions, in particular to what has become known as Lévy theory.

But it took some time before the importance of Lévy processes for Stochastic Calculus and its applications was more widely appreciated.

From my own work on infinite divisibility, growing out of the introduction of the generalized hyperbolic distributions, I was naturally led to join the efforts to develop the field of Lévy processes and their ramifications. In this I was most fortunate to be in contact not only with Gerard Letac but, via the Toulouse-Aarhus collaborations, also with Jean Jacod, Francois Bardou and Jean Bertoin, all leading French probabilists.



**Jean Jacod** [4]

**Jean Bertoin** [5]

These interests and connections were of major importance in relation to the formation of the MaPhySto Centre under which Lévy theory was one of the main topics. Jean Jacod, Francois Bardou and Jean Bertoin provided advice on the research plans for the Centre and participated strongly in its life, as did Gerard Letac.

Another leading figure in the development of the theory of Lévy processes, and indeed in the life of MaPhySto, was Ken-Iti Sato whose monumental monograph 'Lévy Processes and Infinitely Divisible Distributions' appeared in 1999.

**Ken-Iti Sato visiting Aarhus**

At the base of modeling by infinite divisibility lies the concept of Lévy processes, that is processes with independent identically distributed increments. More generally, when it comes to tempo-spatial modeling the key concept is that of Lévy bases; these are random measures that endow finite regions of space-time with random quantities in a homogeneous fashion and such that for disjoint regions these quantities are stochastically independent.

This step from a purely temporal to a tempo-spatial setting opens the way for basic and very detailed and realistic modelling of a wide range of dynamic phenomena, of which turbulence has been and is a main area for study. In particular this has led to the discovery of new universal laws in the Statistical Theory of Turbulence, as described in Section 33. The research in this field falls under the heading Ambit Stochastics. The latter field is discussed in Section ***.

### supOU – an illustrative scenario

Ornstein-Uhlenbeck, or OU, processes are processes that develop in time in a stationary manner. Such a process is driven by a Levy process. In particular, that process may be Brownian motion, the case considered by Leonard Ornstein[2] and George Eugene Uhlenbeck who proposed this as a model for the velocity of a massive Brownian particle under the influence of friction.

OU processes may take only positive values and as such they are potential candidates for modelling volatility or intermittency and have in fact been used for that purpose.

However, in this regard it was recognised that they were lacking in representing the often seen phenomenon that the value at any given time of an empirically determined volatility process depends significantly on previous values far back in time. Thus a suitable generalisation of the concept of OU processes was called for.

By suitable is here meant that the generalised type of process should not only be capable of reflecting the long term dependence but should have a mathematically tractable form, if possible having the property of infinite divisibility.

A generalisation of this kind was proposed in [BN (1998), (2001)] under the name supOU (superposition of OU processes), via the introduction a new concept of 'Levy mixing'.

This type of process has since been applied in many studies in financial econometrics, the area that was my original focus.

A recent unexpected application has been to active galactic nucleus quasar black hole mass estimation, discussed in a very detailed study, published in The Astrophysical Journal 2013 [Kelly, Treu, Malkan, Pancoast, Woo (2013)].

This illustrates the potential of focused modelling to yield widely applicable research tools.

## 18.2 LASER COOLING

A major impetus for the surge of interest in Lévy processes was the revolutionary research on the cooling of atoms that in 1997 lead to the Nobel Prize in Physics being awarded to the three physicists Steven Chu, Willam Phillips and Cohen-Tannoudji. In experimental setups, where a number of laser beams where directed towards a point in space around which a cloud of atoms was held largely concentrated by means of a magnetic field, it was possible to decrease the momentum of the atoms thereby reducing the temperature to a staggeringly low level at the order of nano Kelvins, i.e. near the lowest level possible for temperature (zero Kelvin). The leading journal for the announcement of such break throughs 'Nature' reported on this on the front page as 'Cycling at the speed of light'. The basic explanation of this behaviour rests on quantum theory.


**This photo from my blackboard shows the poster announcement from the Nobel Foundation of the Nobel prize. (In the upper right hand corner of the photo is a 'Kartoffelkind', a popular type of souvenir from the neighbourhood of the Mathematisches Forschungsinstitut Oberwolfach.)**

The techniques of laser cooling opened the way for numerous studies of fundamental questions in particle physics, for instance in regard to atom optics, atom interferometry, atomic clocks, and high resolution spectroscopy.

The lasers have the effect of making the momentums (velocity vectors) of the atoms jump but in such a way as to keep the momentums close to 0. The behaviour is random and proper understanding of what happens calls for probabilistic modelling.

Such modelling turned out to be instrumental in raising the efficiency of the cooling and a key to that was viewing the jumps a 'Lévy flights', i.e. as three-dimensional Lévy processes.

The main contributors to this type of modelling were Claude Cohen-Tannoudji, Jean-Phillipe Bouchaud, Alain Aspect [3] and Francois Bardou. An account of their works was given in their book 'Lévy Statistics and Laser Cooling: How rare Events Bring Atoms to Rest' – a masterpiece, non too technical, of exposition of a fruitful interplay between Stochastics and Physics.

My first PostDoc under MaPhySto was Fred Espen Benth and together we studied to understand the work of Bardou, Cohen-Tannoudji, Aspect and Bouchaud and that led us to consider the relation of that to the concept of renewal and Markov jump processes. The theoretical considerations in the work of Cohen-Tannoudji, Bouchaud, Aspect and Bardou were, as is common in Physics, based various simplifying Ansatze which, as the authors pointed out, could likely be relaxed with the prospect of added efficiency in the cooling process. Motivated by this Fred Espen Benth and I, together with Jens Ledet Jensen, wrote several papers that under other, but related, mathematical assumptions made it possible to give more detailed descriptions of the time an atom spent in a given state and of the probability distribution of the momentum (cf. Section 10.2.1 in 'Lévy Statistics and Laser Cooling: How rare Events Bring Atoms to Rest', [Bardou, Bouchaud, Aspect, Cohen-Tannoudji (2001)]).

# 19 ON THE MEANING OF INDEPENDENCE

Now, one could take a step backwards and ask: In an intuitive sense, what meaning might one give to a concept of independence and can that be given a mathematical formulation; and in that case is there a natural definition of infinite divisibility.

Intriguingly this question has a precise answer according to which there are precisely five types of independence (of which one it the classical probabilistic). One of these has associated with it a concept termed free probability and one speaks of free independence and free infinite divisibility. The mathematical specification of the framework of free probability (and more so of the other three, nonclassical, independence structures) is very abstract and seems at first very foreign.

In 1999 it was shown by H. Bercovici and V. Pata that there exists a one-to-one correspondence between the classes of infinitely divisible laws in the classical and in the free sense, respectively, i.e. to any classically infinitely divisible law there corresponds one and only one freely infinitely divisible law.

I was fascinated by this fact and started speculating to what extent some of the properties of ordinary infinite divisibility on which I had been working might carry over to the free probability setting. The mathematical theory of free probability involved (and still involves) very abstract and advanced mathematics, particularly from algebra, operator and group theory with which I was not familiar. But my friend and colleague Steen Thorbjørnsen has great expertise in those areas and we started a collaboration that has been going ever since, our first joint paper being from 2002. Much of this work was reviewed in [BN and Thorbjørnsen (2006)].

One of our early results was a formula expressing the cumulant function of a freely infinitely divisible law as a simple integral with respect to the cumulant function of the corresponding – in terms of the Bercovici-Pata bijection – classically infinitely divisible law. This made it possible in a simple operational way to establish a large number of results regarding classes of freely infinitely divisible laws, as discussed in [BN and Thorbjørnsen (2006)].

# 20 QUANTUM STATISTICS

> *Attempting to understand quantum mechanics causes you to fall into a black hole, never to be heard of again.*
> Richard Feynman

Quantum Physics and the associated aspects of probability must hold a deep fascination for anyone interested in the principles of classical statistical inference. That was certainly the case for me.

But the intricacies of the inherent physical concepts, the differences to ordinary realism, and the advanced mathematics required – which is unfamiliar to most traditionally educated statisticians – makes the field fairly inaccessible.

In hindsight, and in view of my interests in statistical inference, it seems inevitable that at some stage I would be bound to toy with statistical and probabilistic aspects of quantum physics. A cue for this was the 1997 Nobel prize in physics, awarded to three physicists for their development of methods to cool and trap atoms with laser light; as discussed in Section 18.2.

Also, having grown up near the Niels Bohr Institute in Copenhagen and having studied mathematics there provided an affinity to quantum physics.

**At the Aarhus Harbour with Richard Gill, a student of Richard's and Peter Jupp**

I was especially interested in what roles the concepts of sufficiency, ancillarity and exponential families, from nonquantum statistical theory, might have in a quantum setting.

My efforts to gain some understanding of the basic tenets of quantum physics and its associated concept of quantum probability were strongly helped by intensive discussions with Klaus Mølmer, Professor at the Aarhus University Institute of Physics and Astronomy, and with Richard Gill, Professor at Department of Mathematics, Leiden University.

Some time later Peter Jupp joined Richard and me in pondering the role of statistics for quantum systems and this led to an effort to give a systematic account of some of the recent developments in the area, in a form that would make it relatively easy for some of our colleagues in mathematical statistics to get an impression of the achievements and challenges in the field. A result of that effort was a paper [BN, Gill, Jupp (2003)] presented to the Royal Statistical Society as a read paper, which was followed by an extensive discussion.

We had hoped to develop that paper into a book. However, the book project never came to fruition, due partly to my becoming Director of MaPhySto and partly because Richard became absorbed in a heroic - and successful! - effort to exonerate a Dutch nurse Lucia de Berk from accusations of having killed a number of patients, on the basic of which she was convicted and sentenced to life imprisonment, for which there is no parole under Dutch law. She was in jail from 2003 to 2010 when she was set free after a new trial had ruled injustice. This outcome was due primarily to Richard Gill´s efforts, in which, among other points, he pinpointed misuses of statistical evidence.

**'Schroedinger's Kittens'. Gift from a Japanese colleague.**
**The Yin and Yang symbol nicely reflects the nature of quantum physics, cf.**

# 21 DIFFERENTIAL GEOMETRY AND INFERENCE

Early in the 1980-ies Shun'ichi Amari caused a significant stir in the community of theoretically oriented mathematical statisticians by publishing and lecturing on his recent work connecting advanced aspects of differential geometry with basic concepts of statistical inference and especially the role of exponential families and conditioning. His results brought to full blossom some pathbreaking results from the mid 1970-ies by Bradley Efron and others, that had their roots in some of Fisher's ideas and claims and work by C.R. Rao in the early 1960-ies.

Professor Shun'ichi Amari's main area of research is information geometry and its applications to Brain- and Neuroscience. He has for a long time held leading positions at the RIKEN Brain Institute. RIKEN, founded in 1917, is a Japanese research organization encompassing a network of world-class centres and institutes across Japan.His basic studies were in Mathematical

Engineering and it was already there that his work on information theory and geometry began. But it was not till the early 1980-ies that his research on the relation of these fields to statistical inference became widely known outside Japan. David Cox and I became fascinated by this new vista for likelihood based inference and in 1984 we organised the first international workshop on differential geometry and statistics, at Imperial College in London. It was there that I met Shun'ichi for the first time and where our friendship began.

**Visiting the home of Professor Amari**

The abovementioned developments became instrumental for some of my own work on ancillarity and the distribution of the maximem likehood estimator. What sparked my interest was mainly the possibility I saw for the light the new theoretical results might throw in two areas of my research: likelihood inference and asymptotic expansions. However, this avenue required spending much time and effort in learning the relevant aspects of differential geometry, including the new purely geometrical insights proposed by Amari. Central to much of this was the key geometrical concepts of connections.

This led me to introduce a new concept termed 'observed likelihood geometries'. Earlier work in the field was focused on 'expected geometries' determined by expected (mean) values of the objects concerned. The observed geometries on the other hand build directly on the data and draws on the ideas of ancillarity and conditioning. For those in the know of differential geometry the adduced citation from a paper in the Proceedings of the Royal Society may give an impression of some of the concepts involved.

*Proc. R. Soc. Lond.* A **411**, 155–176 (1987)
*Printed in Great Britain*

# Strings: mathematical theory and statistical examples

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(*Communicated by Sir David Cox, F.R.S. – Received 27 August 1986*)

*Strings*, in the sense of the present paper, are sequences of multiarrays with two types of indices; tensorial and structural, and they are characterized by a transformation law that generalizes those for tensors, affine connections and derivatives of scalars. The original definition Barndorff-Nielsen (*Proc. R. Soc. Lond.* A **406**, 127–137 (1986)) is here extended and a systematic study of the mathematics of strings is undertaken. In particular, a *convolutive multiplication* of strings is introduced and is used in the discussion of *intertwining*, a type of operation that produces tensors from strings and strings from tensors and *connection strings*, a special kind of string. It appears that tensors and connection strings have a role, respectively, as 'coordinates' and 'coordinate frames' in the calculus of strings. A definition of 'covariant differentiation of strings' is proposed and is related to convolutive multiplication and to intertwining. The general theory is illustrated by various examples from the context of statistical inference. Finally, a brief comparison is made between strings and the somewhat related concept of extensors.

## 1. INTRODUCTION

In Barndorff-Nielsen (1986*b*) a differential-geometric concept termed strings was introduced and studied to some extent, and various statistical examples and applications were given. A string is a sequence of multiarrays that satisfies a certain transformation law (see (2.1) and (2.5)), and the concept encompasses those of tensors, (affine) connections, and successive derivatives of scalars.

The present paper extends the definition of strings by abandoning a symmetry condition and it discusses further properties of and operations on strings; associated statistical illustrations are also presented. In particular, a definition of covariant differentiation of elements of strings is proposed and this leads to a concept of canonical strings, which are obtained by successive covariant differentiation.

The interest in the relation between Differential Geometry and inference (in a broad sense) and its many applications has grown greatly since the early period. A manifest sign of this is the creation of a new mathematical journal 'Information Geometry', published by Springer. The scope and aims of the journal are set out in the following quotation, taken from the journal.

## ENDNOTES

1 Augustin Louis Cauchy (1789-1857) is among the most famous French mathematicians of the Nineteenth century, a pioneer in particular of mathematical analysis.

2 Leonard Salomon Ornstein (1880-1941), a Dutch physicist, was Professor at University of Utrecht. George Eugene Uhlenbeck (1900-1988) was a Dutch-American physicist holding positions as Professor at a number of Dutch and American Universities, including Columbia University and the Rockefeller Institute.

2 Alain Aspect is in fact most well known from the experiment he carried out, while still a PhD student, showing that Niels Bohr, and not Einstein, was right in his interpretation of the basic nature of quantum behaviour. Together with Boris Podolsky and Nathan Rosen, Einstein had constructed a thought experiment – the EPR paradox – which if it could be carried out would decide whether Bohr's thesis of 'spooky action at a distance' was correct or not. In colloquial terms the thesis says that if one has a pair of photons and one of them changes its quantum state then the other will change its state at exactly the same moment irrespective of their distance, even if the distance is so large that the photons could not 'communicate' by the speed of light.

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BERNOULLI
SOCIETY

Among the most pleasant, stimulating and scientifically rewarding activities in my scientific life has been my involvement with the Bernoulli Society for Mathematical Statistics and Probability, in its formation and later life.

# 22 THE ORIGIN OF THE BERNOULLI SOCIETY


**Signatures of BS presidents**

Up through the late 1960-ies and early 1970-ies there was, within the International Institute of Statistics (ISI) and also outside, a growing feeling of necessity to invigorate and greatly extend the Institute's range of activities, particularly in regard to the increasing importance of advanced theories in applied and theoretical probability.

This led to creation of an, largely independent, 'International Association for Statistics in the Physical Sciences' and a 'Committee for Conferences on Stochastic Processes', later accompanied by a new journal carrying the name 'Stochastic Processes and Their Applications'. Another aspect of these concerns was the establishment of the 'European Meetings of Statistics'.

In 1975, following a debate at the 1975 meeting of ISI in Warsaw, these new activities were formally combined and extended under the umbrella of the 'Bernoulli Society', with David Kendall as its first President. One of his first initiatives as President was to write to the Bernoulli family asking permission to adopt the Bernoulli heraldic tinctures for the new Society. Permission was

*Bernoulli* **1(1/2)**, 1995, 001–002

# The foundation of the Bernoulli Society

DAVID G. KENDALL

*Department of Pure Mathematics and Mathematical Statistics, Statistical Laboratory, University of Cambridge, Cambridge CB2 15B, UK*

I have been asked as the founding President of the Bernoulli Society to write a few words welcoming the first issue of our journal *Bernoulli* and to recall some of the circumstances of the Society's creation.

For many years the only international statistical society was the International Statistical Institute, but its membership was rather small and consisted to a great extent of government statisticians: mathematical statisticians could be and occasionally were elected, but they remained as it were a fringe. Things began to change after World War II, when one began to see names like that of Peter Whittle appearing in the list of members, but the actual meetings of the ISI had rather little to offer them. It was Jerzy Neyman who started the campaign to put things right by creating a sister or daughter body as a home for statisticians in the broadest sense – one that would be open to any serious candidate, and that would impose no 'national quotas'.

The time soon came when we perceived a ground swell of support for action in this direction, but it became clear that a step-by-step procedure would be necessary. The first move was to persuade the ISI to establish a Section for 'Statistics in the Physical Sciences' (IASPS) with a relaxed attitude to election procedures. But other events had a great influence also, notably the creation of the 'European Meeting of Statisticians' and the 'Conference on Stochastic Processes and their Applications'. Each of these groups had its partisans, and a little time had to pass before it was possible to set up a comprehensive solution to the general problem.

Neyman already realized what form this would have to take, and made his target the creation of a Bernoulli Society – an inspired choice because the Bernoullis as a group had betweeen them such a wide range of interests. By the time of the Warsaw meeting of the ISI in 1973 there was a very general demand for such a body, and in Warsaw it was born.

Symbolically during that meeting a Nova (new star) flashed out in the Polish heavens. I enjoyed pointing this out to some of the delegates at midnight. They did not all believe me, so we agreed to assemble again at the following midnight to see if (as I had predicted) the star would by then have become fainter, or have disappeared. Happily the Nova behaved as predicted, and having done its work, vanished.

Immediately afterwards the Assembly of the ISI voted the Bernoulli Society into existence as a Section of the ISI accommodating all the other statistical groupings that had been in search of a home. I became its first President with the daunting task of writing appropriate Statutes together with Bart Lunenberg (a veritable tower of strength) – and that was that.

Now at last we are to have our own journal, and one can confidently predict that it will be a dazzling success, supported as it is by a wealth of good will.

One of the most pleasant of my duties as President was to locate and write to the Bernoulli family



readily granted and followed up by presenting to David Kendall a book on the history of the Bernoulli family. He then started the tradition of having the successive presidents of the Society writing their signatures on the end-papers of the book.

For more details, see the article on the 'History of the Bernoulli Society' by Jef L. Teugels, which appeared in 'Bernoulli News' on the occasion of the 25th Anniversary of the Society.


**Campaigning for the Bernoulli Society on the tennis court, summer 2014.**

# 23 THE FIRST WORLD CONGRESS OF THE BERNOULLI SOCIETY

One of the major events in the life of the Bernoulli Society was the first World Congress of the Society which was held in Tashkent in 1986, with Yuri Vasilyevich Prokhorov as the main organizer and with strong support in particular by Professor S. Kh. Sirazhdinov who was in charge of the local organization. This was first ever fully international conference embracing the whole span of probability and mathematical statistics and had an outstanding scientific programme as well as a fantastic excursion – by airplane! – to the marvels of Tashkent. Most remarkably, accommodation for a number of those most heavily involved in the Society had been reserved in the residential compound of the local Government, where Leonid Brezniew stayed when visiting Tashkent. Each of us were given a large apartment with Persian carpets everywhere, and our transport to and from the Congress events we secured by the Prime Minister having made available around 60 cars with drivers for the invited speakers.

How this came about is described in the following citation from the interview with Albert Shiryaev on the preparations for the Conference, published in Bernoulli News, Volume 21-#2-2014:

(…) I arrived in Tashkent three weeks before the congress. We soon understood that there were problems with accommodation. Even those hotels that promised to accommodate the participants of the congress informed us that they had reduced the number of available rooms. In fact, at that time Gorbachev had started his struggle with corruption in Uzbekistan (the so-called Gdlyan-Ivanov deal) and a great number of the investigators in charge arrived in Tashkent and occupied the rooms. So, it was necessary to resolve this problem.

As sometimes happens, a random event helped us. I returned to the hotel, or, to be more precise, to the residence in which I stayed with my son who was thirteen at that time (this was a residence of the Government of Uzbekistan and there were only three guests: I, my son and cosmonaut Dzhanibekov). It was a late evening and I saw that my son pointed out something in the sky to a well-dressed man. I should point out that my son had a great interest in astronomy and knew a great deal about stars and planets. The man invited us to his nearby dacha and asked me what I was doing in Uzbekistan. I explained to him emotionally the situation with accommodation and spoke of how the congress would be an important and prestigious event for Uzbekistan and the USSR. The man replied, "OK, I think that I am able to help you. Please, come to me tomorrow". I asked him: "Where to?" He pointed at two silent men who had been following us all the time: "These two men will explain." It turned out that this was Usmankhodjavev – the First Secretary of the Communist Party of Uzbekistan at that time!

The next morning all our problems with the accommodation were resolved: we were given the residence of Brezhnew in Uzbekistan and a very comfortable hotel "Shelkovichnaya". Moreover, we were supplied with 40 black "Volga" cars, very prestigious Soviet cars. These cars (with drivers!) were attached to the members of the International Program, and Local Committees as well as to the members of the Executive Committee of the Bernoulli Society. (…)


**Photo of bowl presented to participants of the First World Congress of the Bernoulli Society in Tashkent 1986.**

The Congress was opened by welcoming words by Kolmogorov, who was Chairman of the organizing Committee, but was too ill to attend, and by Shiryaev.

Their welcoming words can be found on the Bernoulli Society website under: http://www.bernoulli-society.org/index.php/history/53-general/203-opening-words-of-the-first-bernoulli-congress.

As reported there the Congress had 35 scientific sections, 100 forty-minute talks, 181 fifteen-minute contributions, 430 stand posters, 15 non-formal discussions, 3 round tables on the topics: "Computational methods and tools in theoretical and applied statistics", "Relationship between theory and applications", and "Historical aspects of development of probability theory and mathematical statistics".

# 24 BERNOULLI

In 1990 it was decided by the Council of the Bernoulli Society to start an own scientific journal, to be called *'Bernoulli'.* The ambit of the journal is Mathematical Statistics and Probability, in short Stochastics.

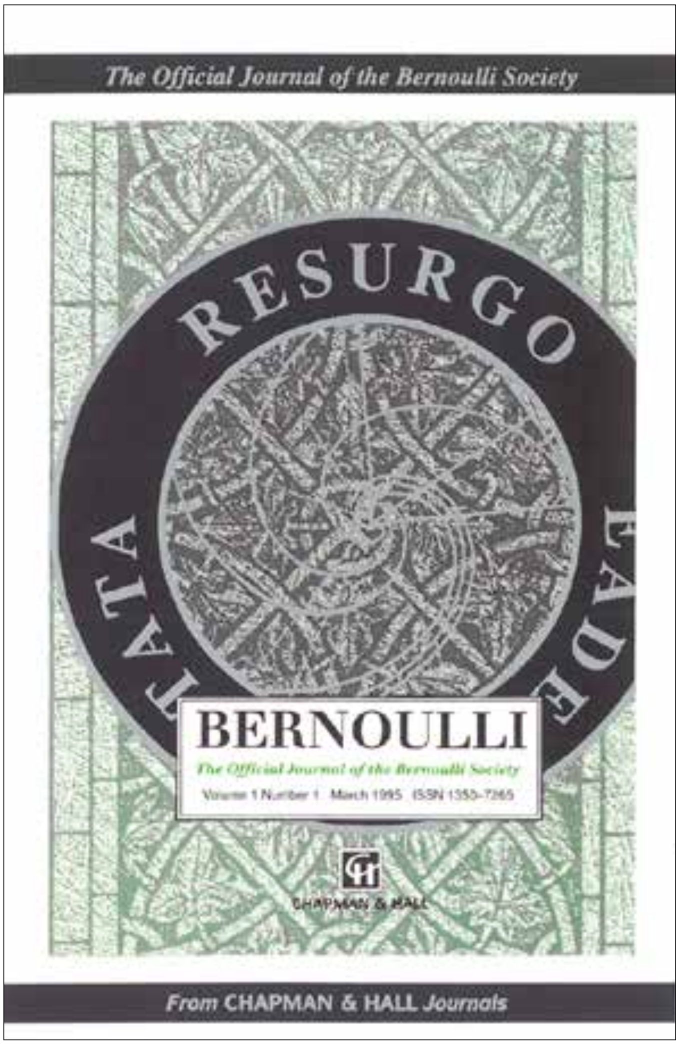
I was asked by the Council to act as the founding Editor of 'Bernoulli'. In spite of my enthusiasm for the creation of the journal, it was with some reluctance that I accepted, finding it embarrassing to take up this challenge while at the same time being incoming President of the Society, a point that I mentioned in my opening address as President at the Conference in Istanbul 1993 and where I could cite from a letter written by the outgoing President Peter Bickel stressing the unanimous decision of the Council in this matter.

In designing the front cover for 'Bernoulli' it was natural to seek to relate it to Jacob Bernoulli but without being too specific about his achievements. What came to mind was his fascination with the logarithmic spiral, or *Spira Mirabilis* as he called it. He was not the first to consider this mathematical curve, it was introduced by Descartes, but Jacob Bernoulli focused on the 'self similarity' of this spiral, i.e. the property that if you zoom in on a picture of the curve it looks the same.

Something close to this kind of self similarity is, in fact, seen in many contexts in Nature. Spiral galaxies and extratropical cyclones show this pattern, and it is seen in biological growing forms, for instance in nautical shells and in sunflower heads.

Whirlpool Galaxy

Cutaway of a nautilus shell




Front cover of Bernoulli

Jacob Bernoulli wanted to have the spiral engraved on his tombstone together with the words 'Eadem Mutata Resurgo', that is 'Though changed I shall rise again'. His wish was followed though the form of the spiral there is far from being self similar.

The front of the 'Bernoulli' cover, which was designed in consultation with an artist at the publisher Chapman and Hall, is reminiscent of that part of his tombstone and is in the Bernoulli family colours.

It seems to me that the citation 'Eadem Mutata Resurgo' eminently represents the unique character of Mathematics, i.e. its inherent capability of generating new concepts and theories of basic importance.

The frame shows one of the introductory pages from Volume 1, Issue 1 (1995) of Bernoulli. The lists of Editors and Associate Editors testifies to the degree of backup and enthusiasm with which the initiative of the Bernoulli Society to start its own journal were met.

I find it difficult to imagine how the journal could have gotten off ground in a convincing way without the invaluable assistance of the managing Co-Editor Jens Ledet Jensen and our Secretary Oddbjørg Wethelund.

Review for Nature.
Best wishes,
Anthony

## James, Nicholas and Daniel

*A.W.F.Edwards*

**Bernoulli: Official Journal of the Bernoulli Society for Mathematical Statistics and Probability.**

It is a bold step to claim the name *Bernoulli* for a journal limited to mathematical statistics and probability, for the members of this famous Basle family made many other important contributions to mathematics, including inventing the calculus of variations (in deriving the catenary), polar coordinates, the Bernoulli numbers of analysis (actually due to Johann Faulhaber in 1631, but then forgotten), and the Bernoulli equation of hydrodynamics. Yet there is some justification, for James (Jacob; 1654-1705) gave the first limit theorem in probability in his posthumous *Ars conjectandi*, Nicholas (Nikolaus; 1687-1759) substantially improved the limits of his uncle's theorem, and Daniel (1700-1782) was one of several people who made early proposals of what is now known as the method of maximum likelihood. The new journal, moreover, is the official journal of the Bernoulli Society, founded in 1973 as a section of the International Statistical Institute.

The many existing journals devoted to probability and statistics will soon feel the effects of the newcomer, for *Bernoulli* has started strongly under its distinguished editors, and is exceptionally well-printed on high-quality paper (oddly, the name of the printer is nowhere given). The early issues carry a promising mix of papers on stochastic processes and the mathematics of statistics, both areas which are developing strongly at the present time, with continuing major implications in many branches of science. There are no book reviews or (as yet) letters, but one or two papers are followed by discussions.

In an introductory note D.G.Kendall, the first President of the Society, writes 'This is a Society that one confidently feels will live for ever'. The same can be said of their journal.

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**WEBSOURCES**

https://mcdonaldobservatory.org/sites/default/files/images/news/gallery/m51.jpg

https://upload.wikimedia.org/wikipedia/commons/0/08/NautilusCutawayLogarithmicSpiral.jpg

# LATE MIDDLE YEARS

# 25 CIMAT — AARHUS

## 25.1 AN INVITATION

Shortly before leaving for Japan in 1993 I received an invitation to a Workshop on Biostatistics and Statistical Inference to be held in March at a place called CIMAT in Mexico that I had never heard of. The organisers were Professor David Sprott, whom I knew well, and another person whom I had never heard of. The topic of the workshop was of interest to me and the prospect of having the chance to visit Mexico was irresistible. Even more so after I had visited the home page of CIMAT and discovered that it was the Mathematical Research Centre of Mexico, located near the historical town of Guanajuato.

Fortunately, it was feasible for me to accept the invitation. That was the start of my close friendship and collaboration with the 'unknown person' Víctor Pérez-Abreu who was then Scientific Director of CIMAT.

CIMAT is located on the slopes of a small mountain near the historical town Guanajuato. It has a fascinating and pleasing architecture. I have been told that the architect said ''You mathematicians love Escher (the famous Dutch lithographical artist) so I will build you an Escher house'', and indeed it gives much of such an impression both inside and from the outside. And the working conditions for doing mathematics are excellent.

Outside and inside CIMAT

## 25.2 VÍCTOR PÉREZ-ABREU

Víctor Pérez-Abreu pioneered the building up of research in Statistics and Probability in Mexico, in particular in relation to CIMAT, the Centre for Research in Mathematics in Mexico, with which he was associated from 1987. He has been very active on the international scene in Stochastics, among other roles as Vice-President of the International Statistical Institute and as President of the Bernoulli Society for Probability and Mathematical Statistics. He has organised numerous international research meetings and was one of the driving forces in establishing the International Year of Statistics. Moreover, he has served as Editor or Co-Editor of a number of leading international journals.

OLE: Er det ikke muligt at kontakte Víctor og bede ham sende et portrætfoto i høj opløsning?

Interspersed between my visits to CIMAT, Víctor Pérez-Abreu has made numerous visits to Aarhus University, collaborating among other things with Steen Thorbjørnsen and me on free probability.

During one of these visits I was committed to give a talk at the University of Vienna. This gave Bente and me a welcome opportunity to see our grandson Rasmus who, in connection with his studies at Aarhus University, was spending some months at the Danish Embassy in Vienna. Victor and Oddbjorg joined us and we had some marvelous days there, including watching a performance of Götterdämmerung given at the Staatsoper, but in this case we were outside, the performance being also shown on a huge screen on the side of the opera building; regrettably we had to cut our viewing short as it was both raining and cold.

However, most noteworthy, there is the fact that in connection to a visit by Victor during his term 2009-2011 as President of the Bernoulli Society he took initiative to have a film made, entitled 'Stochastics in Science: The Aarhus Sand Project', describing the studies of sand transport carried out at Aarhus University in the wake of my contact and collaboration with Ralph Bagnold, as described elsewhere in these notes.

Among the students of Victor who have spent time in Aarhus are Ulises Marquez and Orimar Sauri, as Ph.D. students and Orimar subsequently as postdoc. They both came well prepared, having been thoroughly introduced to Ambit Stochastics (see Section XIII) under guidance of Victor.

## 25.3 CONFERENCE ON STOCHASTICS IN SCIENCE 2006

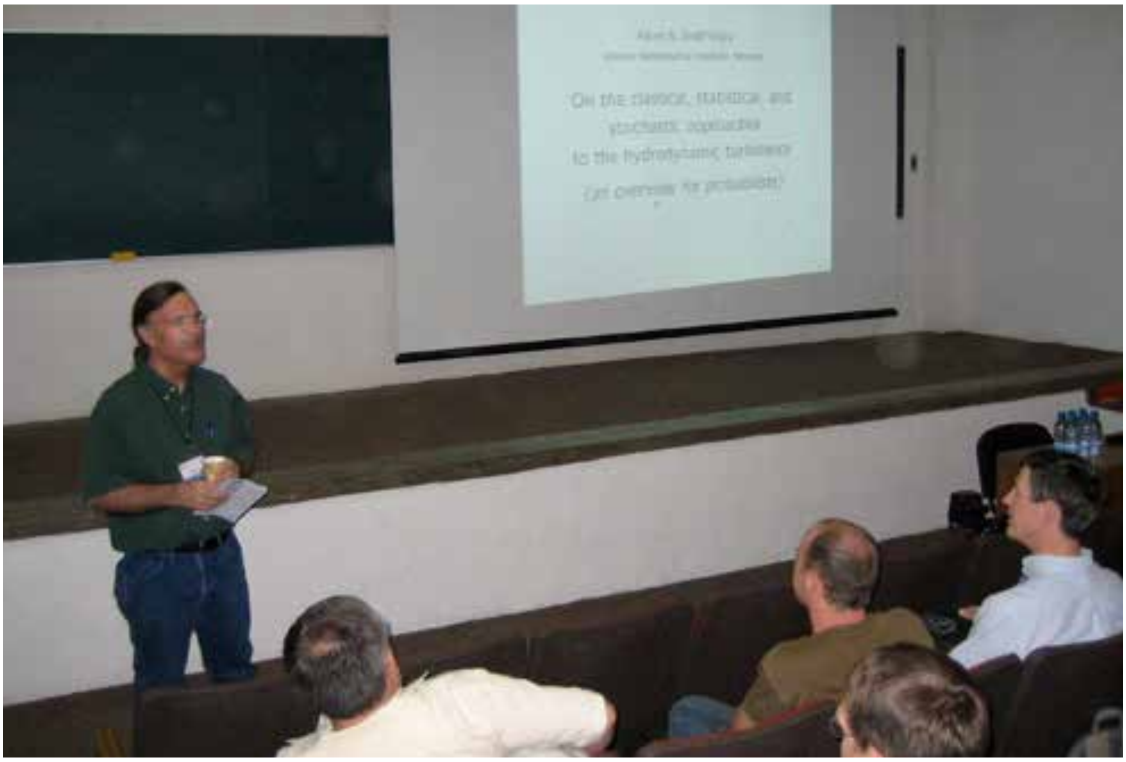
On the initiative of Víctor Pérez-Abreu and his colleagues at CIMAT an international Conference on 'Stochastics in Science' was held at CIMAT in Guanajuato.

A purpose of the Conference was to focus on aspects relating to my work and interests in the area of Stochastics in Science. Many leading probabilists and statisticians convened for the Conference, which turned out to be a marvelous occasion – scientifically, culturally and socially.

To be together with so many outstanding senior and junior colleagues and close collaborators made this event a high point in my life.

With Claudia Klüppelberg and Jean Jacod

With Friedrich Hubalek, Jeanette H.C. Woerner and Neil Shephard (2003)



**Ed Waymire introducing the key note talk by Albert Shiryaev**


**Albert Shiryaev presenting his talk on 'The Classical, Statistical and Stochastic Approaches to the Hydrodynamical Turbulence'**

The Sponsors of the Conference

**Callejoneada**

Guanajuato is a charming city nestled in the mountains of the Sierra de Guanajuato. Abundant silver deposits were discovered by the Spanish in the 1500's, and the vast fortunes extracted from the capricious coal mines built the colonial city of Guanajuato which was officially declared a city in 1741. The city played a significant historical role in 1810 when the Alhondiga de Granaditas was the site of a fierce battle between rebels and royalists, beginning the struggle for independence. Guanajuato was named a World Heritage Site by UNISCO in 1988.

A unique cultural experience in Guanajuato – the callejoneada, a musical tradition imported from Spain, featuring local university students dressed as medieval troubadours. Playing music on stringed instruments, the ensemble – known as an estudiantina – leads the people through a song-guided tour of the beautiful narrow, colonial streets of the city. This activity is sponsored by the Municipality of Guanajuato.

A popular tourist attraction, the estudiantina owes its popularity to a character named "Tuno", who creates a world of magic and surprise through romantic, jolly and picaresque music, full of his good spirit, with the city providing a magnificent backdrop.

During the walk-around the participants were presented with ceramic jugs that were amply supplied with red wine.

**Jürgen Schmiegel and Kristiana Jonsdottir with their wine jugs**

**Neil Shephard, Mark Podolsky and Peter Tankov**

Fernando Avila, Zaida Gonzalez, Albert Shiryaev and Jürgen Schmiegel

Neil Shephard,
Jean Jacod and
Michael Sørensen


Nuffield College
Oxford OX1 1NF
3 April 2006

Dear Ole,
I was very disappointed not to come to your meeting at CIMAT. As was to be expected I hear that it was a very happy and fruitful occasion.
As I hope you have always known, I value the period when we worked intensively together as a high-point in my life, for scientific and personal reasons.
With very best wishes to you, Bente

David

**Letter from David Cox**

# 26 CHANGING TIMES

> *Come writers and critics, who prophesize with your pen*
> *And keep your eyes wide, the chance won't come again*
> *And don't speak too soon, for the wheel's still in spin*
> *And there's no tellin 'who that it's namin'*
> *For the loser now will be later to win*

As recounted in Chapters 1 and 2, Statistical Inference in the sense of R. A. Fisher, with the tenets of likelihood and sufficiency and ancillarity and associated model classes, was a major research area of Mathematical Statistics in the period from 1950 to 1995. The methods in question arose initially out of the need to extract a maximum of insight into problems studied from experimental data of limited extent, sometimes down to a few hundred observations. The basic concepts are so rich that they led to the build up of mathematically advanced and sophisticated theoretical structures.

However, the development of Science generally, with the greatly expanded possibilities for extended experiments and observations, and for storing and managing enormous data sets, poses a whole new range of problems concerning inference, where the classical concepts and methods are no longer sufficient. The modern byword for this field is 'Big Data', but the term is unfortunate as it seems to indicate a scientific area of some established substance whereas the term is close to vacuous, covering an extreme wide set of issues that bear little or no relevance to each other. Nevertheless, the questions of how best to extract valid conclusions from large data sets poses a major challenge to the scientific communities.

To develop concepts and methods of a significance similar to the classical the way ahead must be to select some focal areas of very substantial scientific interest where mathematical concepts are judged to have a major potential effect.

In addition to Stochastics, mathematical fields like geometry, topology and graph theory are called on as tools for the inferential analysis of complex data.

Throughout the scientific world initiatives of a similar kind are under development and these promise to lead to exciting new developments of much scientific interest and importance for Society.

# 27 WIEN — AARHUS

My first professional contact to Wien was a visit to the Zentralanstalt für Meteorologie und Geodynamik (also known as Hohe Warte) around 1980, that arose out of my work on turbulence and hyperbolic distributions.

A good deal later, around 2001, this time from my work on Financial Econometrics with Neal Shephard, I became acquainted with Walter Schachermayer who at that time was Professor and Chairman of the Institute of Financial Mathematics at the Technical University (TU) Vienna while from 2008 he has held a Professorship at the Mathematical Faculty. Universität Wien.

At the TU Institute I met a PhD student of Walter's, Friedrich Hubalek, with whom I shared interest in various problems related to Financial Econometrics. This led to our working together on those problems and, significantly, to a long term affiliation of Friedrich to the Institute of Mathematical Sciences, Aarhus University, as Associate Professor.

Conveniently the TU Wien is situated near the centre of Vienna, close to the Naschmarkt, the Secession, the Hofburg and the Wiener Staatsoper.
Of the many attractions of Wien the Staatsoper has been of particular joy to Bente and me, with many splendid instances.

In June 2013 Bente and I were in Vienna for a few days, in connection with my participation in an evaluation for the Austrian Research Council, and we had been fortunate enough to obtain tickets for a performance at the Wiener Staatsoper of Gounod's 'Romeo and Juliette', with Placido Domingo as conductor. We took tram D from our hotel to the Oper in what we thought were ample time. We missed the first connection and the next was a bit late and only arrived some 7 or 8 minutes before the performance was to begin, which is still quite sufficient time when you are on the Ringstrasse just across from the Oper. Unfortunately, there was a traffic jam there with several trams ahead of ours that could not draw up to the step off platform so we could not get out of the tram. It was obvious to me that Bente and I were not the only passengers for the opera so I asked the driver to kindly let us out, which would be completely risk free as we just would have to step out onto a peaceful piece of lawn. But the driver let us know that opening the doors in such a situation was allowed only in 'Notfall' (an emergency). Another fellow opera enthusiast, an Austrian lady, then stepped in exclaiming 'Aber es ist ein Notfall: Placido Domingo dirigiert!' That persuaded the driver to open the doors and we just made it. Goes to show what importance the Viennese attach to opera.

# 28 MUNICH – AARHUS

**Claudia Klüppelberg**

## 28.1 CLAUDIA KLÜPPELBERG

In the early 1990-ies I paid several visits to Paul Embrechts and his group at the Department of Mathematics in ETH, where he had become Professor in 1989.

One of Paul's PhD students at the time was Claudia Klüppelberg with whom I quickly found a warm accord which has lasted since. During one of my visits we went together to see the movie 'Meeting Venus' which is about a famous Conductor that goes to the Paris Opera to set up Wagner's *Tannhauser*. The movie describes, in a humouress fashion the many difficulties that are prone to occur during such an endeavour, including problems with trade unions, primadonnas, and such. The climax comes at the time of the premiere when the person who has the responsibility to raise the ironcurtain refuses to do so by order of his trade union. The leading soprano, played by Glen Close, then says – much to my liking – that the magic of Wagner's music and text will carry over even if the opera is performend in front of the iron curtain, which duly happens, to a great success.

A sequel to this occurred some ten years later during one of my numerous visits to Professor Walter Schachermayer, at the Technische Universitaet Wien, who is also a lover of opera. I told him of my enthusiasm for the movie and he went to one of be big and well assorted DVD stores in Vienna, seeking the film in vain till, in an inspired moment, he went to the porno section, as he reported, well amused.

Coming back to my friendship with Claudia, following her appointment in 1997 to the Chair of Mathematical Statistics, at the Center for Mathematical Sciences of the Munich Technical University, I spent numerous visits to Munich, greatly enjoying both my collaborations with her and her group of young researchers, and the many cultural opportunities that Munich has to offer.

In particular I was fortunate enough to establish long term productive relations to two of her PhD students, Robert Stelzer and Alex Lindner, both of whom have since become Professors at the Institute of Mathematical Finance in the University of Ulm.

## 28.2 AN OPERATIC FORTUITY

To our great joy Bente and I were invited to Anna and Alex Lindner's wedding which took place 27 August in Vazzola, located in the Italian Treviso Province, and was hosted by Anna's parents at Castel Brando in the village Cison di Valmarino. This was a magnificient occasion.

Bente and I chose to drive to Italy and that led to a completely unsuspected and remarkable addition to the trip. Quite by chance, and how I cannot recall, we discovered that Wagner's Ring would be performed at a place named Erl of which we had never heard. This a small village, with about 1500 souls, located in the Northern part of Austria near the German border, and it was virtually on the way to Vazzola. It turned out that the Pre-evening of the Ring – *Das Rheingold* – was to be given just two days before the wedding and so we decided to give things a chance and obtain tickets for the performance, which turned out to be possible.

Arriving to Erl, we realised that the four operas were going to be performed in a Passionsspielhaus building which at first sight looked a bit like a grain silo. The program for the whole tetralogy was presented in the form of a cartoon replica of the London Underground system, each of the parts of the Ring constituting a station. The whole program was modestly produced and one of the first things we noticed was that it stated that there were eight musical assistants. So we were prepared for an enjoyable evening of 'Wagner ultralight'.

However, on entering the Passionsspielhaus we realised that our impression was completely off the mark. There was the largest Wagner opera orchestra that we have ever, before or after, experienced. Being constructed for Festspiele there was no scene in the ordinary sense of the word. Instead the orchestra was seated up the back wall which meant that the whole universe of Wagner sounds was broad out in a way which is not possible in an ordinary opera theatre.

Accordingly the singers were placed and acted on a platform in front of the orchestra.

The orchestra and many of the singers, of whom we had never heard before, were relative young, highly talented people from Eastern Europe. That gave an appealing freshness to the performance, which was of very high quality.

The conductor, also not known to us, was Gustav Kuhn who, as we have since realised, has had an illustrious international career and who was and is the driving force behind the Erler Festspiele.

The whole atmosphere was nicely relaxed and members of the orchestra mingled with the audience in the intermissions, with cows grass-eating next by. And many of the inhabitants of Erl, including the fire corps and school children, participated in the Ring event, the latter as *Nibelungen*.

Every Summer, since the start in 1998, the Ring, as well as other Wagner operas and musical performances, has been given, at first in the Passionsspielhaus but from 2012 in the newly erected Festspielhaus, an opera and symphony building in the traditional sense of the word, but in a stunning new architecture. The fact that it has been possible to obtain support for the erection of an opera house 'in the middle of nowhere' is a manifestation of the tremendous success that Kuhn's initiative has had, and the Erler Festspiele are now viewed as 'another Bayreuth'.

**Gustav Kuhn at Master Class – Judit für NEUE STIMMEN**

**Gustav Kuhn**

Gustav Kuhn was born in 1945. He studied conducting with Herbert von Karajan at the conservatories in Vienna and Salzburg and graduated from Salzburg University in philosophy, psychology and psychopathology.

He has been conducting at a great many of the leading opera houses and symphony orchestras, including la Scala, Covent Garden and the Paris and Rome Operas, and the Berliner Philharmonic Orchestra, the Royal Philharmonic Orchestra and the Vienna Philharmonic Orchestra. And from the Mid-Eighties he has often at the same time directed and conducted operas.

Among his initiatives perhaps the most remarkable besides the Erler Festspiele is his creation of the Accademia di Montegral – a 'Monsalvat – which furthers the developments and careers of highly talented young musicians and opera singers. The Academia is located in a former cloister in the vicinity of Lucca, Italy.

# 29 OTHER GERMAN TIES

## 29.1 HUMBOLDT PRIZE

In 2001, thanks to a magnanimous initiative taken by Claudia, I was awarded the Humboldt Research Prize, thereby becoming a Fellow of the Alexander Humboldt Foundation.

Alexander von Humboldt
Stiftung / Foundation

*Symposium for Research Awardees*
*Conferment of Awards and Concert*

*Former Dominican Library*
*Friday, March 21, 2003*

*Programme*

*Welcoming Address*

*Conferment of Awards*
*with musical intermezzo:*
*L. v. Beethoven: Allegretto alla Polacca aus der Serenade*
*für Violine, Viola und Violoncello, op. 8 D-Dur*

*Professor Dr. Wolfgang Frühwald*
*President of the Alexander von Humboldt Foundation*

*Concert*

*Wolfgang Amadeus Mozart (1756 – 1791)* — *Divertimento für Violine, Viola und Violoncello Es-Dur KV 563*

*Allegro*
*Adagio*
*Menuetto (Allegretto)*
*Andante*
*Menuetto (Allegretto)*
*Allegro*

*Armand String Trio*

*Raúl Teo Arias - Violine*
*Lois Landsverk - Viola*
*Markus Mayers - Violoncello*

**Receiving the Humboldt Prize**

## 29.2 JOHANNES RAU

**Bundespräsident Johannes Rau**

Shortly after the Award Ceremony, the President of the German Republic, Bundespräsident Johannes Rau paid a brief official visit to Denmark. He had expressed the wish, rather than meeting journalists, to talk to some of the Danish Humboldt Prize Winners and prominent cultural personalities. A meeting was then set up, by the Chairman of the Danish Humboldt Association, Professor Jens Holger Schiørring, and he asked me to say some words on that occasion.

The meeting took place in the cafeteria at Lousiana, a well known museum of modern art, located close to Øresund, the strait between Denmark and Sweden, north of Copenhagen. Surprisingly, there was no sign of police or military guards in or around the cafeteria, and the museum and cafeteria were open as usual.

Johannes Rau and his party of about ten people, including his wife and a single high ranked military officer, arrived on foot through the cafeteria area, where a small part, shielded from the rest by large windows, was set off for the meeting. Coffee and Danish pastry was set out on a number of small tables and the atmosphere of the meeting was relaxed and informal throughout.

At the end of his brief introductory speech, where he speculated about the possible role of Denmark as a cultural bridge between the Anglo-Saxon and the German Cultural spheres, I was given the word and took the opportunity to say that the Mathematisches Forschungsinstitut Oberwolfach (MFO), in the Schwarzwald mountains, was a jewel among the German research institutions and that I felt that I could speak for mathematicians generally, around the World, in stating that the Forschungsinstitut is considered an absolutely unique meeting place for fruitful exchange of information and fostering of forefront mathematical research, the secluded location being highly conducive to mathematical research work. The Institute organises weekly workshops and the hosting of small working groups of mathematicians, providing full access to the excellent facilities and accommodation of the Institute, including the outstanding library – one of the best and most complete in the world. However, as a recent visitor to MFO I had noted with regret how the buildings were gradually deteriorating due to lack of funds.

At this stage in the short speech, Johannes Rau made a sign to one of the persons in his entourage, and in his concluding words, finishing the meeting, the Bundespräsident said that immediately upon arriving back to Germany he would contact the German Minister for Research and the Ministerpräsident of Baden-Württemberg concerning the state of MFO.

By misfortune this concern was pushed into the background by a tragic event (a shooting at a highschool). But eventually some added means were given to MFO through the initiative of the Bundespräsident, as acknowledged in an email to me from the scientific director of MFO, Gert-Hans Martin Greuel.

## 29.3 MATHEMATISCHES FORSCHUNGSZENTRUM OBERWOLFACH

Besides participating in many workshops, the first in 1961, at MFO (Mathematisches Forschungsinstitut Oberwolfach), the German mathematical research centre, located wonderfully in mountainous Schwarzwald and treasured by mathematicians all over the world, I have twice stayed under their magnanimous scheme 'Research in Pairs', which offers stays, with full accommodation and scientific facilities, for two, and occasionally three, researchers to spend about a month working on a joint project, such as a book. One stay was with Richard Gill and Peter Jupp for a study in quantum probability, and one with Neil Shephard to work on our joint book project. During the former stay, Richard Gill and I took a long walk in the neighbourhood and came across a small butcher shop which had good-looking sausages on display. We went in to buy some and on entering the young woman there exclaimed 'Ah zwei Mathematiker!' thus demonstrating that mathematicians are easily recognised figures as such in that area.

**ENDNOTES**

Photos from Oberwolfach Photo Collection

Gustav Kuhn Master Class, CC BY 2.0, https://commons.wikimedia.org/w/index.php?curid=42054299

# FROM SAND TO TURBULENCE AND FINANCE

My involvement in the study of the Physics of Blown Sands led naturally to an interest in the statistical theory of turbulence. A trait that immediately struck me was the relative similarity between the empirical distributions of log size of sand grains, as reported by Bagnold and illustrated in Section 5.1, and the distributions of velocity differences as observed generally in turbulence. However, the latter showed considerably heavier tails than the hyperbolic. My first paper related to the latter phenomenon was published in Proceedings of the Royal Society, London [BN (1979)]. There I constructed the NIG law, considered already above, in Section 17.5.

It turned out much later, that the NIG has numerous applications elsewhere, in particular in financial econometrics. It is often said that the financial markets are turbulent, which is a quite telling phrase. However, that should not be construed to mean that at a deeper level the driving forces in the markets are similar to those present in turbulent fluids. Nevertheless, to a noteworthy extent, stochastic models from one of the two fields has in many cases been found to have a useful analogue in the other.

An example of this is the classes of Brownian and Levy semistationary processes. The former were introduced in [BN, Schmiegel (2009)], in the first place aimed at modelling the time-wise behaviour of velocities in homogeneous turbulence. It represented a breakaway from traditional representations of stationary processes, in particular by incorporating volatility or intermittency effects. But it was found that this type of stochastic process, and the closely related Levy semimartingales, have widespread uses in Financial Econometrics. Other examples of this kind of symbiosis are found in the wider context of Ambit Stochastics, cf. Section 58.

# 30 FINANCIAL ECONOMETRICS

## 30.1 A SURGE OF INTEREST

The period starting around 1990 saw an explosive development and interest in Mathematical Finance and Financial Econometrics, regarding both basic research and applications, triggered to a large degree by the increasing availability of high frequency data.

One sign of this was the creation of the Bachelier Finance Society which took place in 1996 at the Institute of Mathematics, Aarhus University, at a Conference organised by Jørgen Aase Nielsen.

## 30.2 A HYPERBOLIC BEGINNING

Among my first encounters with the modern developments of mathematical finance and financial econometrics was by hearing a talk by Ernst Eberlein (Professor at the Department of Mathematical Stochastics at the University of Freiburg) at a Conference in France in 1994 where he described his studies of the empirics of financial time series. Such studies take their que from the behaviour of increments of log asset prices. At the time the prevailing thinking, following the work of Benoit Mandelbrot, was that the time series of such increments would follow what is termed fractional laws.

However, as demonstrated decisively by Eberlein, that is generally not the case. From the graphs he showed I could see that the hyperbolic laws might provide a much better description and analysis. I suggested this to Ernst who took up the idea and has since, with a number of collaborators, pursued the possibilities in this regard with great success, as documented in the book 'Advanced Modelling in Mathematical Finance. In Honour of Ernst Eberlein' (2016).


**Ernst Eberlein**

That honourary volume contains a splendid interview with Ernst Eberlein, carried out by Jan Kallsen and Antonis Papapanteleoni, from which the adduced citation is taken. It gives another example of the scepticism with which the hyperbolic laws were met at first; an early instance of this was mentioned in Section ##. Also, the interview provides a fine impression of the breadth of Eberlein's contributions not only to mathematical finance but also other parts of mathematics.

*You were one of the pioneers in the application of Lévy processes in finance. How did you start working in this direction?*

This began with statistics and data analysis. In 1987 we had started an interdisciplinary seminar in our University where the people involved in statistics and probability met. The formal foundation of the Freiburg Center for Data Analysis and Modeling (FDM), which grew out of this seminar, had to wait until 1994. The talks in this seminar inspired me or, rather, put some pressure on me to contribute and to start with data analysis myself. It was clear that I wanted to analyze financial markets. I acquired daily stock price data from a data bank service. After consulting with Stan Pliska, I got access to some of the relevant literature and one of my talented students, Ulrich Keller, agreed to do the work on the computer. The results were ready to be presented in January 1994 in Paris at the first conference where our newly established European network met. Jean Jacod acted as head during the first four years of funding for this network. That was the reason why we met in Paris. Ole Barndorff-Nielsen was in the audience and when he saw our graphs of empirical return distributions from German stock price data he commented '*This looks very much like hyperbolic distributions*'. So, back at home, I read the papers on this class of probability distributions, which Ole had introduced in the seventies in the context of the so-called sand project. In this interdisciplinary project, people in Aarhus studied the drift of sand under the impact of waves and this sort of distribution turned out to be useful for the statistical description of the particle size. Preben Blaesild was kind enough to send us a program for parameter estimation. Once we had parameters, the question that I posed myself was: Is there a model such that the return distributions from this model—let us say at time 1—are hyperbolic? Being used to the Black-Scholes- or rather the Samuelson—setting, I tried it for several months with diffusion processes. To illustrate how difficult it sometimes is to abandon the thinking in which you were trained, let me tell you the following story. I presented the empirical findings again, but this time including parameter estimates, during a conference in Cortona in Italy in May, to which a good part of the then-élite of mathematical finance had been invited by Wolfgang Runggaldier. In the discussion following my talk one of the prominent members of the community commented '*This looks very interesting, but you will never be able to develop a suitable theory based on these distributions*'. I had gone through other difficulties and it would have taken more than that to discourage me. At some point I realized that diffusions do not and cannot work to reach the goal. With the exception of the simplest cases, one does not even know the distribution which is produced by a diffusion on a given time horizon. A diffusion equation or, equivalently, a stochastic exponential was just not the right starting point to get what I was looking for. Something more radical had to be done. Hyperbolic distributions are infinitely divisible and thus they generate Lévy processes, where you get the generating distribution back at time one. In Bauer's book in my student days these processes had just been called processes with stationary and independent increments. The name Lévy process came only later. Since we used log-returns for the statistics, one had to take the ordinary exponential of the corresponding Lévy process instead of a stochastic exponential, in order to get exactly the distribution that comes from the data. In the classical case of Brownian motion the difference between the ordinary and the stochastic exponential is not that crucial, but for jump-type Lévy processes it is. It was so simple once I had seen this point. The joint paper with Ulrich on the hyperbolic Lévy model appeared in October 1994 as No. 1 of the newly created FDM-preprint series and was published in the first volume of *Bernoulli* a year later. Let me finish by making one more remark. That meeting of the EU network in Paris helped showing the many avenues of research in this field, and many network members, none of whom had been involved in financial models before, were inspired by my talk to start working in stochastic finance. DYNSTOCH, as we called it later for the second funding period, headed by Michael Soerensen, became a very successful working platform and the group has continued with an annual meeting for more than twenty years already.

## 30.3 NEIL SHEPHARD

During a visit to David Cox around 1992 I had occasion, more or less by chance, to hear a young man, Neil Shephard, give a talk on a topic in Financial Econometrics, a field where I had rather little knowledge at that time. However, I was much impressed by Neil's presentation and it was clear to me from the way he presented his talk that our views on modelling were very similar. And his emphasis on the importance of stochastic volatility interested me particularly in view of my own interest in volatility/intermittency in turbulence.

In Financial Econometrics, at the time, stochastic volatility was recognised as a ubiquitous phenomenon, described as a 'stylised feature', but realistic modelling of this, in mathematical terms, was essentially non existing.

I took the opportunity to invite Neil to visit the MaPhySto Centre. This was followed by many mutual visits, during one of which we had lunch at 'Mellem Jyder', the oldest inn in the small town Ebeltoft on Djursland near Aarhus. It was there and then that we came on the key idea in what has come to be known as the BNS model, presented as a Read Paper to the Royal Statistical Society in 2002. The key point was the modelling of volatility in financial markets by stationary Lévy-driven processes and incorporating the models in a suitable way into the existing standard frameworks in Mathematical Finance and Financial Econometrics. On the statistical side analysis was, at the time, still based primarily on time series analysis, that is where the models consist of discrete-time series of random observations, whereas in Mathematical Finance continuous-time models were adamant, and the model we proposed is of the continuous-time type.

The BNS model is a continuous-time model constructed so as to reflect the main stylised features of typical log asset prices; it specifies the log price Y of a given asset as determined by the stochastic differential equation

$$dY(t)=\mu dt+\sigma(t)dB(t)+\beta\sigma^2(t)dt$$

where $\mu$ and $\sigma$ are parameters, B is Brownian motion and $\sigma^2$ is a stochastic process expressing the volatility, the emphasis being on cases where $\sigma^2$ follows a particular type of stationary process, defined as superpositions of Lévy-driven Ornstein-Uhlenbeck processes. The model was presented as a Read Paper to the Royal Statistical Society in 2001 [BN, Shephard (2001)]. A Read Paper is made available in advance of the meeting where the paper is presented, allowing interested colleagues to familiarise themselves with the material prior to the actual discussion of its contents. The discussion consists of both invited and contributed comments, subsequently published together with the paper. In this case the Discussion was extensive, with comments from a large number of probabilists and experts in Financial Econometrics, and references to a wide range of related work and subjects.

In view of the interest and of the projections for further developments of the modelling strategy engendered by our Read Paper Neil and I decided to embark on writing a book on 'Lévy-driven Volatility Models', originally with focus on financial econometrics as far as applications were concerned. However, although by the end of 2011 the book was near to completion our work on the book came for various reasons to a standstill, one of them being that Neil left his Professorship at Oxford University and the Directorship of the Oxford-Man Institute of Quantitative Finance to take up a Chair at Harvard University.

Highly productive for our joint work was a longer stay at Mathematisches Forschungszentrum Obwerwolfach (MFO) under their 'Research in Pairs' programme where food and lodging – and warm hospitality – is provided for the visitors. The library at MFO is one of the best in the world.

**With Neil Shephard at Mathematisches Forschungszentrum Oberwolfach**

Another important element in Neil's and my work on Financial Econometrics has been our association to CREATES (Centre for Research in Econometric Analysis of Time Series). This Centre, started in 2007 with Professor Niels Haldrup as Director and funded by the Danish National Research Foundation, has had a major influence on the development internationally of Financial Econometrics through the holding of conferences, fostering of new talent and creating a very active research environment for visitors and locals. Neil and I have been Fellows of CREATES from its start and some of our early work was presented in two lectures that may be seen on the CREATES podcast archive.

Neil was elected to the British Academy for the Humanities and Social Sciences and in 2009 he was awarded an Honorary Doctorate by Aarhus University.

**Honorary Doctor Neil Shephard**

Professor Neil Shephard, Nuffield College/Oxford-Man Institute, University of Oxford, UK, was appointed Honorary Doctor at the Faculty of Social Sciences and the Faculty of Science.

Professor Shephard has been awarded several academic distinctions and prizes. In 2006, he was elected Fellow of the British Academy, just as he is Fellow of the Econometric Society. In addition, he is co-editor of Econometrica – absolutely the most renowned journal in the field of economics – where he has also published a number of research articles.

Professor Shephard is indisputably one of the leading and most acknowledged researchers in the world in the fields of time series econometrics and financial econometrics. His interdisciplinary research covers fields such as mathematics, statistics, econometrics, economy and financing.

For the last ten years, the British professor has had excellent collaboration with Professor Ole Barndorff-Nielsen, Department of Mathematical Sciences and the Centre for Research in Econometric Analysis of Time Series (CREATES), Aarhus University. Together, they have made remarkable and significant contributions to the analysis, modelling and understanding of the variability and "jumps" in financial markets.

Professor Shephard has an exceptional list of publications in all the leading international journals in statistics, stochastics and econometrics. His work over the last ten years is mainly the result of his collaboration with the Danish professor.

**From the Aarhus University Newsletter**

The concept of volatility and the question of its modelling continued to be a Focal point for our collaborations cf., among other things, the papers [BN, Shepard (2002)] and [BN, Shephard (2004)].

# 31 PHYSICS OF TURBULENCE

> *As it unfolded, the structure of the story began to remind me of one of those Russian dolls that contain innumerable diminishing replicas of itself inside. Step by step the narrative split into a thousand stories, as if it had entered a gallery of mirrors, its identity fragmented into endless reflections.*
>
> (Carlos Ruiz Zafón: The Shadow of the Wind)

The study of windblown sand, spanning from the generation, shapes and dynamics of sand dunes to questions regarding transport and packing of sand grains in relation to their sizes, inevitably entails a need for understanding the nature of the physics of wind dynamics and hence to the statistical theory of turbulence. That theory centers around the time-wise and directional variations of the velocity vectors of the wind, considered as a fluid, as well as for other fluids.

The main name associated to the statistical theory of turbulence is that of Andrey Nicolaevich Kolmogorov (1903-1987), known also for his contributions to many other areas of theoretical and applied mathematics.

Besides the statistical theory of turbulence, he is primarily known for his work in probability theory, in particular for establishing the rigorous mathematical basis for probability theory. Two key universality concepts on turbulence, the 2/3 and 4/5 laws, were introduced by Kolmogorov in 1941 and are at the centre of most turbulence studies. Interestingly, it was later pointed out by the Russian physicist Lev Davidovich Landau (1908-1968; Nobel Prize 1962), one of the most influential physicists of the 20th Century, that Kolmogorov's hypotheses needed an additional assumption, to take account of the effect of a phenomenon termed 'intermittency'. This and the closely related concept of 'volatility' are discussed in Section ##, and universality is the subject of Section ##.

In turbulence the shape of the distributions of velocity increments is a central issue, subject to numerous experimental studies. The shape is reminiscent of the hyperbolic but deviates in showing that the tails of the distributions are significantly heavier. The precise character of these deviations is a key fingerprint of turbulence. The shape is precisely represented by the normal inverse Gaussian law, discussed particularly in Section 17.5.

Absolutely essential and decisive for my later work in turbulence have been my close collaborations and friendships with Albert Shiryaev and Jürgen Schmiegel.

## 31.1 JÜRGEN SCHMIEGEL

In 2001 I attended a Conference on Quantum Stochastics at the Max Planck Institute for the Physics of Complex Systems in Dresden. There, by a fortuities incident, I came to talk to a Ph. D. student who, as it turned out, was having some ideas regarding the modelling of volatility and energy dissipation which were fairly close to some thoughts I had had. This student was Jürgen Schmiegel.

The Humboldt Society has an arrangement whereby it is possible for a Humboldt Fellow to apply for a so-called Feodor Lynen stipend under which a young German scientist may spend a prolonged period in another country in collaboration with that Fellow. To my great joy Jürgen agreed to come to Aarhus for a period - and he has been in Aarhus since. This has been absolutely decisive for the development of the studies in turbulence that have taken place at the Department of Mathematics in Aarhus, studies that may be seen as part of the general field of Ambit Stochastics, discussed in Section ##, which in fact has grown out of the original papers that Jürgen and I wrote in the years from 2002 onwards. The first of these papers, on Lévy-based tempo-spatial modelling with applications to turbulence was presented at the International Conference "Kolmogorov and Contemporary Mathematics" organised in Moscow in 2003 by the Russian Academy of Science to mark the Centenary of the birth of Kolmogorov. The paper appeared in print in 2004, in Uspekhi Matematicheskikh Nauk.

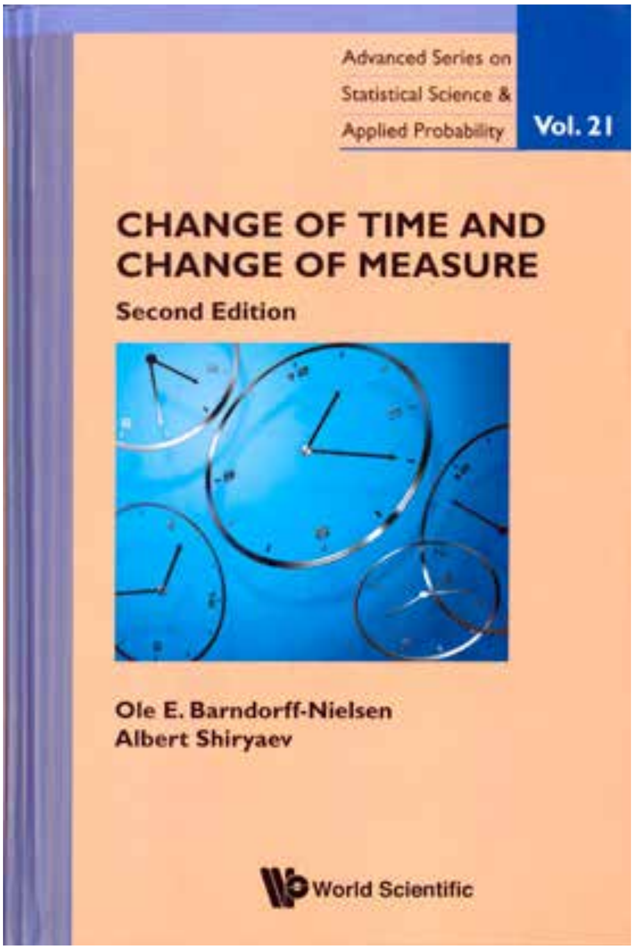
That type of modelling, central to the theory of Ambit Stochastics, represented a break away from the then prevailing attempts to model by means of multifractality. However, the Lévy-based approach may be seen as encompassing aspects of multifractality, as was indicated in our second joint paper from 2005, written jointly with Hans C. Eggers, Professor of Physics at the University of Stellenbosch, South Africa.

Our main aim has been to develop realistic models of aspects of turbulent flows, based on recent advances in the theory of probability, in particular those relating to the concept of infinite divisibility and adhering to Kolmogorov's view that turbulence should be understood as a statistical phenomenon.

## 31.2 ALBERT NIKOLAEVICH SHIRYAEV

My acquaintance and friendship with Albert Shiryaev was initially mainly rooted in our involvement in the founding of the Bernoulli Society and in having a common interest in Mathematical Finance and Financial Econometrics.

But at a Conference "Probability towards 2000", held 1995 at Colombia University, New York, I gave a talk entitled "Probability and Statistics; self-decomposability, finance and turbulence' where I drew attention to the striking similarities between some of the important empirical findings in finance on the one hand and turbulence on the other, something that Albert also had noted.




This led to our extensive collaborations on problems in these two fields. In particular, it resulted in our writing a book on the similarities and key differences between Financial Econometrics and the Statistical Theory of Turbulence and on the associated mathematical techniques. The book has the title *Change of Time and Change of Measure*; the front cover of the book, cleverly designed by the publisher World Scientific, is a nice comment on the content.

The following is the introduction to a presentation that I wrote for a Conference held in Moscow on the occasion of Albert Shiryaev's 80$^{th}$ birthday.

'When I think of Albert, as I often do, it is not only as an outstanding mathematical scientist but equally as an exceptionally generous human being.

Those who know Albert well will immediately appreciate what I mean. Others may discern this from knowledge of how Albert, alone or with fellow stochasticians, has shared his enormous insight and expertise, not least through the writing of numerous advanced monographs in a broad range of topics in stochastics. But also from his contributions to the international life in statistics and probability, in particular through his work as President of the Bernoulli Society for Mathematical Statistics and Probability.'

**Petrujska and her boyfriend and photo of Kolmogorov, given to me by Albert**

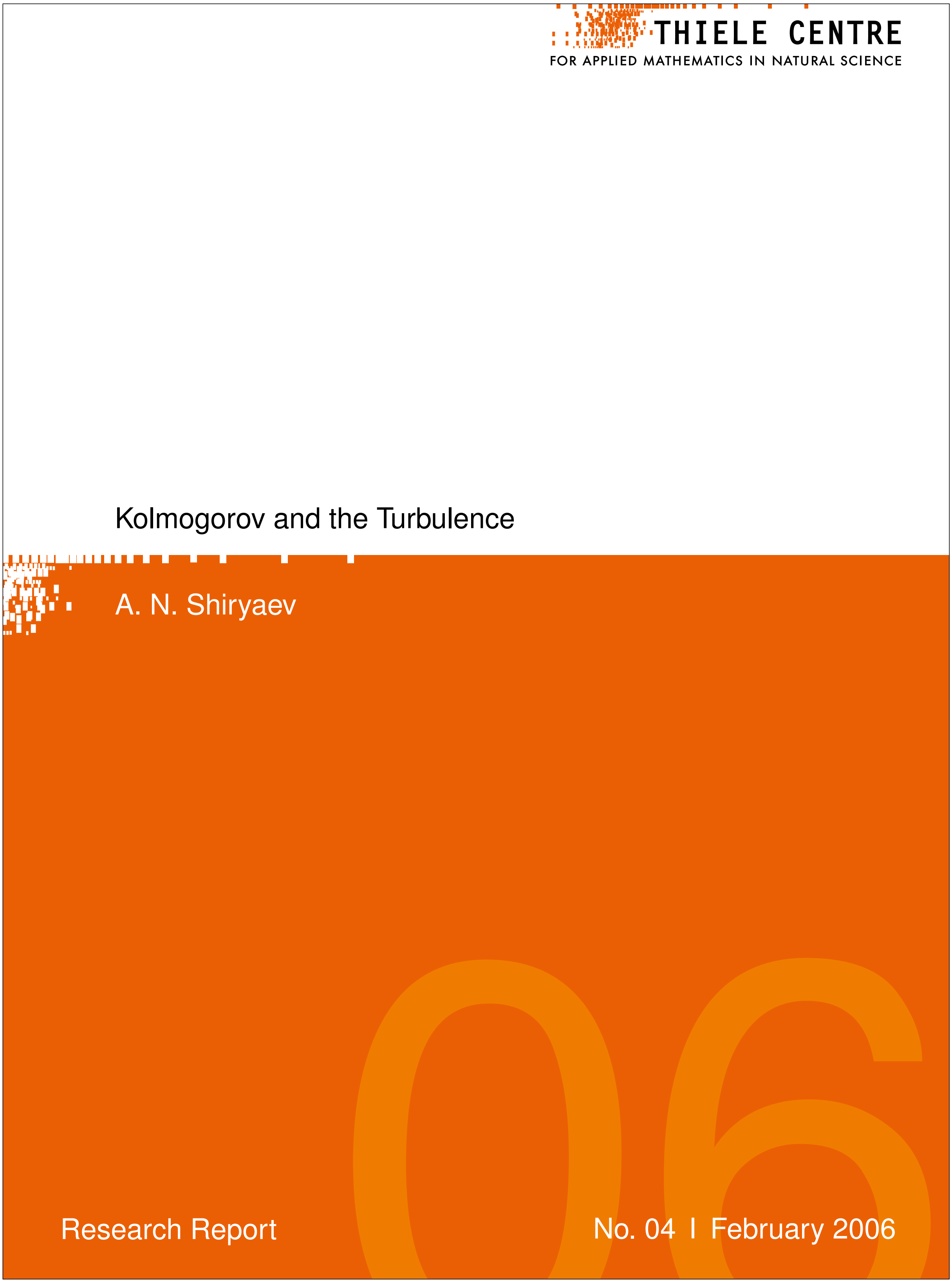
Albert Shiryaev's interest in turbulence was rooted in his close relation to Andrey Nikolaevich Kolmogorov of whom he was a student and later a close collaborator and friend.

Kolmogorov (1903-1987) was a master of both pure and applied mathematics and ranks among the greatest mathematicians of all times. He founded the basis of Probability in the strict mathematical sense and he established the Statistical Theory of Turbulence through pathbreaking work referred to as K41 and K62.

Kolmogorov participated actively in empirical studies of turbulence in the Pacific Ocean, aboard the research vessel 'Dimitri Mendeleev'.

The following Section 31.3 is an account by Shiryaev of Kolmogorov's involvement in the study of turbulence. It was written as a MaPhySto research report and is reproduced here with Albert's consent. This is an important historical document, throwing light on a part of Kolmogorov's work that is little known.

## 31.3 KOLMOGOROV AND THE TURBULENCE



THIELE CENTRE
FOR APPLIED MATHEMATICS IN NATURAL SCIENCE

Kolmogorov and the Turbulence

A. N. Shiryaev

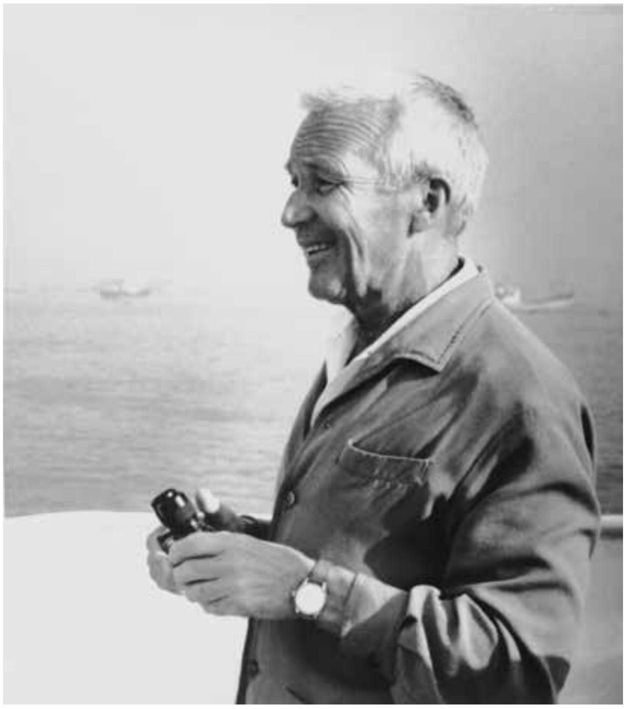
Research Report

No. 04 | February 2006

# Kolmogorov and the Turbulence

Except for the frontispiece this Thiele Research Report is a reprint of MaPhySto Miscellanea no. 12, 1999. Printed with permission.

This Thiele Research Report is also Research Report number 472 in the Stochastics Series at Department of Mathematical Sciences, University of Aarhus, Denmark.



Kolmogorov on board the *Dimitry Mendelyeev*

# Kolmogorov and the Turbulence

A. N. Shiryaev

May 20, 1999

## 1 Introduction

In August, 1990, at the Second International Congress of the Bernoulli Society in Uppsala (Sweden), I made a short report entitled "Everything about Kolmogorov was unusual ... ". I said: "*Andrei Nikolaevich Kolmogorov was one of a selected group of people, who made you feel that you had met an unusual, great and extraordinary person. That was the feeling of having met a wonder.*

*Everything about Kolmogorov was unusual: his entire life, his school and university years, his pioneering discoveries in many areas of mathematics and in such disciplines as meteorology, hydrodynamics, history, linguistics, ... pedagogy ... His interests were unusually diverse including music, architecture, poetry and travelling. His erudition was unusual: he had a qualified opinion about everything*".

This paper was prepared for the workshop "Turbulence and Finance" organized by MaPhySto (5-7 May, 1999, Aarhus) with a view to "discuss the striking similarities as well as the differences between key empirical features observed in the financial markets and the turbulence of fluids".

In the last few years the connection between these two disciplines has been stressed by several authors. In October 1995 O. E. Barndorff-Nielsen gave at Columbia University a talk "Probability and Statistics: Self-decomposability, *Finance* and *Turbulence*" where he discussed "key features of empirical data from finance and turbulence" which "are widely recognized as being essential for understanding and modelling within these two, quite different, subject areas", {10}.

In unison, with these words the journal "Nature" (Vol. 38, June,

1996) used on its cover the term

FINANCIAL TURBULENCE.

publishing in this issue the article "Turbulent Cascades in Foreign Exchange Markets" by five authors S. Ghashghaie, W. Breymann, J. Peinke, P. Talkner and Y. Dodge. The article shows the similarity in the statistical behaviour of financial data and turbulent data. In the abstract to the article, the authors note that: "*The availability of high frequency data for financial markets has made it possible to study market dynamics on timescales shorter than a day. For foreign exchange (FX) rates, V. A. Muller et. al. (J. Banking Fin., 1990, 14, 1189-1208) have shown that there is a net flow of information from long to short timescales: the behaviour of long-term traders (who watch the markets only from time to time) influences the behaviour of short-time traders. Motivated by this hierarchical feature, we have studied FX market dynamics in greater detail and will show here an analogy between the dynamic and the hydrodynamic turbulence. Specifically, the relationship between the probability density of FX price changes ($\Delta X$) and the time delay ($\Delta t$) (Fig. 1a) is much the same as the relationship between the probability density of the velocity differences ($\Delta V$) of two points in a turbulent flow and their spatial separation $\Delta r$ (Fig. 1b).*

*Guided by this similarity, we claim that there is an information cascade in FX market dynamics that corresponds to the energy cascade in the hydrodynamic turbulence ... The analogy gives a conceptual framework for understanding the short-term dynamics of speculative markets*". At the end of their article the authors conclude optimistically: " ... *We have reason to believe that the qualitative picture of turbulence that has been developed during the past 70 years, will help our understanding of the apparently remote field of financial markets*". (See also Chapters III and IV in {11}.)

In the present report, I am not going to compare the statistical data of the turbulence and of the finance. I would like to give a review of Kolmogorov's different periods of the turbulence study as well as of his main conceptions and the results achieved in this field, that influenced the later development of hydrodynamics.

There are *three* periods when Kolmogorov studied the turbulence.

The *first* period lasted from the late 30-s to the early 40-s. In that period he published his classical works (see also {1}):

[1] The local structure of turbulence in an incompressible fluid at very high Reynolds numbers. Dokl. Acad. Nauk USSR, 30 (1941), p. 299-303.

[2] The logarithmically normal distribution of the size of particles under the fragmentation. Dokl. Acad. Nauk USSR, 31 (1941), p. 99-101.

[3] The decay of isotropic turbulence in an incompressible viscous fluid. Dokl. Acad. Nauk USSR 31 (1941), p. 538-541.

[4] Energy dissipation in locally isotropic turbulence. Dokl. Acad. Nauk USSR, 32 (1941), p. 19-21.

On January 26, 1942, Kolmogorov presented his works on the turbulence at the Joint Meeting arranged in Kazan by the Department of Physics and Mathematics of the USSR Academy of Sciences.

[5] Equations of turbulent motion of an incompressible fluid. Izv. Acad. Nauk USSR, ser. Fiz. 6(1942), p. 56-58.

P.A. Kapitza and L.D. Landau took part in the discussion. Landau pointed out that "*A.N. Kolmogorov was the first to give the right conception of the local structure of the turbulent flow*".

Kolmogorov explained his interest in the turbulence as follows: "I took an interest in the study of turbulent flows of liquids and gases in the late thirties. It was clear to me from the very beginning that the main mathematical instrument in this study must be the theory of random functions of several variables (random fields) which had only then originated. Moreover, it soon became clear to me that there was no chance of developing a purely closed mathematical theory. Due to the lack of such a theory, it was necessary to use some hypotheses based on the results of the treatment of the experimental data. Therefore, it was important to find talented collaborators who were able to combine theoretical studies with the analysis of the experimental results. In this respect, I was quite successful". (Kolmogorov mentioned his students A. M. Obukhov, M. D. Millionshchikov, A. S. Monin and A. M. Yaglom; see {1})

The "**Law of two-thirds**" is the pearl of the first investigations by A.N. Kolmogorov.This is a universal law of the turbulence nature, supported by the experiments made for the fluids with high Reynolds numbers (see {1}).

The *second* period of Kolmogorovs's investigation of the turbulence started in the early 60-s. It was mainly related to his participation (with a group of his followers A. N. Obukhov, M. D. Millionschikov, A. M. Yaglom) in the two International Meetings on the Mechanics of Turbulence, arranged in Marseilles by the IUTAM (The International Union of Theoretical and Applied Mechanics) and the IUGG (The International Union of Geodesy and Geophysics).

The main ideas of Kolmogorov's report were presented in his two articles (see also {1}):

[6] Les Précesions sur la structure locale de la turbulence dans un flux visqueux aux nombres de Reynolds elevés. En Méchanique de la Turbulence. Coll. Int. du CNRS à Marseille, p. 447-458, Paris, CNRS, (1962).

[7] A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds numbers. J. Fluid Mech, 13 (1962), 82-85.

It can be seen from the titles of these articles that they are concerned with the refinement of the results obtained in the early 40-s. The verifications were mostly due to L.D. Landau's remarks made on the report [5] in 1942. Landau emphasized the fact that "the velocity rotor in the turbulent flow exists only in the restricted part of the space". These articles are closely connected with the report by A.M. Obukhov {3} that included new ideas about the development of the locally isotropic turbulence theory (see the last section of the article [5] and §6.4 in the book {3}).

Finally, the *third* (not widely known) period of Kolmogorov's study of the turbulence is related to his participation in the expeditions on board the "Dmitry Mendeleev" research ship (the $2^{nd}$ and the $5^{th}$ voyages).

The first expedition lasted from June 23 to September 18, 1969. Its duration was 87 days. The route was as follows:

Kaliningrad → Reykjavik (the capital of Iceland) → Rio-de Janeiro (with a call at Konakri (Guinea)) → Dakar (Senegal) → Gibraltar → Kaliningrad.

The second expedition lasted from January 20 to May 12, 1971 (26 132 sea miles were covered). In fact, that was a round-the-world journey: by train from Moscow to Kaliningrad and a long voyage aboard the "Dmitry Mendeleev" ship. The route was as follows:

Kaliningrad → The Kiel Canal → Pas de Calais → The Bay of Biscay → The Sargasso Sea → across The Gulf Stream to Cape Canaveral (The State of Florida) → The port of Kingston (Jamaica) → The Panama Canal → The Galapagos Islands (a territory possessed by Ecuador where Ch. Darwin elaborated his theory of the Origins of the Species) → The port of Honolulu (The Hawaii) → the passage to the South along the $159^o$ meridian of the western longitude → the coral Atoll of Fanning (Brit.) → the crossing of the Equator → the bay of Avarua on the island of Rapotong (the central island in the Cook Archipelago protected by New Zealand) → the passage to the West → the port of Suva (the Fiji Islands) → the port of Vila (Esratos Island), Touman Island, Malecula Island (the New Hebrides) → the port of Yokohama (Japan) → Vladivostok. Then Andrei Nikolaevich came back to Moscow by train. The picture on the front-page of this paper was taken at the Moscow train station.

In both voyages A. N. Kolmogorov was the Assistant Director in research. Prof. A. Monin, the Director of the expedition in 1971, wrote (see {5}): "Andrei Nikolaevich was responsible for the geophysical oceanic investigations. Five teams were engaged in that work, equipped with various devices. Some of the devices were new and did not function properly. Kolmogorov spared no effort and time in checking the measurement accuracy and the calibration of the devices as well as in revealing the interferences that distorted the readings."

It is a pity that a great mathematician had to waste time on solving insignificant problems. But he rejected any attempts to release him from his duties and would check himself the quality of the measurements.

I have known before that such an attitude was natural for Andrei Nikolaevich. He expressed it best in his last interview with a documentary-film maker A.N. Marutyan: "In fact, when the mathematician solves, for example, a hydrodynamics problem (I myself was dealing with the hydrodynamics of the ocean), it means that a hydrodynamics problem is solved by mathematical means. The mathematicians always want that their mathematics should be pure, that is, strict and provable, wherever possible. However, the most interesting and realistic problems could not usually be solved in that manner. Therefore, it is very important that the mathematician should be able to find the approximative (not necessarily strict but effective) ways of solving such problems. At any rate, I've always done it by this means... If turbulence is an object of my studies, I am dealing with the **turbulence**. I rate highly those mathematicians who, as a matter of fact, cannot be called "pure" mathematicians. They just solve applied problems by strict methods, if possible, or by making "hypotheses".

– So, you are in favor of the flexibility of thinking?

– And in favor of the direct participation, where possible, in the experiments together with the physicists".

## 2 Locally Isotropic Turbulence. The Law of 2/3.

The articles [1]–[5] were published during the *first* period of Kolmogorov's study of the turbulence in the early 40-s. These articles were preceded by the following two papers (see also {1}):

[8] Curves in a Hilbert space that are invariant under the one-parameter group of motions. Dokl. Acad. Nauk USSR, 26 (1940), p. 6-9.

[9] Wienersche Spiralen und einige andere interessante Kurven in Hilbertschen Raum. Dokl. Acad. Nauk USSR, 26 (1940), p. 115-118.

These articles are closely connected both with the turbulence and the general theory of random processes as well as with the stochastic finance.

We will now give some definitions and facts concerning homogeneous random processes and fields, in order to describe the mathematical notions of these papers.

Let $S = \{s\}$ be a homogeneous space of points $s$ with a transitive group $G = \{g\}$ of transformations mapping the space $S$ into itself $(S \to gS)$.

Let $(\Omega, \mathcal{F}, \mathsf{P})$ be a probability space and let $X = X(s)$, $s \in S$ be a random field on this probability space, that is, $X(s) = X(s, \omega)$, $\omega \in \Omega$, $s \in S$ is a family of (complex-valued) random variables.

The random field $X = X(s)$, $s \in S$ is called (wide-sense) *homogeneous* if

$$\begin{aligned}&\mathsf{E}\,|X(s)|^2 < \infty,\\ &\mathsf{E}X(s) = \mathsf{E}X(gs),\\ &\mathsf{E}X(s)\overline{X(t)} = \mathsf{E}X(gs)\overline{X(gt)}\end{aligned}$$

for all $s, t \in S$ and $g \in G$.

The following special case is of particular importance: $S = \mathbb{R}^k$ and $G$ is the group of parallel shifts. The homogeneous random field is often defined as a field of this type.

If $G$ is a group of isometric transformations on $S = \mathbb{R}^k$ (generated by the parallel shifts, the rotations and the reflections), then $X = X(s)$, $s \in \mathbb{R}^k$ is called the *homogeneous isotropic random field.*

The special case of the homogeneous field is a (wide-sense) stationary process $X(s)$, $s \in \mathbb{R}$ where $\mathsf{E}\,|X(s)|^2 < \infty$, $\mathsf{E}X(s) = m(= const)$ and $\mathsf{E}X(s)\overline{X(t)}$ depends only on the difference $t - s$.

Assume $m = 0$ and denote by

$$R(t) = \mathsf{E}X(s+t)\overline{X(s)} \tag{1}$$

the *correlation* function of the process $X$.

Due to the Bochner-Khinchin theorem (for the mean-square continuous processes $X$), this function admits the *spectral representation*

$$R(t) = \int_{-\infty}^{\infty} e^{i\lambda t}\, F(d\lambda) \tag{2}$$

Here, $F = F(A)$ is a finite measure on the Borel sets $A \in \mathcal{B}(\mathbb{R})$. This measure is called the *spectral measure*, and the function

$$F(\lambda) = \int_{-\infty}^{\lambda} F(d\nu) \quad (= F(-\infty, \lambda]) \tag{3}$$

7

is called the *spectral function*. If $F'(\lambda)$ exists, then $f(\lambda) = F'(\lambda)$ is called the *spectral density* or the *energy spectrum*, or simply the spectrum. The magnitude $f(\lambda)\,d\lambda$ can be described as follows: it is a contribution to the "energy" of the harmonics whose frequencies are within the interval $(\lambda, \lambda + d\lambda)$.

It is important to note that (2) implies

$$\mathsf{E}[X(t) - X(s)]^2 = 2 \int_{-\infty}^{\infty} \left(1 - e^{i\lambda(t-s)}\right) F(d\lambda) \tag{4}$$

If $X$ is a *real-valued* process, then $R(t) = R(-t)$ and

$$R(t) = \int_{-\infty}^{\infty} \cos(\lambda t)\, F(d\lambda), \tag{5}$$

$$\int_{-\infty}^{\infty} \sin(\lambda t)\, F(d\lambda) = 0. \tag{6}$$

Thus, the spectral function is symmetric with respect to the point $\lambda = 0$.

Set

$$G(\lambda) = F(\lambda) - F(-\lambda) \quad (= \int_{|\nu| \le \lambda} F(d\nu)).$$

Then we obtain

$$\begin{aligned} G(\lambda) &= 2F(\lambda) - R(0), \\ G(\lambda) &= \frac{2}{\pi} \int_0^{\infty} \frac{\sin(\lambda t)}{t} R(t)\, dt \end{aligned}$$

and

$$\begin{aligned} R(t) &= \int_0^{\infty} \cos(\lambda t)\, G(d\lambda), &(7) \\ \mathsf{E}\,|X(t) - X(s)|^2 &= 2 \int_0^{\infty} (1 - \cos(\lambda(t-s))\, G(d\lambda). &(8) \end{aligned}$$

Essentially, the same is true for the *homogeneous one-dimensional random real-valued fields* $X(s)$, $s \in \mathbb{R}^k$ (with the group $G$ of parallel shifts). Here, the spectral representation (2) is reformulated in the following way: for the mean-square continuous complex-valued random field $X = X(s)$, $s \in \mathbb{R}^k$, the covariance function (or the correlation function in the case of centered random fields) $R(t)$ admits the representation

$$R(t) = \int_{\mathbb{R}^k} e^{i(t,\lambda)} F(d\lambda) \tag{9}$$

where $F = F(d\lambda)$ is a finite (uniquely determined) non-negative measure on $\mathcal{B}\left(\mathbb{R}^k\right)$.

For the homogeneous *isotropic* fields, the covariance function $R(t)$, $t = (t_1, \ldots, t_k)$ is defined as a function of $\|t\| = \sqrt{t_1^2 + \ldots + t_k^2}$: $R(t) = R(\|t\|)$. In this case, the integration over $\mathbb{R}^k$ in (9) is replaced by the integration over $\mathbb{R}_+$. To be more precise, we have

$$R(u) = 2^{\frac{k-2}{2}} \Gamma\left(\frac{k}{2}\right) \int_{-\infty}^{\infty} \frac{I_{\frac{k-2}{2}}(\lambda u)}{(\lambda u)^{\frac{k-2}{2}}} Q(d\lambda) \tag{10}$$

where $I_\nu(x)$ is the Bessel function of index $\nu$, $Q$ is a non-negative random measure on $\mathcal{B}(\mathbb{R}_+)$ such that $Q(\mathbb{R}_+) = G(\mathbb{R}^k) = R(0)$.

Similar results are true for the homogeneous isotropic *vector fields* (defined on $\mathbb{R}^k$ with values in $\mathbb{R}^l$) (see, for example, {6}, {7}).

It is remarkable that for the (wide-sense) stationary mean-square continuous random processes $X = X(t)$, $t \in \mathbb{R}$, the spectral representation is valid both for the correlation function $R(t)$ and for the process $X$. The following result is due to Karhunen, Kolmogorov and Cramér: there exists a complex-valued random measure $Z = Z(A)$ (or $Z = Z(A;\omega)$) with orthogonal values (that is, $\mathsf{E} Z(A)\overline{Z(B)} = 0$ if $A \cap B = \emptyset$), such that

$$X(t) = \int_{-\infty}^{\infty} e^{i\lambda t} Z(d\lambda). \tag{11}$$

Moreover, $\mathsf{E}\left|Z(A)\right|^2 = F(A)$.

The similar representation is valid for the homogeneous (both one-dimensional and vector) random fields on $\mathbb{R}^k$ with values in $\mathbb{R}^l$.

9

The role of the homogeneous random fields in the turbulence was for the first time emphasized by Taylor in {8}. This paper deals with the statistical theory of the turbulence. It was in this paper that Taylor introduced the important notion of the *isotropic* turbulence.

A.N. Kolmogorov pointed out in [1] that "Taylor's isotropy hypothesis has a good experimental support for the turbulence generated by the flow passing through the grating. In most other cases of practical interest, this hypothesis can be considered as a very rough approximation even for small areas ... and for extremely high Reynolds numbers".

It was mentioned above that the first of Kolmogorov's work on the turbulence was preceded by the papers [8], [9], in which he made the first step towards introducing the notion of *locally* homogeneous random fields. This notion (together with the notion of *local isotropy*) became the main mathematical means in analyzing the turbulent phenomena (especially in the case of high Reynolds numbers).

In [8], A.N. Kolmogorov considered a random field on $\mathbb{R}^1$ with values in $\mathbb{R}^1$. Such fields $Y(s)$, $s \in \mathbb{R}$, are the random processes with (wide-sense) stationary or homogeneous increments.

For such processes, the increments

$$\Delta_r Y(t) = Y(t) - Y(t-r)$$

(rather than the values of $Y(t)$) are supposed to be such that the function $\mathsf{E}\Delta_r Y(t)$ depends only on $r$ ($\mathsf{E}\Delta_r Y(t) = m(r)$) and the function $\mathsf{E}\Delta_{r_1} Y(t+s)\Delta_{r_2} Y(s)$ does not depend on $s$ for any $s, t, r_1, r_2$. This will be stressed by the notation

$$\mathsf{E}\Delta_{r_1} Y(t+s)\Delta_{r_2} Y(s) = D(t, r_1, r_2)$$

The function $D(t, r_1, r_2)$ is called the *structural* function of the process $Y$.

Due to the equality

$$(a-b)(c-d) = \frac{1}{2}\left[(a-d)^2 + (b-c)^2 - (a-c)^2 - (b-d)^2\right],$$

the structural function $D(t, r_1, r_2)$ can be represented as a function of one variable:

$$D(r) \equiv D(0, r, r) = \mathsf{E}\left|\Delta Y_r(t)\right|^2, \tag{12}$$

which is also called the structural function.

Kolmogorov obtained in [8] the following spectral representation for $D(r)$ and $D(t, r_1, r_2)$:

$$D(r) = 2 \int\limits_{\mathbb{R}\backslash\{0\}} (1 - \cos \lambda r)\Phi(d\lambda) + ar^2 \tag{13}$$

and

$$D(t, r_1, r_2) = \int\limits_{\mathbb{R}\backslash\{0\}} e^{i\lambda t} \left(1 - e^{-i\lambda r_1}\right) \left(1 - e^{i\lambda r_2}\right) \Phi(d\lambda) + ar^2. \tag{14}$$

Here, $a$ is a constant, $a \geq m^2$, $m$ satisfies the equality $m = m(r)$ $(= \mathsf{E}_{\Delta_r} Y(t))$ and $\Phi$ is a measure on $\mathbb{R}\backslash\{0\}$ such that

$$\int\limits_{\mathbb{R}\backslash\{0\}} \frac{\lambda^2 \Phi(d\lambda)}{1 + \lambda^2} < \infty. \tag{15}$$

**Remark.** *The comparison of formula* (13) *for* $a = 0$ *and formula* (8) *shows that they are very similar (this is not true for the covariance functions of* $X$ *and* $Y$*).*

A.N. Kolmogorov obtained in [8] the following spectral representation for the process $Y$:

$$Y(t) = \int\limits_{-\infty}^{\infty} \left(e^{i\lambda t} - 1\right) Z(d\lambda) + ut + v \tag{16}$$

where $u$ and $v$ are random variables with finite second moments; $Z = Z(A)$, $A \in \mathcal{B}(\mathbb{R})$ is a random measure with orthogonal values and such that

$$\mathsf{E}\, |Z(A)|^2 = \Phi(A), \qquad A \in B(\mathbb{R}).$$

If the process $Y$ is (wide-sense) stationary, then the spectral decomposition (11) can be derived from (16). Similarly to the passage from the stationary processes (which are automatically isotropic in the one-dimensional case) to the homogeneous fields, one can pass from the processes with the stationary increments to the locally homogeneous and locally isotropic $\mathbb{R}^l$-valued vector fields

$$\overline{Y} = \overline{Y}(\overline{t}) = (Y_1(\overline{t}), \dots, Y_l(\overline{t})).$$

11

Let

$$\left\|D_{ij}(\overline{t},\overline{r}_1,\overline{r}_2)\right\| = \left\|\mathsf{E}\Delta_{r_1}Y_i(\overline{t}+\overline{s})\Delta_{r_2}Y_j(\overline{s})\right\|$$

and

$$D_{ij}(\overline{r}) = D_{ij}(0,\overline{r},\overline{r}).$$

For the locally homogeneous and locally isotropic fields, one has

$$D_{ij}(\overline{r}) = \left[D_{ll}(r) - D_{kk}(r)\right]\frac{r_i r_j}{r^2} + D_{kk}(r)\delta_{ij}$$

where $r = \|\overline{r}\|$ and $D_{ll}(r)$, $D_{kk}(r)$ are the longitudinal and transverse structural functions:

$$D_{ll}(r) = \mathsf{E}\left|Y_l(\overline{t}+\overline{r}) - Y_l(\overline{t})\right|^2, \quad D_{kk}(r) = \mathsf{E}\left|Y_k(\overline{t}+\overline{r}) - Y_k(\overline{t})\right|^2.$$

Here, $Y_l(\overline{t})$ is a projection of the vector $\overline{Y}(\overline{t})$ on the direction $\overline{r}$ and $Y_k(\overline{t})$ is a projection of the same vector on the direction orthogonal to $\overline{r}$.

**Remark.** *The functions $D_{ij}(\overline{r})$ are simpler than the functions $D_{ij}(\overline{t},\overline{r_1},\overline{r_2})$. Therefore, it is important to find out whether the latter functions can be expressed by the former ones (that is, through $D_{ll}(r)$ and $D_{kk}(r)$). This can be done under the condition $D_{ij}(\overline{t},\overline{r}_1,\overline{r}_2) \to 0$, $|\overline{t}| \to \infty$ (see {6}). In particular, this can be done in all the applications to the turbulence of the locally homogeneous and locally isotropic random vector fields (for details, see {6}, p. 315 ).*

We will now consider the paper [9], which is closely connected with the paper [8]. A.N. Kolmogorov investigates in [9] the structure of the continuous Gaussian processes $X(t)$, $t \geq 0$ with stationary increments and with the self-similarity property, that is, for any $a > 0$, there exists $b > 0$ such that

$$Law\left(X(at); t \geq 0\right) = Law\left(bX(t); t \geq 0\right).$$

It turns out that such processes with the zero mean have a special correlation function:

$$\mathsf{E}X(t)X(s) = \frac{1}{2}\left(|s|^{2H} + |t|^{2H} - |t-s|^{2H}\right) \tag{17}$$

where $0 < H < 1$.

12

Kolmogorov called such Gaussian processes the "Wiener Spirals". Later they were named the Fractional Brownian Motion.

Note that

$$\mathsf{E}\|X(t) - X(s)\|^2 = |t - s|^{2H}. \tag{18}$$

The parameter $H$ was called the Hurst parameter.

Let us now turn to the main results of the papers [1]–[4], which contain the "law of two-thirds".

Let $\overline{u}(\overline{x}) = (u_1(x_1, x_2, x_3), u_2(x_1, x_2, x_3), u_3(x_1, x_2, x_3))$ be a field of velocities of the turbulent flow at the point $x = (x_1, x_2, x_3)$.

Kolmogorov assumes that this field is locally homogeneous and is locally isotropic. So, unlike Taylor, he introduces the idea of *locality*. This reduces the analysis to the study of the structure of *increments*

$$\overline{u}(\overline{x} + \overline{r}) - \overline{u}(\overline{r}).$$

Further, Kolmogorov introduces "*the first hypothesis*" of similarity. Using this hypothesis, he deduces that the longitudinal structural function $D_{ll}(r)$ has the form $r^m$ for a wide range of values $r$ (compare with (18)).

In order to find the value $m$, Kolmogorov introduces "*the second hypothesis*" of similarity, which, together with the first one, yields $m = 2/3$. Therefore,

$$D_{ll}(r) \sim r^{2/3}. \tag{19}$$

Kolmogorov formulates this result more precisely in the following way: *if the dissipation rate $\varepsilon$ of the kinematic energy is constant, then for the turbulent movements with a very high Reynolds number ($Re = \frac{Lv}{\nu}$ where $L$ and $v$ are length and velocity scales for the whole movement, the so-called typical scales, and $\nu$ is kinematic viscosity) and in the "inertia" interval of scales*

$$\lambda \ll r \ll L$$

*where $\lambda = \varepsilon^{1/4}\nu^{-3/4}$ ("Kolmogorov's interior scale"), one has the following approximation:*

$$D_{ll}(r) \approx C(\varepsilon r)^{2/3} \tag{20}$$

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*where $C$ is a constant. Moreover, $D_{kk}(r) \approx \frac{3}{4} D_{ll}(r)$.*

Along with the above "*correlational formulation*", the fundamental law of the small-scale turbulence (20) permits the *spectral formulation.* For $D_{ll}(r)$, one can obtain the spectral representation similar to (13) (with the spectral measure $\Phi_{ll}(dk) = E_{ll}(k)dk$). It turns out that

$$E_{ll}(k) \sim k^{-5/3} \tag{21}$$

for a wide range of frequencies $k$.

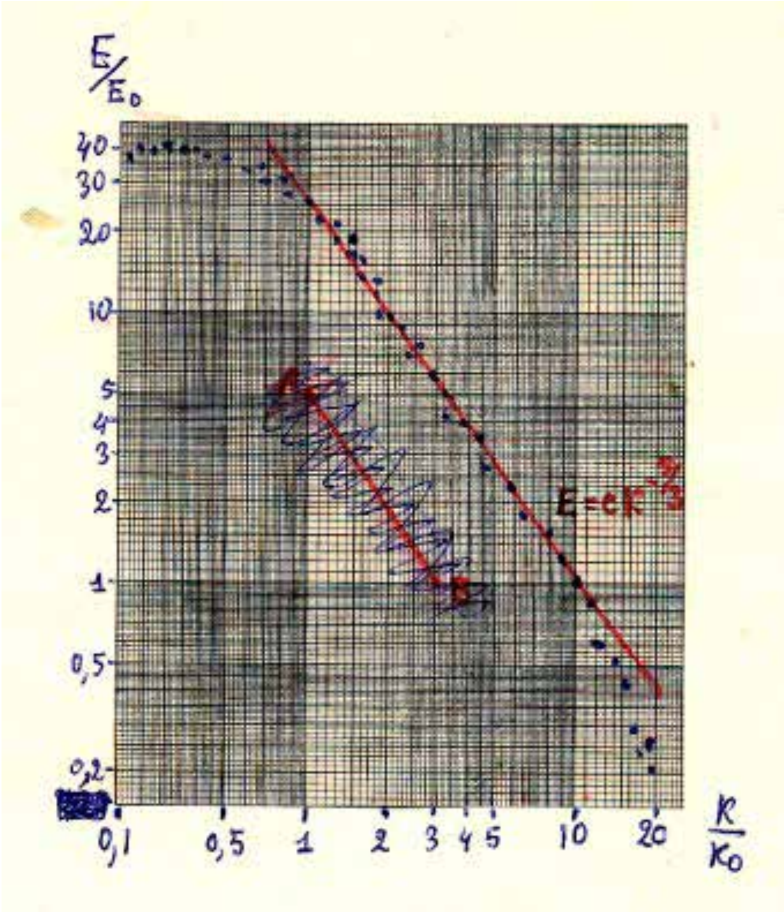


Figure 1: To the law "$E_{ll}(k) \sim k^{-5/3}$" (Kolmogorov's drawing)

In connection with (20) and (21), note that due to the properties of the Fourier transform for $f(\lambda) = |\lambda|^{-\alpha}$ with $1 < \alpha < 3$, one has

$$\int_{-\infty}^{\infty} (1 - e^{i\lambda\tau}) f(\lambda) d\lambda = \tau^{\alpha-1} \int_{-\infty}^{\infty} (1 - e^{i\lambda}) |\lambda|^{-\alpha} d\lambda,$$

and therefore, $\tau^{\alpha-1} = \tau^{5/3-1} = \tau^{2/3}$ for $\alpha = 5/3$.

At the end of the analysis of the main "turbulent" results published in Kolmogorov's papers in the 40-s, I would like to stress that Kolmogorov made these studies, in fact, as a physicist employing clear and natural physical assumptions.

Kolmogorov always used the experimental data in order to justify and verify his hypotheses.

As mentioned above, the *second* period of Kolmogorov's study of the turbulence was in the 60-s. The results of those studies were published in the papers [6] and [7].

These articles as well as the previous ones are notable for their physical style. Here, neither mathematical proofs nor complicated analytical calculations are present, but *three* new hypotheses of similarity are proposed instead of the *two* previous hypotheses.

It is interesting to note that the two previous hypotheses of similarity are related to the velocity increments while two of the new hypotheses are formulated in terms of velocity increments ratios.

These two new hypotheses were supplemented by the third hypothesis postulating the logarithmic normality of the energy dissipation rate $\varepsilon$ and indicating the form of the logarithmic variance of the averaged dissipation rate $\overline{\varepsilon}_r$ (for details, see formulas (1) and (2) in [6]).

It turns out that these three hypotheses make it possible to account for L.D. Landau's remark that the variation of the energy dissipation

$$\varepsilon = \frac{\nu}{2} \sum_{\alpha,\beta} \left( \frac{\partial v_\alpha}{\partial x_\beta} + \frac{\partial v_\beta}{\partial x_\alpha} \right)^2$$

should infinitely increase if the ratio $L : l$ (between the "exterior" scale $L$ and "interior" scale $l$) increases.

## 3 Kolmogorov's Oceanographic Expeditions.

In the first expedition on board the "Dmitry Mendeleev" ship in 1969 and in the second expedition in 1971, A.N. Kolmogorov took an active part in performing the most of the experiments, in carrying out the programs of the measurements and in making the statistical analysis of the collected data.

It should be noted that difficulties are usually encountered in recording the oceanic turbulent fluctuations of flow rate, temperature, electric conductivity, sound velocity, refraction factor and other hydrodynamic parameters. This requires high-sensitivity and low-inertia devices. Serious problems arose in making the experiments during the voyage since the records of natural fluctuations were distorted due to the vibration of the towed devices, caused by the rocking of the ship and due to electric noises in a high-frequency range.

Another problem was that the frequency ranges of the turbulent fluctuations and those of the surface and internal waves largely overlapped. Thus, it was necessary to filter out mechanical and electric noises as well as the fluctuations created by the waves.

Kolmogorov wrote in his report: " My duty, as the Research Director of the hydro-physical investigations in the expedition (1971), was the coordination of the works performed by:

1. the hydrology team;
2. the turbulence team;
3. the small-scale turbulence team;
4. the team of acoustic methods;
5. the team employing the devices to measure the vertical flow of the moisture."

During the voyage in 1971, Kolmogorov wrote some articles (unpublished), for instance, "*The notes about the mathematical treatment of the observation results", "On the techniques employed to obtain integral spectra*".

In the first of these papers, one of the sections is called "The interpolation in depth and the calculation of gradients". Here, Kolmogorov writes that "these methods were used to treat the data of the conventional hydrologic station and to test the thermal sonde. The same methods were useful in restoring the shape of the isotherms from the data obtained. In all the cases, the scale of the observations is much larger than the "inner scale" of the turbulence. Therefore, it is necessary to account for the fact that the variation of the measured magnitudes will be irregular in depth and hence, that the linear interpolation will be useless. However, the linear interpolation could be considered, at least, as “harmless” since it gives intermediate values for the intermediate horizons. Here we do not lose useful information if we change the values obtained with the irregular intervals of the order of 1 m. for the linearly interpolated values on the scale with a 1 m. step.

The quadratic interpolation is also “harmless” in the measurements made with a conventional hydrologic station when non-standard horizons of the direct observations are close to the corresponding standard horizons. For example, if we restore the temperature at the 500 m. horizon from the temperatures at the 407 m., 492 m., 601 m. horizons, we could make the linear interpolation for the 492 m., 601 m. horizons and the extrapolation for the 407 m., 492 m. horizons. The quadratic interpolation gives the excessive average of these two results. By applying the quadratic interpolation, we can gain a lot in accuracy in the “good” case of smooth temperature curves and lose absolutely nothing in the “bad” cases.

The situation is different when the interpolation is made at the center of the interval between the horizons of the direct observations. We have examples of real data where the quadratic interpolation gives much worse results than the linear interpolation for rather typical temperature curves.

One should be even more careful when the interpolation is made by using the polynomials with the degree higher than two. It is notable, however, that the temperature curves are very smooth at great depths. The following exampleis not a single one:

| | | | | | |
|---|---|---|---|---|---|
| 50 | 14.54 | | | | |
| | | 1.24 | | | |
| 100 | 13.30 | | .54 | | |
| | | .70 | | .18 | |
| 150 | 12.60 | | .36 | | −.02 |
| | | .34 | | .20 | |
| 200 | 12.26 | | .16 | | −.00 |
| | | .18 | | .20 | |
| 250 | 12.08 | | −.04 | | |
| | | .22 | | | |
| 300 | 11.86 | | | | |

Here, the third differences are almost constant and the fourth ones are almost equal to zero. In this case, we recommend that the interpolation should be made with the polynomials of the third degree.

If we restore the temperature at the 150 m. horizon by the linear interpolation between the 100 m. horizon and 200 m. horizon, we will get the value of 12.78.

The quadratic interpolation between the 50 m., 100 m., 200 m. horizons will give the value of 12.54.

The interpolation by the polynomial of the $3^{rd}$ degree along the 50 m., 100 m., 200 m., 300 m. horizons gives a precise result (with the accuracy up to 0.01) for the 150 m. horizon.

It would be of interest to find the reasons for getting "smooth" and "unsmooth" curves. But we recommend that the polynomials of the third degree should not be used in standard programs of interpolation.

The gradient must be calculated from the linear or the quadratic approximation of the curve. If the magnitude is measured with a small step in depth and with a noticeable error of each measurement, then it is reasonable that the linear or the quadratic approximation of the curve should be determined by the excessive number of points, using the least squares method".

We will now consider in greater detail the mathematical aspects of the second article "On the techniques employed to obtain integral spectra".

The integral spectrum is the following function of the frequency $k$:

$$J(k) = \int_k^\infty E(k')\,dk'. \tag{22}$$

Here, $E = E(k)$ is the spectral density. By setting

$$H_{k_0}(k) = \begin{cases} 0 & \text{if} \quad k < k_0 \\ 1 & \text{if} \quad k \geq k_0, \end{cases} \tag{23}$$

we get

$$J(k_0) = \int_0^\infty |H_{k_0}(k')|^2 E(k')\,dk'. \tag{24}$$

The equations like (24) naturally arise in connection with the linear transformations of the stationary random processes $X = X(t)$, $t \in \mathbb{R}$

having the following spectral representation (compare with (11)):

$$X(t) = \int_{-\infty}^{\infty} e^{itk}\, Z(dk) \tag{25}$$

where $\mathsf{E}|Z(dk)|^2 = E(k)\, dk$.

Indeed, let $Y(t) = \mathcal{L}\{X\}(t)$ be a linear transformation

$$Y(t) = \int_{-\infty}^{\infty} h(u)\, X(t-u)\, du \tag{26}$$

determined by the *weight* function $h = h(u)$ with the following Fourier transform:

$$H(k) = \int_{-\infty}^{\infty} e^{-iku}\, h(u)\, du. \tag{27}$$

For the physically realizable systems, one has $h(u) = 0$ if $u < 0$.

It follows from (25)–(27) that

$$Y(t) = \int_{-\infty}^{\infty} e^{itk}\, H(k)\, Z(dk) \tag{28}$$

and, obviously,

$$\mathsf{E}|Y(t)|^2 = \int_{-\infty}^{\infty} |H(k)|^2 E(k)\, dk. \tag{29}$$

Comparing (24) and (29), we conclude that it would be possible to estimate the integral spectrum $J(k_0)$ properly (using estimates for $\mathsf{E}|Y(t)|^2$ $(= \mathsf{E}|Y(0)|^2)$) if the frequency characteristic (transfer function) $H(k) = H(k_0, k)$, of a physically realizable filter is "close" to the function $H_{k_0}(k)$ from (23) which corresponds to the non-physically realizable filter. Let us denote by $\hat{J}(k_0)$ an estimate of the integral in (29) using some empirical estimator (of type of the arithmetic mean) for $\mathsf{E}|Y(t)|^2$ $(= \mathsf{E}|Y(0)|^2)$.

Kolmogorov points out that the appropriate selection of functions $H(k) = H(k_0, k)$ should be made with consideration for, at least, a rough idea of how the spectrum $E(k)$ or the integral spectrum $J(k)$ behaves.

A characteristic feature of the turbulence is that the following approximation is rather good for large intervals of the frequency variation.

$$J(k) = C_\beta \cdot k^\beta.$$

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Thus, for the locally isotropic turbulence in the "inertial" range of the frequencies, one has

$$J(k) = C \cdot k^{-2/3}$$

since $E(k) \approx k^{-5/3}$.

In practice, it is very easy to apply the physically realizable filters for which the transfer function $H(k_0, k)$ has the following form:

$$H(k_0, k) = H\Big(\frac{k}{k_0}\Big)$$

where $H = H(\xi)$ is a function of $\xi = k/k_0$.

The spectral density $E(k)$ is usually several orders of magnitude greater for the low frequencies than for the high ones. Therefore, it is necessary that the filter characteristic $H(\xi)$ should sharply fall at low frequencies.

The filters $H_1(\xi)$ of the DISA company were used in the expedition in 1971. Kolmogorov compiled the following table for these filters:

| $\xi$ | 0,01 | 0,02 | 0,05 | 0,1 | 0,2 | 0,5 | 1 | 2 | 5 |
|---|---|---|---|---|---|---|---|---|---|
| $H_1^2(\xi)$ | 0,0001 | 0,0004 | 0,0025 | 0,012 | 0,044 | 0,21 | 0,56 | 0,86 | 1,00 |

It is easily seen from this table that the asymptotics $H_1^2(\xi) \approx \xi^2$ is rather good for $\xi \downarrow 0$. If these two filters $H_1(\xi)$ are placed in series, we obtain the filter $H_2(\xi)$ for which $H_2(\xi) = H_1^2(\xi)$. Then we have

| $\xi$ | $H_2^2(\xi) = H_1^4(\xi)$ |
|---|---|
| 0,01 | 0,00000001 |
| 0,02 | 0,00000016 |
| 0,05 | 0,0000062 |
| 0,1 | 0,00014 |
| 0,2 | 0,0019 |
| 0,5 | 0,044 |
| 1 | 0,31 |
| 2 | 0,74 |
| 5 | 1,00 |

Figure 2 illustrates the behaviour of $H_1^2(\xi)$ and $H_2^2(\xi)$.

The curve $H_2^2(\xi)$ is offset to the right with respect to $H_1^2(\xi)$. Therefore, if $H_1(k_0, k) = H_1\Big(\frac{k}{k_0}\Big)$ and $H_2(k_0, k) = H_2\Big(\frac{k}{k_0}\Big)$, then for a wide

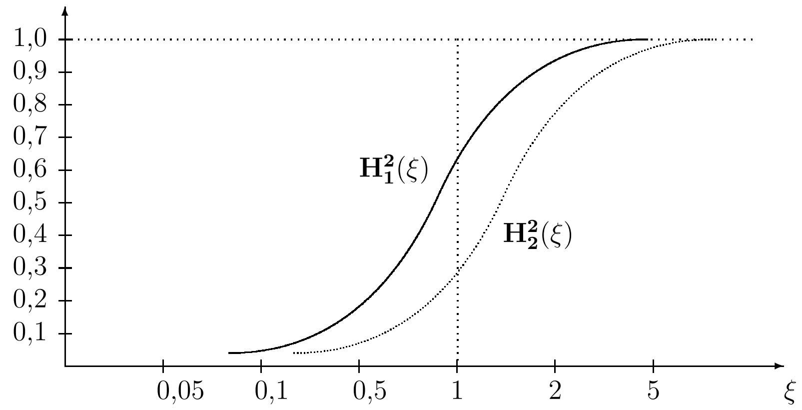


Figure 2: Behaviour of $H_1^2(\xi)$ and $H_2^2(\xi)$

range of frequencies $k$,

$$H_1(k_0, k) \approx H_2(1,35k_0, k).$$

In other words, when using the characteristic $H_2$, we deal with the new "nominal" frequency $k_0' \approx 1,35k_0$ instead of $k_0$ (nominal frequency for $H_1$). This should be remembered when one compares the results of employing different filters.

Furthermore, Kolmogorov gives a better method to make corrections for the filter properties.

Set

$$\Delta(\xi) = \begin{cases} H^2(\xi), & \xi < 1 \\ H^2(\xi) - 1, & \xi \geq 1 \end{cases}$$

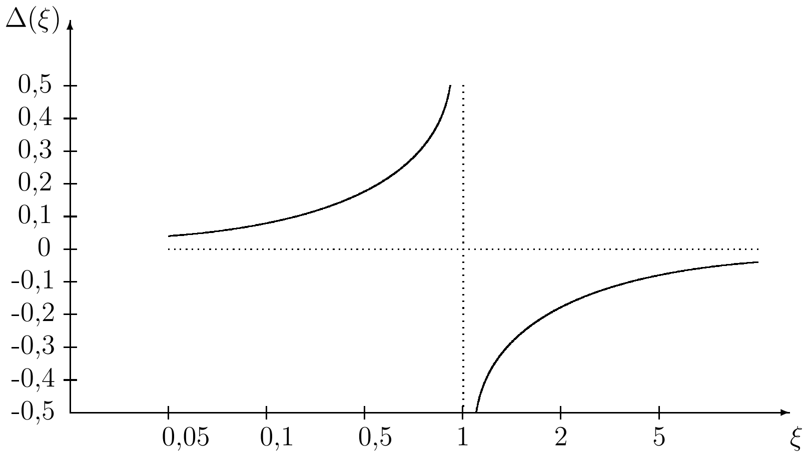


Figure 3: Graph of $\Delta(\xi)$

Then

$$\hat{J}(k_0) - J(k_0) = \int_0^\infty \Delta\Big(\frac{k'}{k_0}\Big)\, E(k')\, dk'.$$

The filters under consideration cut the low-frequency components of the spectrum and pass high frequencies with little distortion. The factor $\Delta\Big(\frac{k'}{k_0}\Big)$ significantly differs from zero in the limited range of the frequencies $k'$ close to the frequency $k_0$. If $J(k) \approx C_\beta \cdot k^\beta$ $(\beta < 0)$ in this frequency range, then $E(k) \approx \beta \cdot C_\beta \cdot k^{\beta-1}$. Set

$$M_\beta = \int_0^\infty \Delta(\xi)\, \xi^{\beta-1}\, d\xi.$$

Then

$$\int_0^\infty \Delta\Big(\frac{k'}{k_0}\Big)\, E(k')\, dk' = \int_0^\infty \Delta(\xi)\, E(k_0\xi)\, k_0\, d\xi \approx$$

$$\approx \int_0^\infty \beta\, C_\beta\, \Delta(\xi)\, (k_o\xi)^{\beta-1} k_0\, d\xi = \beta\, C_\beta\, k_0^\beta\, M_\beta$$

and, therefore,

$$\hat{J}(k_0) - J(k_0) \approx \beta\, C_\beta\, k_0^\beta\, M_\beta.$$

On the other hand, $J(k_0) \approx C_\beta \cdot k_0^\beta$. Consequently,

$$\hat{J}(k_0) \approx k_0^\beta\, C_\beta\, (1 + \beta M_\beta).$$

Since $J(k_0) \approx C_\beta \cdot k_0^\beta$, we deduce that

$$\hat{J}(k_0) \approx J(k_0')$$

if

$$k_0' = k_0\, (1 + \beta M_\beta)^{1/\beta}.$$

In other words, by getting the empirical value of $\hat{J}(k_0)$, we obtain the value of the integral spectrum at the "shifted" point $k_0'$ rather than at $k_0$.

Let

$$\Delta^{(1)}(\xi), \quad M_\beta^{(1)}$$

and

$$\Delta^{(2)}(\xi), \quad M_\beta^{(2)}$$

denote the variables $\Delta(\xi)$ and $M_\beta$ for the filters $H_1(\xi)$ and $H_2(\xi)$, respectively. Then we find the following values for $C_\beta^{(i)} = (1+\beta M_\beta^{(i)})^{1/\beta}$:

| $\beta$ | -2 | -1,5 | -1 | -0,5 | 0 |
|---|---|---|---|---|---|
| $C_\beta^{(1)}$ | 0 | 0,52 | 0,62 | 0,72 | 0,83 |

and

| $\beta$ | -4 | -3 | -2 | -1 | 0 |
|---|---|---|---|---|---|
| $C_\beta^{(2)}$ | 0 | 0,75 | 1,00 | 1,20 | 1,36 |

As for the evaluation of $\beta$, we have

$$\ln J(k_0) \approx \ln C_\beta + \beta \ln k_0$$

assuming that $J(k_0) \approx C_\beta \cdot k_0^\beta$. Therefore, $\beta$ is evaluated by the slope at the point $k_0$.

In conclusion, I would like to express my hope that I have illustrated some of Kolmogorov's ideas in the statistical theory of turbulence and also his views and methods of working with real statistical data in practical situations.

P.-S. Laplace used to address the mathematicians with the words: *"Read Euler, read Euler, he is our common teacher."* We may rightfully refer these words to Kolmogorov too.

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## 31.4 FROM NAVIER-STOKES TO NIG

It was and still is a matter of debate whether a detailed understanding of the nature of turbulence will be best achieved via the Navier-Stokes equations driven by random forces or whether turbulence is inherently of a basic random nature that one should attempt to model directly. To quote from Tsinober's book An Informal Conceptual Introduction to Turbulence: "Whatever the origins of real turbulence, (whether turbulence is a priori random/stochastic because Nature is such or the intricacy of turbulent flows arises out of deterministic equations like NS or any other unknown reason), turbulent flow states are so complicated that the use of statistical tools is unavoidable."

However, the two approaches are leading to complementary insights. An instance of the synergy of the two approaches is provided by the work of Björn Birnir on a stochastic version of the Navier-Stokes equations.

In 2013 Björn Birnir, who is from Iceland and Professor of Mathematics and Director of Centre for Complex and Nonlinear Science at the University of California at Santa Barbera, gave a talk at a number of Universities in North America on the Navier-Stokes Millenium Problem. This talk was based on his paper: The Kolmogorov-Obukhov Statistical Theory of turbulence. Nonlinear Science (2013) 23, 657-688, cf. also his book The Kolmogorov-Obukhov Theory of Turbulence (2013). In those works, he shows that by a certain choice of stochastic forcing of the Navier-Stokes equations the evolution of the increments of the velocity in the longitudinal direction follows the normal inverse Gaussian law exactly.

Some slides from that talk are reproduced on the next pages, with his permission.

Turbulence

Birnir

The Millennium Problem

Laminar versus Turbulent

The Stochastic Navier-Stokes Equation

The Invariant Measure of Turbulence

Comparison with Simulations and Experiments

Conclusions

# The Navier-Stokes Millennium Problem: Laminar versus Turbulent Flow

Björn Birnir

Center for Complex and Nonlinear Science
and
Department of Mathematics, UC Santa Barbara

*UC Santa Barbara, May 2nd, 2013*

## The Deterministic Navier-Stokes Equations

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Birnir

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- A general incompressible fluid flow satisfies the Navier-Stokes Equation

$$\begin{aligned} u_t + u\cdot\nabla u &= \nu\Delta u - \nabla p \\ u(x,0) &= u_0(x) \end{aligned}$$

with the incompressibility condition

$$\nabla\cdot u = 0$$

- We impose periodic boundary conditions:

$$u(x+e_{x_i},t) = u(x,t),\ e_{x_i} \text{ unit vector in } \mathbb{R}^2 \text{ or } \mathbb{R}^3$$

- Eliminating the pressure using the incompressibility condition gives

$$\begin{aligned} u_t + u\cdot\nabla u &= \nu\Delta u + \nabla\Delta^{-1}\text{trace}(\nabla u)^2 \qquad (1) \\ u(x,0) &= u_0(x) \end{aligned}$$

## The Millennium Problem



- The Millennium Problem is: prove that there exists a smooth solution $u(x,t)$ of the initial value problem (1) in three-dimensions, $x \in \mathbb{T}^3 = \mathbb{R}^3/\mathbb{Z}^3$
- In other words, show that there exists a divergence free function $u \in C^\infty(\mathbb{T}^3 \times [0,\infty))$ that satisfies the initial value problem, IVP (1)
- We need an infinitely differentiable function that both solves the Navier-Stokes equation and takes on the initial condition $u(x,0) = u_0(x)$, when $t = 0$

## The Difference between Laminar and Turbulent

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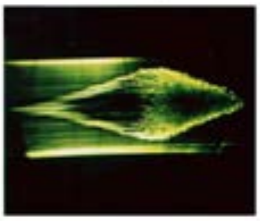
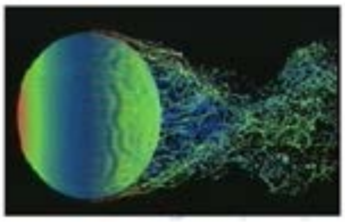
- If the Reynolds number is small, only the laminar solution exists
- In this case the ambient noise is quelled
- If the Reynolds number is large, the laminar solution exists but is unstable
- The ambient noise is magnified by the instabilities of the laminar flow and becomes large
- Then the turbulent solution satisfies a stochastic partial differential equation (SPDE)





## The Kolmogorov-Obukhov Theory



- In 1941 Kolmogorov and Obukhov [6, 5, 9] proposed a statistical theory of turbulence
- The structure functions of the velocity differences of a turbulent fluid, should scale with the distance (lag variable) $l$ between them, to the power $p/3$

$$E(|u(x,t) - u(x+l,t)|^p) = S_p = C_p l^{p/3}$$


A. Kolmogorov


A. Obukhov

## Onsager's Observation

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Lars Onsager

- In 1945 the physicist L. Onsager made a profound observation [11]
- Fluid velocity that satisfied the Kolmogorov-Obukhov scaling can not have a a gradient $\nabla u$ continuous in $x$
- The precise mathematical term is that $u$ is Hölder continuous in $x$ with Hölder index $1/3$
- This means that the turbulent solutions are not smooth
- The solution of the Millennium problem must be the laminar solution

## Stochastic Navier-Stokes with Turbulent Noise

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- The stochastic Navier-Stokes equations describing fully developed turbulence

$$du = (\nu\Delta u - u\cdot\nabla u + \nabla\Delta^{-1}\mathrm{tr}(\nabla u)^2)dt$$

$$(\text{additive noise } =) + \sum_{k\neq 0}[c_k^{\frac{1}{2}}\, db_t^k\, e_k(x) + d_k|k|^{1/3}dt]\; e_k(x)$$

$$(\text{multiplicative noise } =) + u(\sum_{k\neq 0}^{M}\int_{\mathbb{R}} h_k\bar{N}^k(dt,dz))$$

$$u(x,0) = u_0(x)$$

- Each Fourier component $e_k = e^{2\pi k\cdot x}$ comes with its own Brownian motion $b_t^k$ and deterministic bound $|k|^{1/3}dt$

## The Probability Density Function (PDF)

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**Lemma**

*The PDF is a Normalized Inverse Gaussian distribution NIG of Barndorff-Nilsen [1]:*

$$f(x_j) = \frac{(\delta/\gamma)}{\sqrt{2\pi}K_1(\delta\gamma)}\,\frac{K_1\left(\alpha\sqrt{\delta^2+(x_j-\mu)^2}\right)e^{\beta(x-\mu)}}{\left(\sqrt{\delta^2+(x_j-\mu)^2}/\alpha\right)} \qquad (7)$$

*where $K_1$ is modified Bessel's function of the second kind,* $\gamma = \sqrt{\alpha^2-\beta^2}$.

$$f(x)\sim\frac{(\delta/\gamma)}{2\pi K_1(\delta\gamma)}\frac{\Gamma(1)2e^{\beta\mu}}{(\delta^2+(x-\mu)^2)},\ x<<1$$

$$f(x)\sim\frac{(\delta/\gamma)}{2\pi K_1(\delta\gamma)}\frac{e^{\beta(x-\mu)}e^{-\alpha x}}{x^{3/2}},\ x>>1$$

O. Barndorff-Nilsen

## The log of the PDF from simulations and fits for the longitudinal direction

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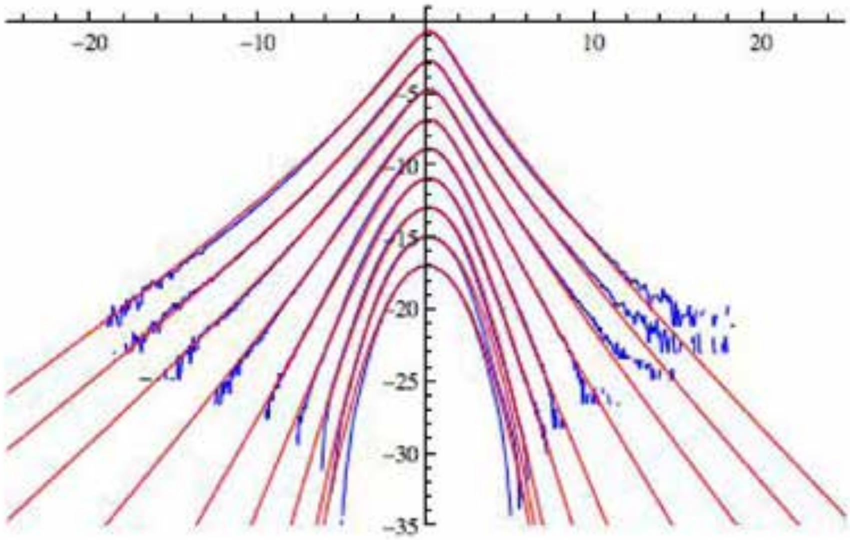


Figure: The log of the PDF from simulations and fits for the longitudinal direction, compare Fig. 4.5 in [13].

## The Artist by the Water's Edge
### Leonardo da Vinci Observing Turbulence

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# UNIVERSALITY

# 32 THE CONCEPT OF UNIVERSALITY

In general, universality of a phenomenon is taken to mean that it is common to all members of a class or classes. A statement of universality may consist in a somewhat rough outline of some trait observed and of the class to which it is thought to apply; or it can be very specific and concrete. In the former case one often speaks of a stylised feature rather than universal feature, having in mind that further investigation may reveal a more definite specification. Stylised features on their own may well be of considerable usefulness, but without theoretical underpinning they tend to be of limited interest and scientific impact.

In most areas of science the development of new theory proceeds via verbal description of stylised features, based on empirical facts, followed by theoretical underpinning, often via mathematical formulations. (Pure mathematics and certain parts of theoretical physics constitute exceptions to this.) Typically, the proper mathematical formulations are arrived at in a series of steps each providing a better representation of the empirical data.

The developments, described earlier, from the study of windblown sands to the establishment of the normal inverse Gaussian as a law with advanced probabilistic properties and the capability to accurately represent empirical distributions from a great variety of fields provides an example of universality.

# 33 UNIVERSAL LAWS IN TURBULENCE

A main goal of Jürgen Schmiegel's and my work has been to represent the universality tenets of the Kolmogorov-Obhoukov Statistical Theory of Turbulence, that is the second and third order scaling laws and the properties of the Kolmogorov variable, from the stochastic modelling point of view.

A further aim has been to detect new universal characteristics of homogeneous turbulence. Two such results are discussed in Section 33.2 below, another will be considered in Section ##.

Throughout, our theoretical investigations have to a large degree been linked to studies of extensive data sets from a wide variety of experimental conditions.

## 33.1 THE KOLMOGOROV-OBHOUKOV TENETS AND BSS MODELLING

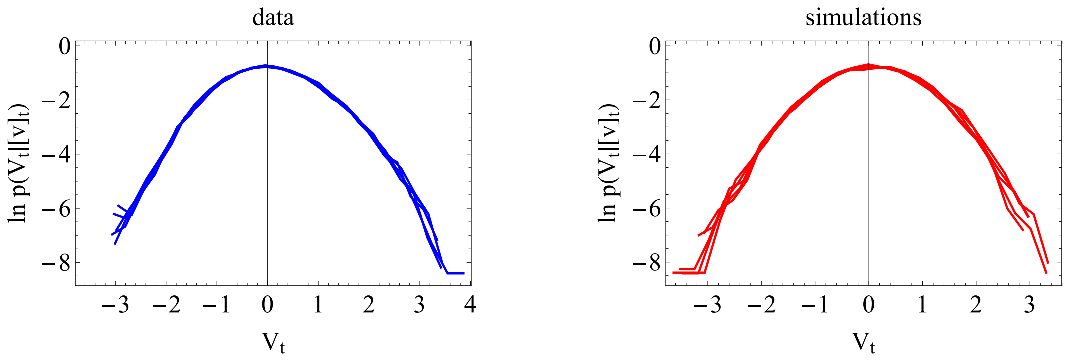
Briefly speaking and in somewhat colloquial language, the Kolmogorov-Obhoukov tenets state:

1. in homogeneous turbulence the second and third order powers of the velocity differences increase, within a certain range of spatial distance r (the inertial range), as r raised to the power 2/3 and 1, respectively
2. standardising the velocity differences by the 1/3 power of the total intermittency over the considered lag interval yields stochastic variables V(r) with distributions that for r much less than the 'spatial' size' L of the experimental setting depend only on the local Reynolds number Re(r). Moreover, for Re(r) much greater than 1 the distribution of V(r) does not even depend on Re(r) and is therefore universal.

The local Reynolds number Re(r) expresses the degree of turbulence; the larger the value of Re(r) the more pronounced the turbulence.

The hypotheses in (ii) were proposed by Kolmogorov in (1962) in a paper referred to as K62. They are based on an uncanny insight, relating large scale behaviour to small scale behaviour in a way that defies ordinary statistical thinking (note that the total intermittency is raised to the one third power) and they encapsulate the unique nature of turbulence.

Our quest for a stochastic model that provides a fullfledged account of the statistical aspects of turbulence as formulated by the Kolmogorov-Obhoukov tenets began with three papers [BN, Schmiegel (2004)], [BN, Blæsild, Schmiegel (2004)] and [BN, Schmiegel (2009)] where the third introduced the concept of BSS processes, described earlier in Sections **. Through a number of steps, the project has been brought to fruition, in the final phase through the work presented in [Hedevang, Schmiegel (2013)] and [Urbina, Schmiegel (2015)]. The Figure is essential in documenting the agreement of the BSS-based model and the character of the distribution of the Kolmogorov variable.




**Comparison of the conditional densities of the Kolmogorov variable from data and from a simulation of the model established in [Marquez, Schmiegel (2015)]. The simulated distributions are superposed on each other, illustrating the collapse of the distributions, as hypothesised by Kolmogorov.**

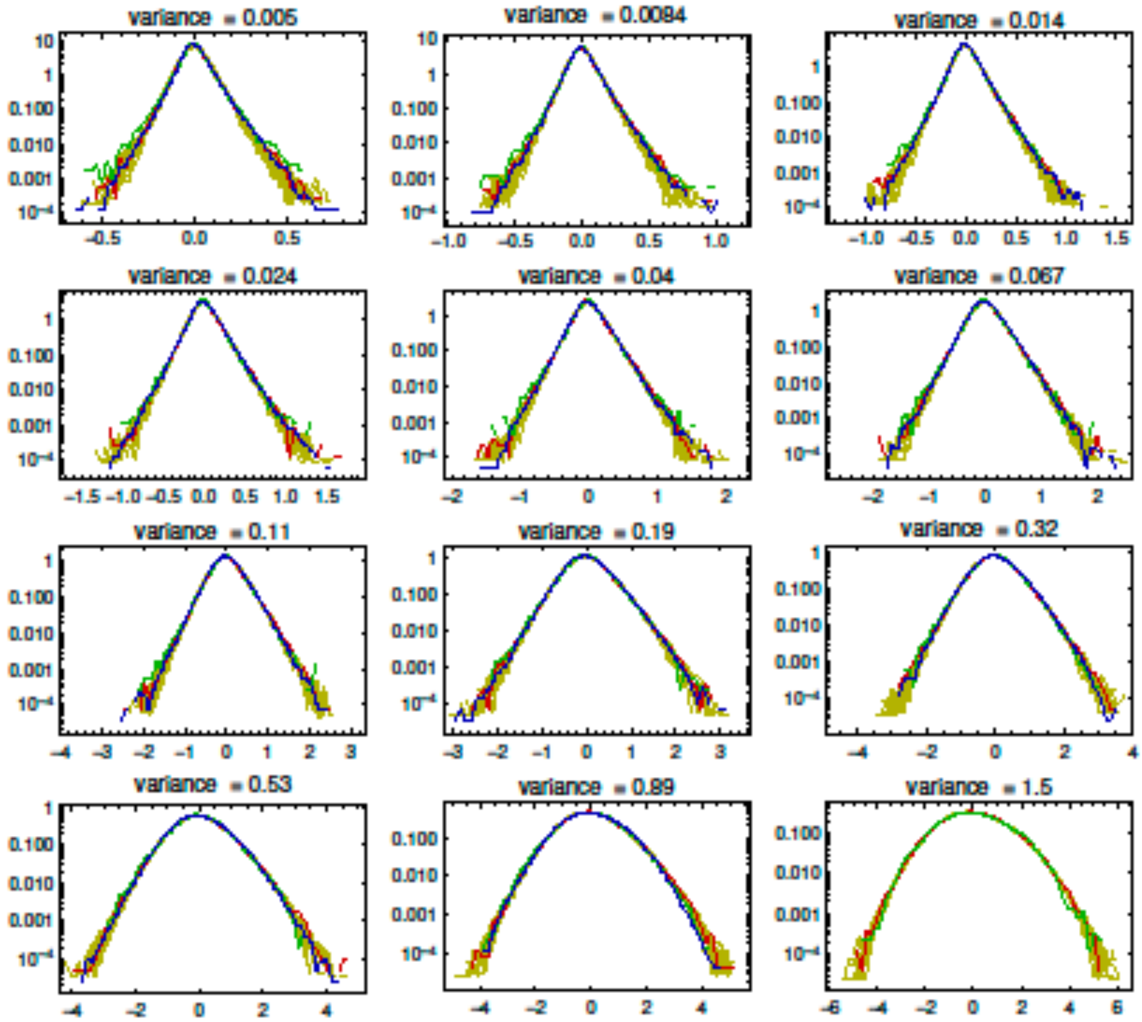
## 33.2 INCREMENTAL SIMILARITY OF VELOCITY INCREMENTS

Extensive measurements are available of velocities in a great variety of experiments where the flows have widely differing Reynolds number and are considered as homogeneously turbulent. Measurements consist of recordings, at a fixed position in the flow, of the time-wise behaviour of the main component of the three-dimensional velocity vector (that is the component in the direction of the fluid flow). Subsequently the timewise increments of the measurements are reinterpreted as spatial increments by means of Taylors Frozen Field Hypothesis

In Turbulence a mainstay is *'Taylor's Frozen Field Hypothesis'* which provides an interpretation of the timewise behavior of the main component of the velocity vector at a fixed point in space in terms of its behavior in space at a fixed point of time. Could this be what Richard Wagner had in mind by the much debated enigmatic words of Gurnemanz in Parsifal: 'Zum Raum wird hier die Zeit'? :)

A detailed analysis of the recordings from a number of such sets of experiments revealed a remarkable phenomenon. To each experiment corresponds a family of distributions, the members of the family corresponding to increments over different time lags. Now consider any pair of experiments, say experiment i and experiment j. To any time lag s under experiment i there is a time lag s' under experiment j such that the corresponding two distributions have the same variance, which is a trivial observation. However, highly not trivial in the fact that not only do they have the same variance but they are in fact identical. The adjacent Figure illustrates this. The 'incremental similarity' thus determined is a striking universality result.

An empirical study and conclusion of this type calls for an explanation of the form of a mathematical model. The study and a model of the kind in question were presented in the paper [BN, Hedevang, Schmiegel (2017)].




Collapse of empirical distributions, demonstrating the phenomenon of incremental similarity. The data are from 16 different experiments, distinguished here by the different colours.

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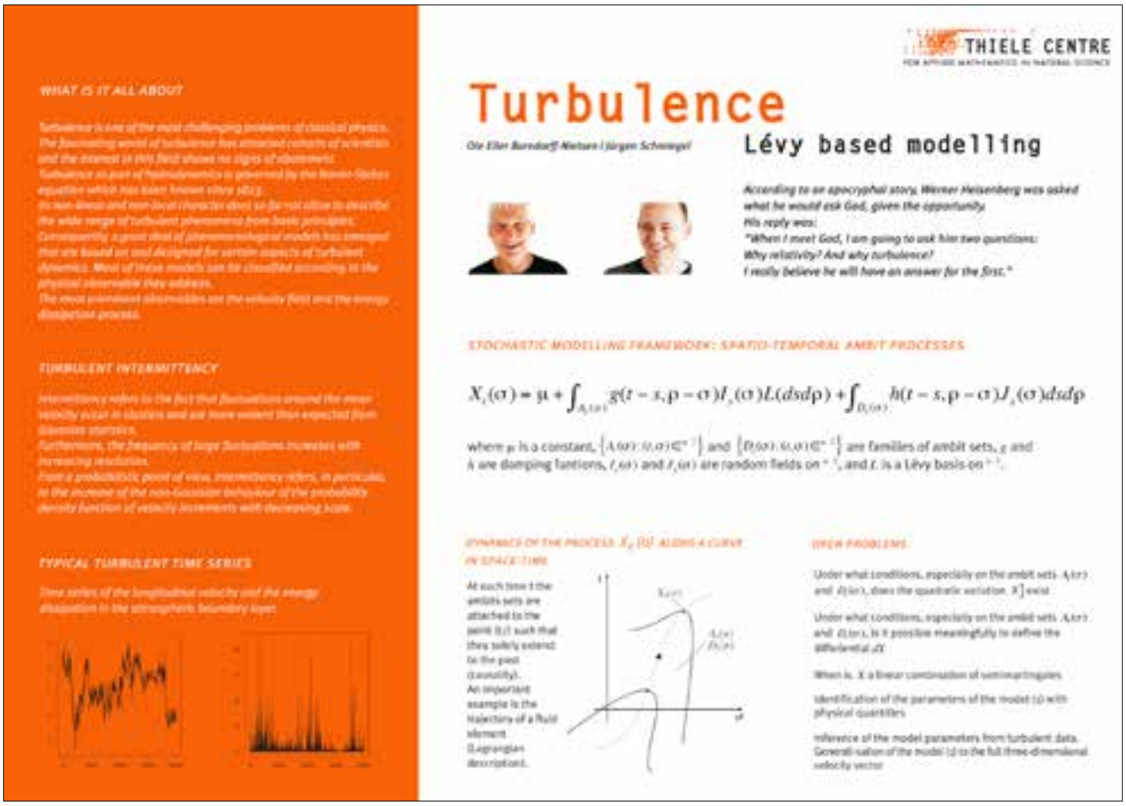
# AMBIT STOCHASTICS

In full generality Ambit Stochastic[1] concerns a class of probabilistically formulated models for describing dynamic processes in space-time.

# 34 CHARACTER AND SCOPE

The concept of Ambit Stochastics grew out of a study of turbulence that Jürgen Schmiegel and I began around the year 2000.

The poster shown here is from a relatively early phase of the study.



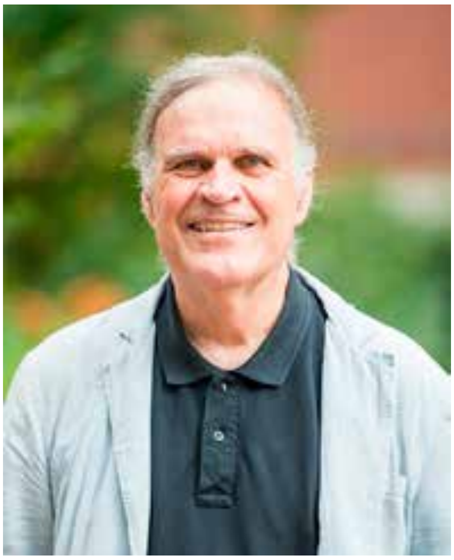

The poster conveys the essence of Ambit Stochastics. Here space is two-dimensional and observations are made along the stipulated curve, as time moves on. To each point on the curve is associated a subset of space, as indicated in the Figure, and those sets are called *ambit sets*.

At the point indicated by a black dot the value of the observation is conceptually the 'sum' (or in technical terms the integral) over quantities indicated by the various circles belonging to the subset of space delineated by the curved boundary that passes through the point.

Two fundamental examples of ambit sets are the light cone and the sound cone, the light cone being the region of the Universe from which, at any given moment, light can reach the observer. The fact that the speed of light is finite implies that part of the Universe is too far away for light to have reached the Earth at the time of observation; and similarly for the speed of sound, which is also finite.

The following quotation, which succinctly indicates the nature of Ambit Stochastics, is from a referee statement written by Edward Waymire in connection with an application to the Villum Foundation for support of a research centre on Ambit Stochastics in Aarhus University, with Mark Podolskij as Director.

> "The key underlying scientific concept is that of 'ambit' stochastic processes and random fields introduced by Barndorff-Nielsen and Jürgen Schmiegel in a decade old publication devoted to turbulence. Their remarkable insight into the essential structure common to this widely observed but poorly understood phenomena has subsequently witnessed remarkable growth in research interest and productivity extending beyond its origins in turbulence theory. Noted applications have already occurred in bio-imaging and in economics. The ubiquitous nature of ambit process makes their study promising for improved understanding and analysis of financial markets, as well as a diversity of biological, chemical and physical processes.
>
> In its most primitive formulation the notion of an ambit process involves stochastic evolution along an embedded space-time curve in such a way that the value at a space-time point on the curve is a random variable depending only on prior values at that point. As I see it, the strength of this idea can be appreciated in relation to another ubiquitous structure that ranks among the most profound ideas of probability theory, yet generally too restrictive for the phenomena addressed by ambit processes; namely, Markov processes. While there have been a number of prominent attempts to adapt Markov process theory to turbulence, for example, it is generally realized to be too restrictive. The notion of ambit processes provides a subtle departure to remedy this fit of theory to observation."



Edward Waymire is Professor of Mathematics at Oregon State University, well known for his work on advanced aspects of Stochastics with particular focus on applications of probability and stochastic processes to problems involving flow and/or dispersion.

Mathematically this involves the interplay between probability and partial differential equations as a two-way avenue. Example areas of application include fluid flows and dispersion of solutes in heterogeneous porous media.

**Edward Waymire**

# 35 THE AMBIT STOCHASTICS PROJECT

The application to the Villum Foundation mentioned above was successful and the resulting Ambit Stochastics Project with Mark Podolskij as leader, was inaugurated in 2016. Mark had recently been called to a Professorship at Department of Mathematics at Aarhus University.

The following is an abbreviated version of the outline of the Project as presented on the project website.

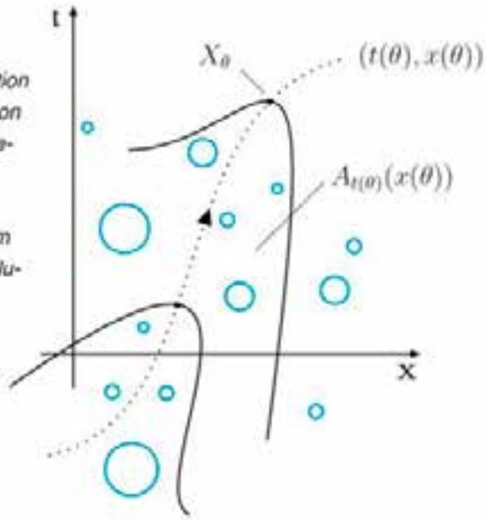
Ambit Stochastics deals with the study of random objects whose properties depend on time and spatial position (or any other type of variables). The variability in space and time is controlled through specific regions in space and time, the so-called ambit sets, and encompasses additional basic stochastic variation, the so-called intermittency/volatility.

Albert Shiryaev and Mark Podolskij

This approach is very general and comprises in particular the basic idea of a causality cone in the past that is fundamental in physics. Accordingly, Ambit Stochastics has the potential to be applied in many fields of sciences where the variability at a certain point can be partly traced back to what happened in a region associated to this point.

The initialising example for the application of Ambit Stochastics to real phenomena is turbulence. Over the past few years a unifying modelling framework has been developed that is able to capture the main stylized features of turbulent flows. The mathematical research in this direction has matured to a stage where more extensive data acquisition, analysis and comparison is called for.

*A conceptual illustration of an ambit process on a path through space-time. The ambit sets define the causality cone: the events from the past that may influence the present.*

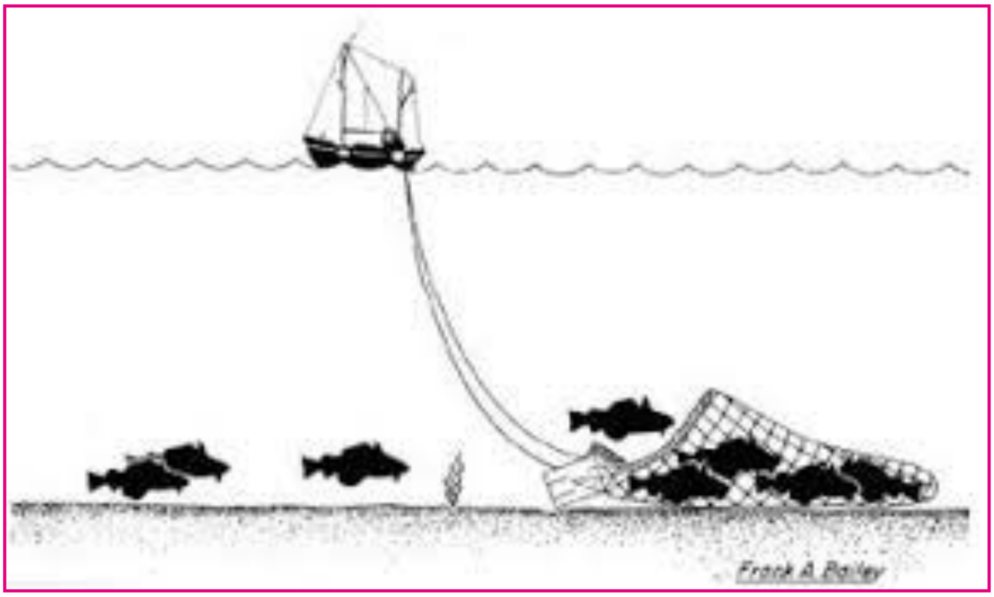


This constitutes an exciting interplay between theory and experiment, typical for the development of the whole field of Ambit Stochastics.

# 36 TRAWL PROCESSES AND ENERGY DISSIPATION

A special class of ambit fields are the trawl processes. Suppose space is two- or three-dimensional and an ambit set of a fixed chosen form is dragged along in a particular direction. At any given time the value of the process is the sum of all 'quantities' contained in the ambit set. These quantities may be random numbers distributed randomly in a homogeneous fashion according to what is called a Levy basis.

Figuratively one may think of trawling for fish, the process value being for instance the number or the collective weight of fish in the net (the ambit set).




One application of trawl processes is to the description and study of energy dissipation in turbulence. There the ambit set is determined by physical considerations, as described in Section ##. Other applications have been to the modelling of stochastic volatility generally, to electricity markets and to integer valued processes of relevance to financial econometrics.

# 37 TUMOR GROWTH

Yet another application of Ambit Stochastics has been to the modeling of tumor growth, as discussed in a paper by Jürgen Schmiegel: 'Self-scaling of tumor growth' published in Physica A (2006). The following citation from that paper indicates the nature of his investigation.

> "We study the statistical properties of the star-shaped approximation of in vitro tumor profiles. The emphasis is on the two-point correlation structure of the radii of the tumor as a function of time and angle. In particular, we show that spatial two-point correlators follow a cosine law. Furthermore, we observe self-scaling behaviour of two-point correlators of different orders, i.e. correlators of a given order are a power law of the correlators of some other order. This power-law dependence is similar to what has been observed for the statistics of the energy-dissipation in a turbulent flow. Based on this similarity, we provide a Lévy based model that captures the correlation structure of the radii of the star-shaped tumor profiles."

The paper illustrates a significant aspect of Ambit Stochastics. The mathematical theory of Fractality is an interesting topic in itself, but as regards applications to modelling of real data it has been found to lack in ability to capture more than a single aspect of the characteristics exhibited by the data. There Ambit Stochastics provides a flexible and encompassing alternative approach.

# 38 VOLATILITY/ INTERMITTENCY

A key characteristic of the Ambit Stochastics framework, which distinguishes this from other approaches, is that beyond the most basic kind of random input it also specifically incorporates additional, often drastically changing, effects referred to as volatility or intermittency.

Such "additional" random fluctuations generally vary, in time and/or in space, in regard to intensity (activity rate and duration) and amplitude. Typically, the volatility/intermittency may be further classified into continuous and discrete (i.e. jumps) elements, and long and short term effects. In turbulence the key concept of energy dissipation is subsumed under that of volatility/intermittency.

The concept of (stochastic) volatility/intermittency is of major importance in many fields of science. Thus volatility/intermittency has a central role in mathematical finance and financial econometrics, in turbulence, in rain and cloud studies and other aspects of environmental science, in relation to nanoscale emitters, magneto- hydrodynamics, and to liquid mixtures of chemicals, and last but not least in the physics of fusion plasmas.

As described here, volatility/intermittency is a relative concept, and its meaning depends on the particular setting under investigation. Once that meaning is clarified the question is how to assess the volatility/intermittency empirically and then to describe it in stochastic terms, for incorporation in a suitable probabilistic model. Important issues concern the modelling of propagating stochastic volatility/intermittency fields and the question of predictability of volatility/intermittency.

# 39 APPLICATIONS TO FINANCIAL ECONOMETRICS

The first applications of ambit stochastics to financial econometrics were given in joint work with Neil Shephard, as a natural continuation of our earlier work discussed in Section 29.3. This application was in fact based on a trawl model and concerned the notion of latency financial econometrics.

Shortly after that I began, together with Fred Espen Benth and Almut Veraart, to consider further applications.

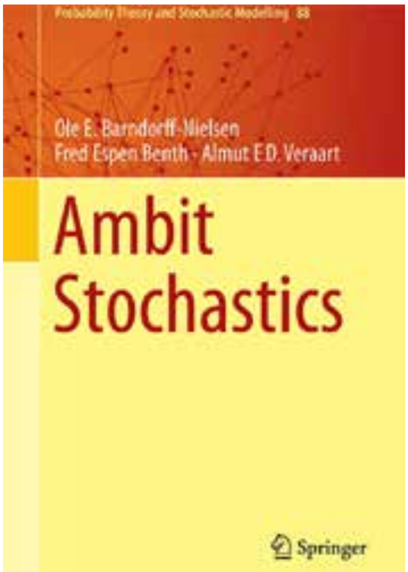
My acquaintance and collaborations with Fred Espen Benth goes back to 1999 where he came to Aarhus from Oslo as the first postdoc under the newly established research centre MaphySto. As mentioned in Section ## we worked together on some problems in Quantum Probability relating to laser cooling. Much later, following my work with Neil Shephard, it turned out that we had some common interests in aspects of mathematical finance and financial econometrics.

I got to know Almut Veraart during a visit to Neil Shephard in Oxford where she was a PhD student of his. This led to her coming to Aarhus shortly afterwards, as a Post Doc Student at CREATES.

The collaborations with Fred and Almut on financial econometrics first concerned questions that lay naturally in the wake of Neil's and my work on building models for financial time series that specifically incorporate stochastic volatility processes. This has included modelling of prices in energy markets.

# 40 THE GENERAL PICTURE

My collaborations with Almut Veraart and Fred Espen Benth in Mathematical Finance developed into considerations of Ambit Stochastics as a field generally, with further research contributions and, eventually, to our writing a book reviewing the field as of 2017-18.




# 41 APPLICATIONS TO TURBULENCE

Sections 36 and 37 above discussed two cases of applications of ambit stochastics to the study of the statistical theory of homogeneous turbulence, both establishing new universal features and both relying on the NIG law.

A third example, relating to energy dissipation, is described in the following Section.

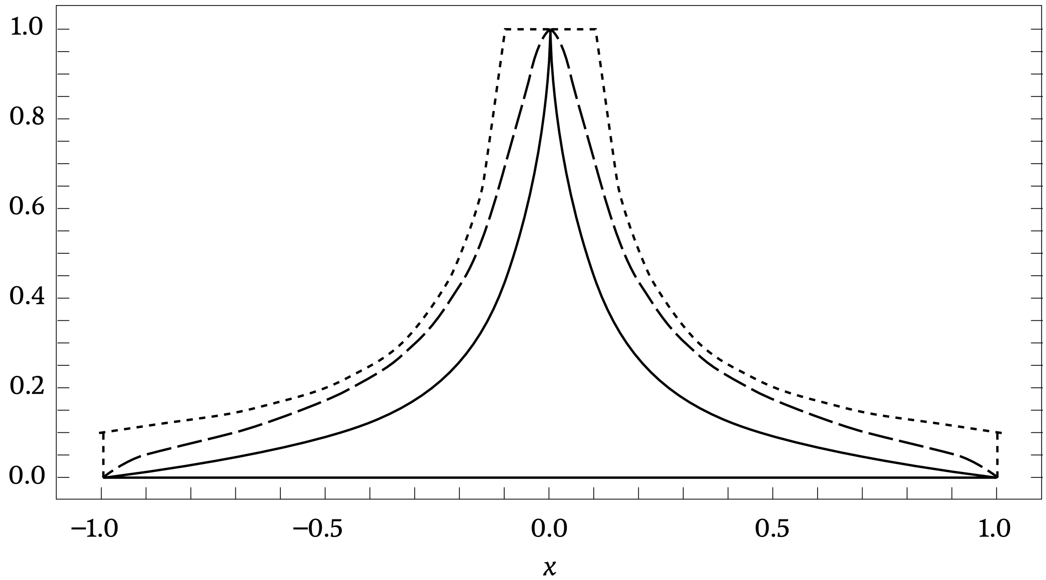
## 41.1 UNIVERSALITY OF THE LAW OF LOGARITHMIC ENERGY DISSIPATION

The below citation is from of a paper by Emil Hedevang and Jürgen Schmiegel from the Journal of Turbulence that establishes a new universality property of the energy dissipation in homogeneous turbulence.

"We discuss continuous cascade models and their potential for modelling the energy dissipation in a turbulent flow. Continuous cascade processes, expressed in terms of stochastic integrals with respect to Lévy bases, are examples of ambit processes. These models are known to reproduce experimentally observed properties of turbulence: the scaling and self-scaling of the correlators of the energy dissipation and of the moments of the coarse-grained energy dissipation. We compare three models: a normal model, a normal inverse Gaussian model, and a stable model. We show that the normal inverse Gaussian model is superior to both, the normal and the stable models, in terms of reproducing the distribution of the energy dissipation; and that the normal inverse Gaussian model is superior to the normal model and competitive with the stable model in terms of reproducing the self-scaling exponents. Furthermore, we show that the presented analysis is parsimonious in the sense that the self-scaling exponents are predicted from the one-point distribution of the energy dissipation, and that the shape of these distributions is independent of the Reynolds number."

The empirical data consisted of 13 data sets each comprising a total of approximately 16 million one-point time records of the velocity component in the mean stream direction in helium gas jet flows.

The basis for the derivation of this result was an ambit stochastics model where the ambit sets are as shown in the adjacent Figure. The curves are constructed so as to satisfy Taylor's frozen field hypothesis.




## 41.2 TWO-DIMENSIONAL TURBULENCE

The investigations discussed above concern studies of three-dimensional turbulence. The character of two-dimensional turbulence is dramatically different from that of three-dimensional settings. Thus, for instance, in two-dimensional turbulence the energy is transported to large scales whereas in the three-dimensional case the energy is spreading continuously to smaller and smaller scales, eventually being lost as heat by dissipation. Another important difference is the fact that vortices are ubiquitous in two-dimensional turbulence.

There is ample scope for realistic stochastic modeling of two-dimensional turbulence, in particular of ambit type.

To a large extent the empirical data on two-dimensional turbulence are of experimental origin. However, the atmosphere, both on Earth and on Jupiter, provides conditions for approximate two-dimensional flows in the form of hurricanes.

**Jupiter Blues Jovian clouds – vortex structures – in shades of blue.**
**View taken from NASA's Juno space craft October 2017.**

Water running down a window pane provide yet another example of two-dimensional turbulence. In my childhood common instances of this were seen around town at the window panes of dairy and cheese stores where the resulting cooling effect was essential for the preservation of the goods.

In applying Ambit Stochastics to two-dimensional turbulence the starting problem is to construct a two-dimensional ambit velocity field which is isotropic and divergence free, but exhibit skewness 0. The basic stochastic input is a Levy basis on a two-dimensional field.

The adduced poster on next page summarises results from an early phase of the project.

# Modelling 2 Dimensional Turbulence via Ambit Fields

O.E. Barndorff-Nielsen[1], Orimar Sauri[1,2], and Jürgen Schmiegel[1,3]

[1]Department of Mathematics, Aarhus University
[2]Department of Mathematical Sciences, Aalborg University
[3]Department of Engineering, Aarhus University

## 1. Aim

Motivated by the 2 dimensional turbulence framework, we build two classes of **isotropic** and stationary ambit fields that are **incompressible**, **rotational** and with **non-null skewness** on its projected increments. We employ two different approaches. In the first, we give a direct specification of the ambit field, while in the second we use the concept of stream functions.

## 2. Ambit fields and stream functions

We propose to model the velocity field of two-dimensional homogeneous fluid by a purely spatial stationary **ambit field**. More precisely, if $u(p)$ denotes the velocity vector, we let

$$u(p) = \int_{\mathcal{R}+p} F(p-q)L(\mathrm{d}q), \quad p \in \mathbb{R}^2, \tag{1}$$

where

- $F : \mathbb{R}^2 \to \mathbb{R}^d$ is a deterministic function, known as **kernel**.
- $\mathcal{R}$ is a measurable subset of $\mathbb{R}^2$ termed as **ambit set**.
- $L$ a homogeneous **Lévy basis**, short for infinitely divisible and independently scattered random measure satisfying that $L(\cdot + x) \overset{d}{=} L(\cdot)$.

If we let

$$F(q) = g(\|q\|)R_\phi q, \quad q \in \mathbb{R}^2, \tag{2}$$
$$\mathcal{R} = \{q : a \leq \|q\| \leq b\}, \quad 0 \leq a < b., \tag{3}$$

where $g$ is a real-valued function, and $R_\phi$ is the rotation matrix on $\mathbb{R}^2$ of angle $\phi \in [0, 2\pi)$, then $u$ has **isotropic increments**, i.e. for any $p_0 \in \mathbb{R}^2$ and $\theta \in [0, 2\pi)$ it holds that

$$\left\{R_\theta^{-1}[u(R_\theta(\cdot + p_0)) - u(R_\theta \cdot)]\right\} \overset{d}{=} \{[u(\cdot + p_0) - u(\cdot)]\}.$$

Another way, perhaps more parsimonious, to model velocity fields in a 2D environment is by using **stream functions**: If $\Psi : \mathbb{R}^2 \to \mathbb{R}$ denotes a smooth functions, then the velocity of a particle $u(p)$ is described as

$$u_\Psi(p) = (-\partial_y \Psi(x, y), \partial_x \Psi(x, y)), \quad p = (x, y). \tag{4}$$

Stream functions helps us to represent the path of the fluid: The collection of curves

$$S_K = \{p \in \mathbb{R}^2 : \Psi(p) = K\}, \quad K \in \mathbb{R},$$

are known as **stream lines**. Thus, the tangent vector at $p \in S_K$ for some $K$, corresponds to $u_\Psi(p)$.

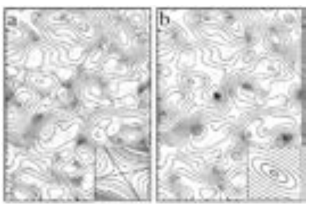


**Figure 1:** *Typical form of stream lines. Source: Rivera et al. (2001).*

By following this framework, we introduce a class of **stream functions of the ambit type** given by the formula

$$\Psi(p) = \int_{D_\rho(p)} \int_0^{2\pi} h(\|p-q\|) f(R_\phi(p-q)) L(\mathrm{d}q\mathrm{d}\phi). \tag{5}$$

where

- $f$ and $h$ are real-valued functions.
- $D_\rho(p)$ is a closed disk with center $p$ and radius $\rho > 0$.
- $L$ a real-valued homogeneous **Lévy basis** on $\mathbb{R}^2 \times [0, 2\pi)$.

In general, fields of the form of (5) are not differentiable. However, the following result holds:

**Proposition 1.** Let $\Psi$ be as in (5) with $f$ and $h$ being $C^1$. If $h(\rho) = 0$, then $\Psi$ has almost surely $C^1$ paths. Moreover, $u_\Psi$ given by (4) is stationary and has **isotropic increments**.

## 3. Incompressible and rotational fields

In classic fluid mechanics, the concepts of incompressibility and rotation of a field $X$ are described in terms of the asymptotic behaviour of the so-called **flux** and **circulation**. In 2D they are defined as

$$\text{Circulation: } \mathscr{C}_r(p; X) := \oint_{\partial D_r(p)} X \cdot n^\perp \mathrm{d}s;$$
$$\text{Flux: } \mathscr{D}_r(p; X) := \oint_{\partial D_r(p)} X \cdot n \mathrm{d}s,$$

where $n$ and $n^\perp$ are the outward and tangent unit vectors of $\partial D_r(p)$.

**The circulation measures the degree of rotation**: The more the fluid is aligned to $\partial D_r(p)$, the larger the circulation, so the more the motion is of rotational type. In this framework, it is said that the fluid rotates locally at $p \in \mathbb{R}^2$ if

$$\lim_{r \downarrow 0} \frac{1}{\pi r^2} \mathscr{C}_r(p; X) \neq 0.$$

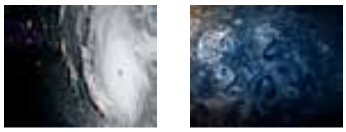


**Figure 2:** *Irma Hurricane and Jupiter Blues. Sources: GOES-16 satellite and NASA.*

On the other hand, **the flux measures the amount of fluid on the region**: The larger (smaller) the flux, the more fluid is leaving (entering) the region. Thus, the concept of **incompressibility** of a fluid, expresses the fact that the density of the fluid is constant, which in terms of the mean flux reads as

$$\lim_{r \downarrow 0} \frac{1}{\pi r^2} \mathscr{D}_r(p; X) = 0, \quad \forall p.$$

The following result describes the asymptotic behaviour of the flux when one consider ambit fields of the form of (1).

**Theorem 1** (Sauri (2018))**.** Let $u$ be as in (1) where $\mathcal{R} \subset \mathbb{R}^2$ is **compact** and $F$ **continuous differentiable** on $\mathcal{R}$. Then the following holds

**i.** If either $F|_{-\partial\mathcal{R}} \equiv 0$ or $L$ is of bounded variation, then

$$\frac{1}{\pi r^2} \mathscr{D}_r(p; u) \overset{\mathbb{P}}{\to} \int_{\mathcal{R}+p} \nabla \cdot F(p-q) L(\mathrm{d}q),$$

**ii.** If $F|_{-\partial\mathcal{R}} \neq 0$, assume in addition that $\mathcal{R} \subset \mathbb{R}^2$ has piecewise $C^{1,1}$ boundary. Under standard conditions on $L$ (domain of attraction of $\beta$-stable distributions), we have that

$$\frac{1}{\upsilon_\beta r^{1+1/\beta}} \mathscr{D}_r(p; u) \overset{\mathcal{F}\text{-}fd}{\longrightarrow} \int_{\partial\mathcal{R}(p)} F(p-q) \cdot u^i_{\mathcal{R}(p)}(q) M^\beta(\mathrm{d}q),$$

where $u^i_{\mathcal{R}(p)}$ is the inward vector of $\mathcal{R}(p)$ and $M^\beta$ is a **strictly** $\beta$**-stable** $(1 < \beta \leq 2)$ random measure with **control measure the 1-dimensional Hausdorff measure**.

Analogous statements for the circulation hold. Hence, by the previous theorem and simple modifications of the arguments in the proof of **i.** in Sauri (2018), we conclude that:

**Proposition 2** (**Rotational and Incompressible ambit fields**)**.** Let $u$ be as in (1) with specifications (2)-(3), and $u_\Psi$ and $\Psi$ be as in (4) and (5), respectively. Then,

**i.** Suppose that $g$ is $C^1$ on $D_\rho$ and either $g(a) = g(b) = 0$ or $L$ is of bounded variation. Then $u$ is **incompressible** if and only if $\phi = \pi/2, 3\pi/2$, or $g(x) \propto x^{-2}$. Furthermore, $u$ is **irrotational** if and only if $\phi = 0, \pi$, or $g(x) \propto x^2$.

**ii.** If $h$ and $f$ are $C^2$ with $h'(\rho) = h(\rho) = 0$, then $u_\Psi$ is **incompressible and rotational**.

## 4. Structure functions

Given an isotropic random field $X$, the **structure functions** are defined as

$$S_m(r) := \mathbb{E}[\langle X(p) - X(p'), n\rangle^m], \quad n \text{ unitary, } r = \|p - p'\|.$$

In 2D turbulence, structure functions exhibit a **double cascade** behaviour, e.g.

$$S_2(r) \propto \begin{cases} r^2 & \text{if } r < l_f; \text{ (Direct enstrophy cascade)} \\ r^{2/3} & \text{if } r > l_f, \text{ (Inverse energy cascade)} \end{cases}$$

and

$$S_3(r) \propto \begin{cases} r^2 & \text{if } r < l_f; \\ r^{2/3} & \text{if } r > l_f. \end{cases}$$

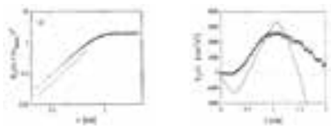


**Figure 3:** *Structure functions measured from thin layer of conducting fluid on a solid substrate. Source: Sommeria (1986)* .

In this context, the structure function of order 3 has been one main objects of study. Due to Proposition 2, $u$ given by (1)-(3), **cannot be incompressible and having non-null third order structure function** simultaneously. Thus, if one want to obtain skewed fields a mixture is needed. Specifically

**Proposition 3.** The stationary ambit field

$$\tilde{u}(p) = \int_{D_\rho(p)} g(\|p-q\|) R_{\pi/2}(p-q) L(\mathrm{d}q) \tag{6}$$
$$+ \int_{\mathcal{R}+p} \|p-q\|^{-2} (p-q) M(\mathrm{d}q), \tag{7}$$

where

- $g$ is $C^1$ with $g(\rho) = 0$;
- $\mathcal{R}$ is as in 3;
- $L$ and $M$ are skewed homogeneous Lévy bases with the latter being of bounded variation,

has **isotropic increments, is incompressible, rotational and its third order structure function does not vanish**.

*Remark* 1. The ambit field $\tilde{u}$ cannot be obtained through a stream function, i.e. there is no $\Psi$ for which $\tilde{u}$ admits the representation (4).

The requirements of incompressibility and $S_3 \neq 0$, lead us to less flexible model. In this matter, the use of stream functions of the form of (5) turned out to be less restrictive.

**Proposition 4.** Let $\Psi$ and $u_\Psi$ be as in Proposition 2 where we additionally put

$$f(x, y) = xy + x.$$

Then $u_\Psi$ has **isotropic increments, is incompressible, rotational and its third order structure function does not vanish**.

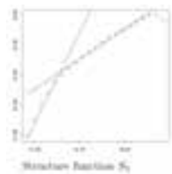


**Figure 4:** *Third order structure function associated to $u_\Psi$ with $\Psi$ as in Proposition 4.* .

*Remark* 2. More general functions $f$ can be considered and necessary condition for non-nullity of $S_3$ have been established by using the concept of **triads**.

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## ENDNOTES

1 According to the Merriam Webster Dictionary the word ambit has the meaning of ‘a sphere of action, expression, or influence’.

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## WEB SOURCES

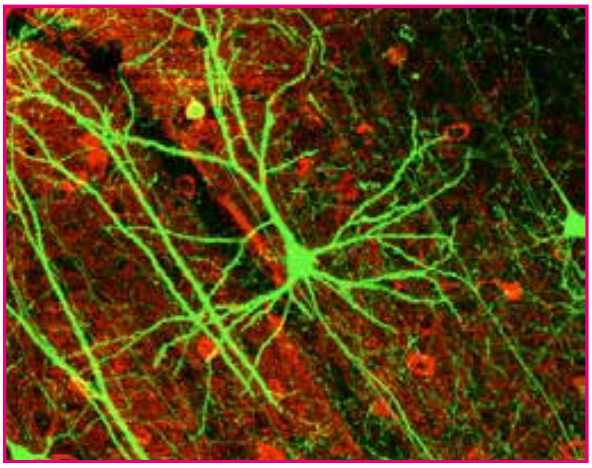

Ambit Stochastics https://www.google.com/imgres?imgurl=http://t2.gstatic.com/images?q%3Dtbn:ANd9GcQ-RqPZaB-0N1WIyhIVWbZuoMBZjqtePu54s_illc-TUR3yAhVb5d&imgrefurl=https://books.google.com/books/about/Ambit_Stochastics.html?id%3DXxd0twEACAAJ%26source%3D-kp_cover&h=389&w=259&tb-nid=uoI099g9bos5wM:&q=Ambit+STo-chastics+bog&tbnh=160&tbnw=106&us-g=AFrqEzcBs2DeLQZdUEt-6Ig27AuEck-UnDA&vet=12ahUKEwjRhsPph7XdAhVK-b1AKHeKGB44Q_B0wCnoECAkQFA..i&do-cid=3VQwebvYspzlaM&itg=1&client=fire-fox-b&sa=X&ved=2ahUKEwjRhsPph7XdAhVK-b1AKHeKGB44Q_B0wCnoECAkQFA

# GIG'S, NEURONS AND INFORMATION THEORY

Sometime in the Fall of 2017 I came on Google and by chance, across a reference to a paper with the title *Barndorff-Nielsen diffusion model neurons in primary sensory cortex.* Having never heard of the Barndorff-Nielsen diffusion I searched the net for that and related references and found several from an internationally leading group in the study of information theory and neurons, located at Virginia University and headed by Professor Tony Berger. Much of their work in the area is presented in the paper Mutual information and Parameter Estimation in the Generalized Inverse Gaussian Diffusion Model of Cortical Neurons [Sungkar, Berger, Levy (2017)].

The history of the generalised inverse Gaussian laws – or GIG's – is described and their roles in my work on the physics of blown sands, financial econometrics and turbulence has been indicated in some of the previous Sections. Most of the work on the theoretical properties of the laws was carried out around 1978. This included a paper, written jointly with Preben Blæsild and Christian Halgreen, where it was shown that the GIG's, for which the parameter $\lambda$ is not positive, can be characterised as the laws of the first hitting times to a constant barrier for a certain class of, explicitly determined, diffusions. This paper was written purely as the answer to what seemed to be an interesting theoretical question and with no particular application in mind. So much more pleasing to see that the results had found application, about 40 years later, in a research area with which I had never before been engaged.

In short nontechnical outline the context is this: In its conceptual origin, a diffusion process aims to describe, in strictly mathematical terms, how a particle moves around according to some physical law but under the influence of random impacts. But such processes, the theory of which was pioneered by Kiyosi Ito, can model many different types of random behaviour. The actual example is the excitation level of pyramidal neurons in the primary sensory cortex. Such a neuron receives electrical impulses from around 10.000 other neurons and its excitation level may both decrease and increase, but with the tendency to the latter. When the excitation reaches a certain level, the barrier, the neuron fires, transmitting the accumulated energy to about 10.000 other neurons.



**Human neocortical pyramidal neuron**

The generalised inverse Gaussian diffusions are then used to model this phenomenon. From the whole range of these diffusions one is selected according to available empirical and theoretical knowledge and it is demonstrated that the model corresponds to optimal transfer of information, in the sense of the Shannon Theory of Communication.

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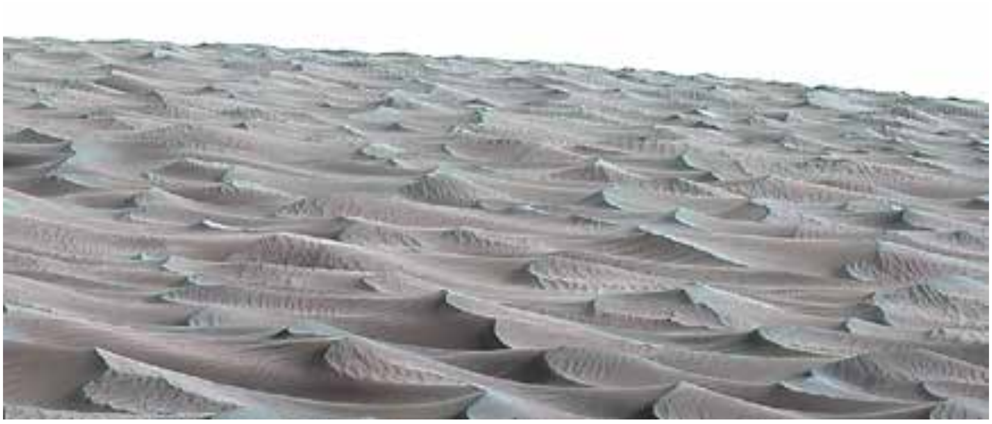
# BACK TO WIND AND SAND

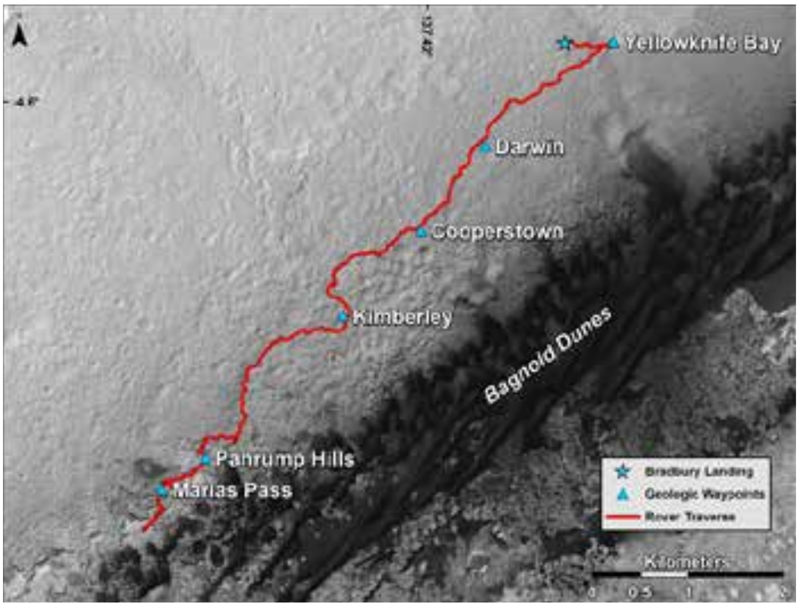
The present Part XV outlines some more recent developments relating to the study of blown sands and wind.

# 42 MARS: THE BAGNOLD DUNES

Following the exploration of the Martian surface by the NASA Rover module, part of the surface was given the name 'The Bagnold Dunes'. Later the Curiosity Mars rover traversed the dunes, as shown on the second display here. Part of the mission of Curiosity was to take samples of sand and study these after separating each sample according to grain size, the separation carried out by sieving, just as was done by Bagnold himself during his pathbreaking investigations.



**Bagnold Dunes on Mars**




**Bagnold Dunes on Mars. Rover Traverse route**

The observations of surface conditions on Mars, Venus and Titan that have taken place over the last 10 to 15 years have enormously stimulated the interest in the physics of blown sands and dust and in the formation of dune structures on Earth as well as on the planets. The paper [Kok. Parteli, Michaels and Karam (2012)] bears ample witness to this. (Much of the work carried out by the researchers mentioned in connection with the workshops and collaborations referred to in Sections #-# is cited in that Report.)

Another documentation of this increased interest is the popularity met by the writings and media contributions of the British geologist Michael Welland, to whom I referred in Section 5. This is true in particular for his books *Sand: a journey through science and the imagination* published by Oxford University Press (2009) (and subsequently by University of California Press under the title *Sand: the never ending story*) and the more recent *The Desert: Land of Lost Borders* published by Reaction Books *(2015).*

# 43 AN UNEXPECTED LINK

Gmail

Carlo Sgarra <sgarracarlo7@gmail.com>

**Un unusual citation of your celebrated work on sand and dunes**

**Carlo Sgarra** <sgarracarlo7@gmail.com> 21 luglio 2015 11:00
Rispondi a: carlo.sgarra@polimi.it
A: oebn@math.au.dk

Dear Ole,

A few weeks after the wonderful experience at the Aarhus conference, I'm writing you in order to communicate an unusual occurrence.

A couple of weeks ago I bought an Italian newspaper, quite popular, suggesting travels and touristic destinations for holidays, and I was positively surprised to find a reportage on the Rabjerg Mile sand dune and the closeby area of Skagen.

More surprising (the journal is devoted to a very broad audience, in principle without any specific nor scientific background) was for me to find a citation of your celebrated work on the mathematical model for the physical phenomenon, and even a small section trying to explain the most relevant achievements in a very basic way...

I'm sending you today by ordinary mail a copy of this journal.

Med Vennig Hilsen,

Carlo Sgarra

Carlo

From: Lello Piazza
Date: Friday 28 August 2015 18:55
To: Ole E. Barndorff-Nielsen
Subject: Lello Piazza has sent you a file via WeTransfer

Dear Ole Barndorff-Nielsen

what a surprise, what a big pleasure and what an incredible honour.
I'm sure you can put in your memories the article. A pdf version of it has been sent via Wetransfer.

Let me know if it is all right with you or if you need something else.

Few lines to present myself. I graduated in 1972 with a thesis on Markov Processes and the Dirichlet problem. After that, as nobody in Milan at that time was working in Probability, I switched to Mathematical Analysis. I started as Assistant Lecturer and after years I became Lecturer. But you know Mathematics is a hard job. I was passionate of wildlife photography. In 1981, one of my friends started with a monthly magazine dedicated to nature, and asked me to become the director of photography of his magazine. From then on, as photography is much easier than Mathematics, my laziness pushed me to take care more of photography than Maths.

But it has been a nice life, like having two lovers.

Now I'm retired but I still teach. Photography now is a very minor part of my life. Teaching Probability and Statistics is the main part. I don't try anymore to demonstrate theorems. But I'm doing a much harder job: making students understand Probability and Statistics.

Much useful work, if it is successful, than produce so so theorems (honestly I'm not a great mathematician).

My work as journalist is to write each month an article nature oriented for Bell'Europa. My article is the only one in the magazine dealing with nature. For instance, I'll be back in Denmark late November - beginning of December, in Saeby to write an article about looking for amber on the beaches after a big storm.

Last but absolutely not least I'm a good friend of Carlo. He's a very nice guy and good mathematician. Some two years ago, after one of my visit in Skagen, while I was telling him about my fascination for Råbjerg Mile, he showed me your article Sand, Wind and Statistics: some Recent Investigations. I thought "when I'll write an article on dunes of Jutland I'll mention it".

Then I found The Fascination of Sand in A Celebration of Statistics (Springer - the chapter 4).

But I couldn't give much space to this subject in the article. The stuff is too difficult for the general audience. In very few lines I had to give an idea of a very complicated phenomenon.

But a sort a recognition to your works seemed to be absolutely due.

My very best.
Lello Piazza

PS: please let me know if you have received the pdf via Wetransfer.
Thanks

There are only 3 kinds of people: those who can count and those who can't

Elio (Lello) Piazza
human being
Dept. of Mathematics – Politecnico
piazza Leonardo da Vinci, 32
20133 Milan, Italy

A piedi

Tre facili percorsi tra le dune di sabbia della penisola di Grenen, a nord di Skagen

Due passi sulla lingua di sabbia

I 3 percorsi

DOVE ORGANIZZARE IL VIAGGIO

# 44 THE WIND IN DENMARK

One of the closest international collaborators of the Sand Gang, from early on, was James Delano Iversen who is of Danish origin, born in Omaha, Nebraska, 1933. He held a Professorship at Iowa State University of Science and Technology and has had a leading role internationally in the research areas of wind tunnels, aeolian processes, aerodynamics, and geophysical fluid dynamics. Together with Ron Greely he published the book ''Wind as a geological process on Earth, Mars, Venus and Titan'' the ultimate purpose of which was an understanding of aeolian processes on Mars.

In the relation to the Sand Gang, James Iversen worked extensively with Keld Rømer Rasmussen. The present Section contains two notes, by Iversen and Rasmussen, respectively, about the pioneering role in wind engineering that Denmark has had.

## 44.1 JAMES IVERSEN: SOME NOTES AND REMINISCENCES

**By James D. Iversen, Professor Emeritus, Iowa State University**

**Introduction**

I was just a young engineering faculty member at Iowa State University in the mid-1960s when the opportunity arose to test in the ISU wind tunnel a scale-model of an architectural design for the proposed coliseum to be built on campus. The first step was a visit to the ISU library to see if anyone else had ever done such a thing, and to my surprise, I discovered that most of the literature on the subject was from Denmark, some in the Danish language and some important enough to have been translated into English.

Upon some reflection, perhaps it is not too surprising that the early pioneers in the field now called "Wind Engineering" were Danes. At times, Denmark can be a windy place. The Jutland peninsula and accompanying islands protrude out into the North Sea from the North coast of mainland Europe to meet the wind, which sweeps across the sea unchecked by any other geographic obstacles. Wind is an important factor in the lives of the people of Denmark. Because of the effects of wind on climate, vegetation, and man-made structures, and its potential for the generation of energy, the wind has long been a subject of study by scientists and engineers in Denmark. Although damaging wind storms are not as frequent as the hurricanes and tornadoes which we experience in the United States, there can be such storms, and one of the most destructive occurred just a few years ago on December 3rd, 1999, when wind gusts of up to 137 mph were measured on the westward island of Rømø, just off the coast of southern Jutland. Seven people were killed in Denmark during that one storm and insurance claims totaled 8.5 billion Danish kroner (approx. 1 billion $US).

There are significant geological features in Denmark, which are due to the wind. There are many square miles of sand dunes along the west coast of Denmark, including the large features of Rubjerg Knude and Råbjerg Mile in the northern part of Jutland. One of the interesting landmarks is the church which was buried by sand in the 19th century (tilsandede kirken), near the northern tip of Jutland at Skagen.

**Technological Advances in the 19th century**

It is just a little over 100 years since the Wright Brothers made their epic flight over the sand dunes at Kitty Hawk. Most of the preliminary achievements prior to their flight occurred in the 19th century, i.e., the invention of the internal combustion engine, and the contributions by others before the Wright brothers, such as Otto Lilienthal and Octave Chanute. What is not generally known is that some people at the time blamed the great mathematician Sir Isaac Newton for a delay in the advent of flight. As part of his work in the application of mathematics to the solution of physical problems, he had attempted to predict the forces on an object as it travels through the air. In this particular case, he made an assumption about the flow of air about an obstacle which was later proved to be false, and the result of this assumption is that a much smaller lift force on a wing is predicted than actually occurs.

A Danish engineer and mathematician, Henrik Christian Vogt (1848-1928), became interested in bird flight during a trip around the world in 1877, as he watched the birds soaring about the ship on which he was a passenger. He worked for a time in England before returning to Denmark, and became a member of the English Aeronautics Club. He worked on the theory of lift on a wing, and became convinced that Newton was wrong1. He wrote to the Smithsonian's Samuel Langley, for example, and Langley answered back that such a great mathematician as Newton couldn't be wrong. Vogt became well enough known that he was invited to the Aeronautical Congress in Chicago in 1893.

H.C. Vogt finally decided that he needed to perform an experiment in order to prove his theory, so he enlisted the assistance of his friend, Johann O.V. Irminger (1848-1938), who was director of the Eastern Gas Works in Copenhagen. The Eastern gas works had a very large chimney. Irminger cut an opening into the side of the chimney, and built on to it his first wind tunnel. The year was 1893; 8 years before the Wright brothers built their wind tunnel. This was not the world's first wind tunnel. Francis Wenham (1871) and H.F. Phillips (1880s) had built wind tunnels in England, but it was the first in Denmark, and started a long history of wind tunnel testing in Denmark, which continues to this day. It was Phillips, in fact, who had suggested to Vogt that he prove his theory by testing an airfoil in a wind tunnel.

Irminger's wind tunnel consisted of a horizontal rectangular tube, 40 inches long and with a 9 inch by 4,5 inch cross section. Powered by the draft of the chimney, it was capable of wind speeds up to about 33 mph. Irminger quickly learned how to measure the pressure distribution on the surface of a wind tunnel model. He built a model of a wing to Vogt's specifications and became the first in the world to measure the pressure distribution on an airfoil, proving in the process that Vogt was right, and Newton was wrong. Irminger published his work in English2 in1894.

### Technological Advances in the 20th century

Irminger built his second wind tunnel in the 1920's, presumably after he had retired (he was now in his 70's). It was much larger than the first, and he enlisted the assistance of Professor Christian Nøkkentved. He had become interested in the pressure distribution on buildings in the presence of wind, and he and Professor Nøkkentved went on to publish the results of the first comprehensive sets of wind experiments on buildings in the 1930s3, 4. Irminger continued to work on building aerodynamics until just a few months before his death in 1938 at the age of 90.

Poul la Cour (1846-1908) was one of the first scientists to tackle the problem of extracting electrical energy from the wind. Born in Aarhus, he was educated in meteorology, and after some time in that field, in 1878 he was hired to become a teacher of science at Askov Folkehøjskolen. He was one of the few Danes who was interested in the work of Vogt and Irminger, and he began to study the wind turbine from a technical standpoint. He was an inventor, and his work with early electrical instruments earned him the unofficial title as "Denmark's Edison". His first electricity-generating windmill was finished in 1891, and in 1897 he built the world's first wind tunnel designed for the purpose of testing windmill designs5. He is known by some as "The Father of Wind Energy." Mostly because of his work, wind generators were fairly common in the early part of the 20th century. One of his students, Johannes Juul, was the designer of the world's first AC (alternating current) wind turbine at Vester Egesborg in the 1950s. Juul designed the 200 kW wind turbine at Gedser, which was built in 1956-7. This turbine was later refurbished in 1975 at the request of NASA which wanted test results for the US wind energy program.

It is interesting to note the close relationships among the people who were advancing the state of the technological art that is now known as "wind engineering". Henrik C. Vogt was responsible for getting J.O.V. Irminger interested in aerodynamics in the 1890s. Vogt, in turn, started working with Professor Christian Nøkkentved in the 1920s, and that collaboration lasted throughout most of the 1930s. Nøkkentved started working with the aerodynamics of snow fences and shelter belts[6,7] in the late 1930s, and he enlisted the help during that time of engineering student Martin Jensen.

Martin Jensen (1914-1991) was an extremely clever and capable scientist-engineer, and his contributions are so significant and well-known that he became known world-wide as the "Father of Wind Engineering". The work by Nøkkentved and Jensen was interrupted by the occupation of Denmark by the Nazis in World War II, but immediately after the War, Jensen started work on a long series of experiments, both in nature and in the wind tunnel[8, 9]. He very early recognized the need to duplicate the characteristics of the natural wind when working at small scale in the wind tunnel laboratory, and eventually discovered that the way to do this was to build a very long wind tunnel, and then model the surface roughness to scale as far upwind of the model as possible (be it snow fence or building or some other obstruction to the wind).

Martin Jensen's work forms the basis for a number of laboratories around the world today (including several in North America), which specialize in modeling the atmospheric boundary layer (wind and turbulent layer nearest the surface). The primary laboratory for this work in Scandinavia today is the wind engineering research laboratory of the Danish Maritime Institute (located in the northern Copenhagen suburb of Lyngby). This laboratory uses three wind tunnels (1. cross-section 0.m x 0.8 m, 80 m/s; 2. cross-section 2.6m x 1.8 m, 25 m/s; 3. cross-section 13m x 1.7m, 8 m/s). Jensen assisted in the design of the new Little Belt Bridge (Ny Lillebæltsbroen), completed in 1971. Like all suspension bridges, the effect of the wind is extremely important in design. The newest suspension bridges in Denmark, the Great Belt Bridge (Storebæltsbroen), and the bridge across the Sound between Denmark and Sweden (Øresundsbroen), were both tested extensively in both the second and third wind tunnels at the Danish Maritime Institute. Allan Larsen was the primary aerodynamicist in the design of the two newest bridges[10]. Bridges built or being built in other countries are also being tested in this laboratory. The first wind tunnel was the only one in existence when this writer spent a very enjoyable 8 months on sabbatical at the laboratory[11] in 1981.

The meteorological study of the atmospheric boundary layer is today called micrometeorology, and there is a large research group at the Risø National Laboratory, which has studied the boundary layer for many years. Niels Busch and Niels Otto Jensen are two of the better-known people in this area[12].

**Wind-blown sand and soil**

The soil in the western two-thirds of the Jutland Peninsula is very sandy and subject to considerable wind erosion, even in the damp Danish climate. This is a problem which intensified after the war of 1864 with the subsequent reclamation of the heath land in Jutland, where the soil is the sandiest. This problem was, of course, the motivation for the wind tunnel experiments of Nøkkentved[6, 7] and Martin Jensen[13, 14] which studied the aerodynamics of shelter. More recently, interest in the physics of sand and soil movement has been led by a group of scientists at the University in Aarhus led by geologists Jens Tyge Møller[15] and Keld Rømer Rasmussen, statisticians Ole Barndorff-Nielsen and Jens Jensen, and physicist Henrik Nielsen. The writer[16] has been fortunate enough to work with these people, primarily with Rasmussen. Keld Rasmussen invented and built a tilting wind tunnel, the only one of its kind in the world, and some significant research[17, 18] has been conducted in this wind tunnel on the effect of slope on sand transport.

**Modern day wind technology in Denmark**

Denmark today is one of the world's primary producers of wind energy and wind energy devices. In the year 2000, there were approximately 6000 electrical generating wind turbines in Denmark. The Danish wind energy industry is among the world's largest. A significant portion of the present global total of almost 40,000 MW wind energy capacity has been provided by the Danish wind energy companies Vestas and NEG Micon. The efforts of those early pioneers in wind technology, i.e., Vogt, Irminger, la Cour, Nøkkentved, Jensen, and others have certainly reaped dividends for Denmark and the Danes.

## 44.2 MORE RECENT ACTIVITIES RELATING TO THE AARHUS WIND TUNNELS

**By Keld Rømer Rasmussen**

I have been asked to amend to Jim Iversen's story about "The Wind in Denmark" by a few paragraphs describing the wind-related research made in the Aarhus wind tunnel from near the start of the 21st century. After Michael Sørensen moved to the University of Copenhagen, only Keld Rasmussen had a somewhat permanent involvement in aeolian research, and one may refer to the period as the post-Sand Gang era. Furthermore, the involvement from KRR was also continuously being "eroded" by activities within the fields of hydrology and hydrogeology which in recent years have had an increasing importance at the Department of Geoscience.

Although discontinuous the joint involvement by MS and KRR has been a linchpin of cooperation in the aeolian research. New connections to international groups have also been formed of which one of the first and still existing links was with colleagues at respectively the Department of Physics at Universitée 1, Rennes and École Polytechnique, Nantes. Initially MS and KRR participated in international workshops in Tunesia (1999) and Mauretania (2001), which in fact became the starting point of the collaboration despite that the first result from the field trips did not emerge until 2007 and only after additional fields activities in Mauretania were made during the intervening period by the French groups.

This resulted in a publication about the dynamics of dune movement in a Mauritanian dune-field (*Journal of Geophysical Research, Earth Surface,* Vol. 112, F02016).

Meanwhile MS&KRR had initiated new laboratory studies of saltation using laser-Doppler measurements to study grain mechanics in combination with classical Pitot-static wind measurements of the flow field (*Journal of Geophysical Research: Earth Surface*, Vol. 113, 2008). One could point at this being essential in formulating an analytical expression for the vertical variation of particle speed and in acquiring simultaneous data of the two velocity populations. These values indicated that in the most intense part of the saltation layer the average grain velocity has similar value than that of the air velocity. As pointed out in 2011 in an extensive review article (Duran, Claudin and Andreotti: *Aeolian Research 3*, 243–270) this is in fact very close to the air velocity in the region of the Bagnold focal point. New detailed grain velocity data were also used in a collaboration with the French colleagues and James Jenkins (Cornell) to understand the dynamics in the saltation layer and to build on the splash information simultaneous investigated by the French group (Journal of Fluid Mechanics, Vol. 625, 2009, s. 47-74).

Alongside with the studies of particle dynamics in a terrestrial environment the planetary aspect was involved in a study of the influence from electrification on transport near the saltation threshold (Rasmussen et al., 2009, Planetary and Space Science, Vol. 57, 804-808), and a couple of other more general contributions on planetary aeolian transport generated in the slip-stream of the new Planetary Wind Tunnel facility set up at the Physics Department (e.g. Rasmussen et al., 2011. In: InTech - Open Access Publisher, p. 51-74).

The elaborate boundary layer control makes the Aarhus wind tunnel unique for studies near the saltation threshold and an interesting study of the role of bursting near the saltation threshold was made jointly with colleagues from ETH in 2014-2015 (Scientific Reports, Vol. 5, 11109).

The next milestone in the aeolian work was founded during the large international workshop Geoflows13 at the Kavli Institute for Theoretical Physics, Santa Barbara where MS and KRR participated for 1 respectively 2 months. An offspring of the workshop was two parallel partly overlapping review papers: one directed towards the geoscience community about the important contributions to aeolian science gained from laboratory studies (Geomorphology 244 (2015), 74–94), and the other more theoretically inclined directed towards the physics community (Valance et al., (2015) Comptes Rendus Physique, 16, pp.1-13.).

Another important outcome of the Kavli workshop was the set-up of an experimental scheme for an advanced laboratory study binding together the saltation dynamics and how it relates to the formation of small scale bedforms like wind ripples. Various wind-ripple models have been proposed in the literature, but they seem incomplete in the sense that so far none of these links the statistical properties of the saltation cloud to the grain splash and its variation, depending on bed texture and the statistical properties of the saltation trajectories. The initial steps in this research program have been taken and the advance of wind ripples as function of grain size and wind speed has been measured. Still, the most challenging parts relating saltation cloud parameters to ripple topography and advance are under investigation.

30. Juni 2016, Keld

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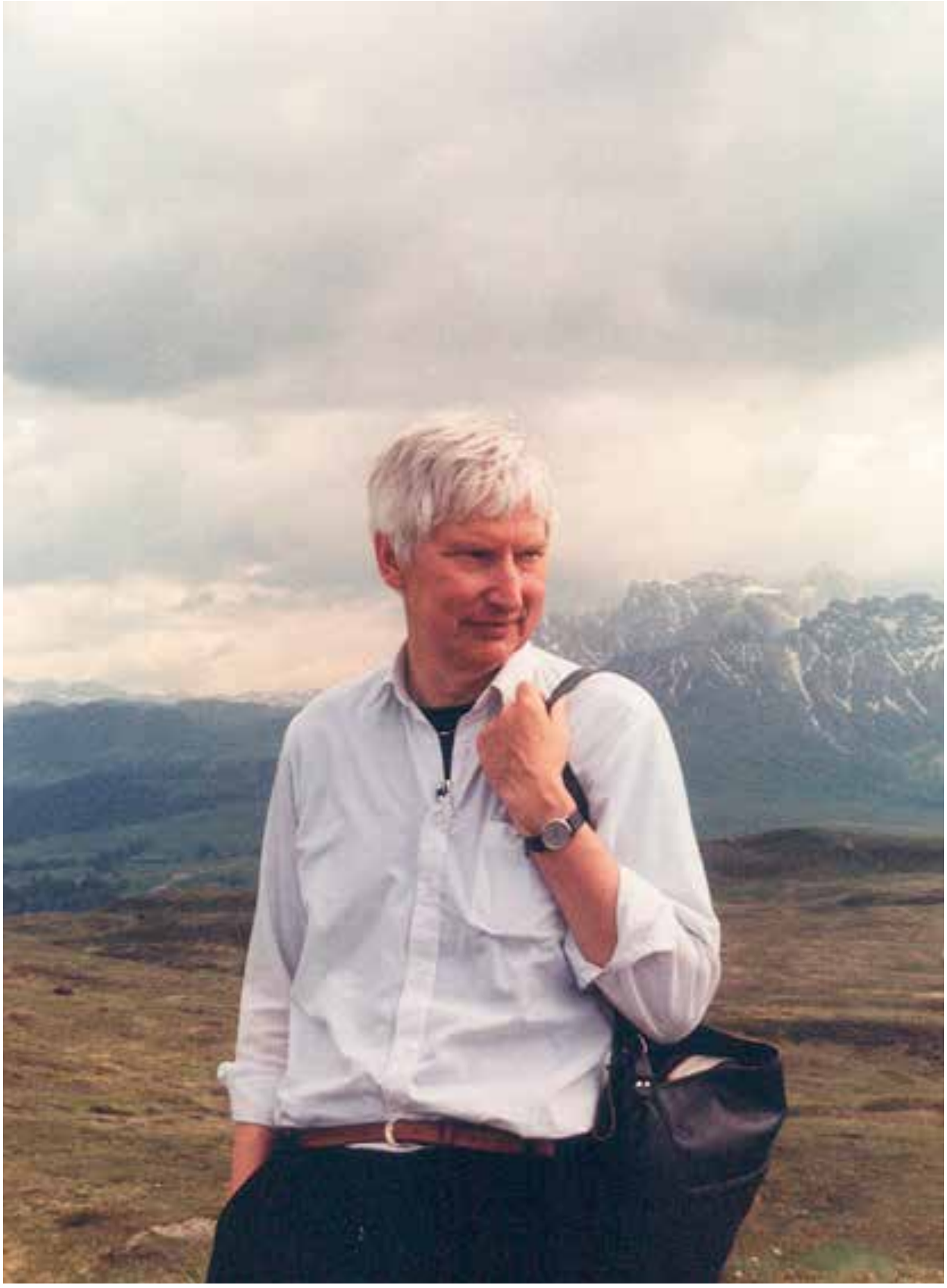
## ADDITIONAL NOTE

MOXIE (https://en.wikipedia.org/wiki/Mars_Oxygen_ISRU_Experiment) is an experiment to be sent on NASAs Mars 2020 mission which is designed to investigate the possibility of extracting oxygen from the Martian CO2 atmosphere. The Principal Investigator of the MOXIE instrument is Michael Hecht from the Massachusetts Institute of Technology(MIT). John McClean from Imperial College London and Morten Bo Madsen from the Niels Bohr Institute at the University of Copenhagen are collaborating with MIT to develop this prototype. The tests within the Aarhus Mars simulator involved critical tests of the pump and filter system in order to determine the pumping efficiency and the susceptibility to dust in the Martian atmosphere.

# SOME TRAVELLING



> *Die Welt durchzog ich*
> *wanderte viel*
> *Kunde zu werben.*
> Wanderer in Richard Wagner: Siegfried, third Act

Having widespread scientific interests naturally implies a great deal of travelling, both in terms of participation in workshops and conferences, and of short visits to university departments to present new results. But equally important are longer stays at leading research institutions. A few of these visits, not referred to elsewhere in these notes, are mentioned here.

# 45 RERF

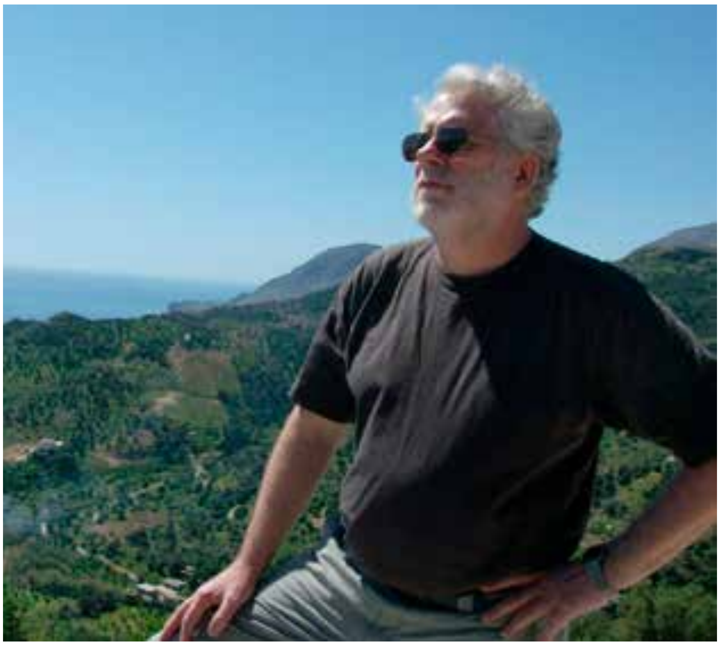
In 1988 I was invited to visit the Radiation Effects Research Foundation (RERF) in Hiroshima where my former student Michael Væth, now Professor at Department of Public Health, Institute of Biostatistics, Aarhus University, was affiliated and where also another acquaintance of mine, Professor Donald Pierce, Statistics Department, Oregon State University, was working. This was the start of my love of the country of Japan. A second visit to RERF took place in 1990 where I travelled together with Preben Blæsild.

# 46 IMPA

In 1990, by invitation, I visited the Brazilian mathematical research centre IMPA (Instituto Nacional de Mathematica Pura e Aplicada), located in Rio de Janeiro, where my former student Bent Jørgensen, later Professor at Odense University, was a fellow. I went there again in January 1991 at the time of the Iran-Irak war, accompanied by Preben Blæsild.

**The view over Rio de Janeiro from IMPA**

**Bent Jørgensen (1954-2015)**



Bent's thesis for his cand. scient. degree consisted in a comprehensive study of the generalized inverse Gaussian distributions. This work, entitled 'Statistical Properties of the Generalized Inverse Gaussian distribution' was published by Springer Verlag in 1982 and has remained the standard reference on the GIG.

At IMPA I met one of Bent's PhD students Rodrigo Laboriau with whom I developed some common interests, which eventually has led to Rodrigo being affiliated to the Department of Mathematics at Aarhus University, where he is now leader of a newly established Laboratory of Applied Statistics.

**Rodrigo Laboriau**

# 47 TOKYO — NAGOYA

**With Makoto Maejima at Nagoya Castle**

One of the long term collaborations that grew out of the MaPhySto programme was with Professor Ken-Iti Sato, Department of Mathematics at Nagoya University, and Professor Makoto Maejima, Department of Mathematics at Keio University, Yokohama. The collaboration was based on our common interest in the theory of Levy processes and led to numerous mutual visits.

More specifically, we developed a number of aspects that, although of theoretical interest in themselves, have relevance in relation to applications. Most pertinent are results on time change and stochastic integral representations.

Among my visits, one was in August 2003 to Keio University where on invitation I was giving a key note talk in connection with the start of the '21st Century COE Programme'. This Programme is set out in the adduced citation, from the web page of COE, on the objective of the program.

> "From 2002, Ministry of Education, Culture, Sports, Science and Technology started the so-called "21st Century COE Program" that aims at promoting the construction of international educational and research bases in several selected universities for each field of sciences. COE stands for "Center Of Excellence". Our program has started in August 2003 in the category of "Mathematics, Physics and Earth Sciences" for five years. The Integrative Mathematical Sciences is to

> open a new horizon to mathematical sciences by challenging to model various natural and social phenomena through data. The core of our program is fundamental mathematics including non-commutative geometry, global analysis, dynamics and nonlinear optimization. The core is wrapped with data science and experimental mathematics. The data science plays a role of an interface to various phenomena by means of data. The experimental mathematics supports experimental aspect of mathematical sciences, including computer experiments and numerical evaluation. The main objective of our program is to integrate such three different aspects of mathematical sciences."

Other visits included collaborations with Professor Shun-Iti Amari at the RIKEN Brain Science Institute, with whom I shared an interest in Information Geometry, and a stay at the Institute of Mathematical Statistics in Tokyo, collaborating with Professor Nakahiro Yoshida.

In 2003, I again was invited to come to Japan, this time to do research together with Professor Ken-Iti Sato, Department of Mathematics, Nagoya University, and Professor Makoto Maejima, Department of Mathematics, Keio University, Yokohama.

# 48 MPI LEIPZIG

Max-Planck-Institut für Mathematik in den Naturwissenschaften

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Inselstr. 22
04103 Leipzig
Deutschland

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**Symposion on Money Value and Capital Transfers**

**October 04 - 06, 2012**
**Universität Leipzig - Hörsaalgebäude - Hörsaal 2**
Is a mathematical model for the value of money and capital transfers possible? - The historical view, statistical and econometrical aspects and the policy environment.

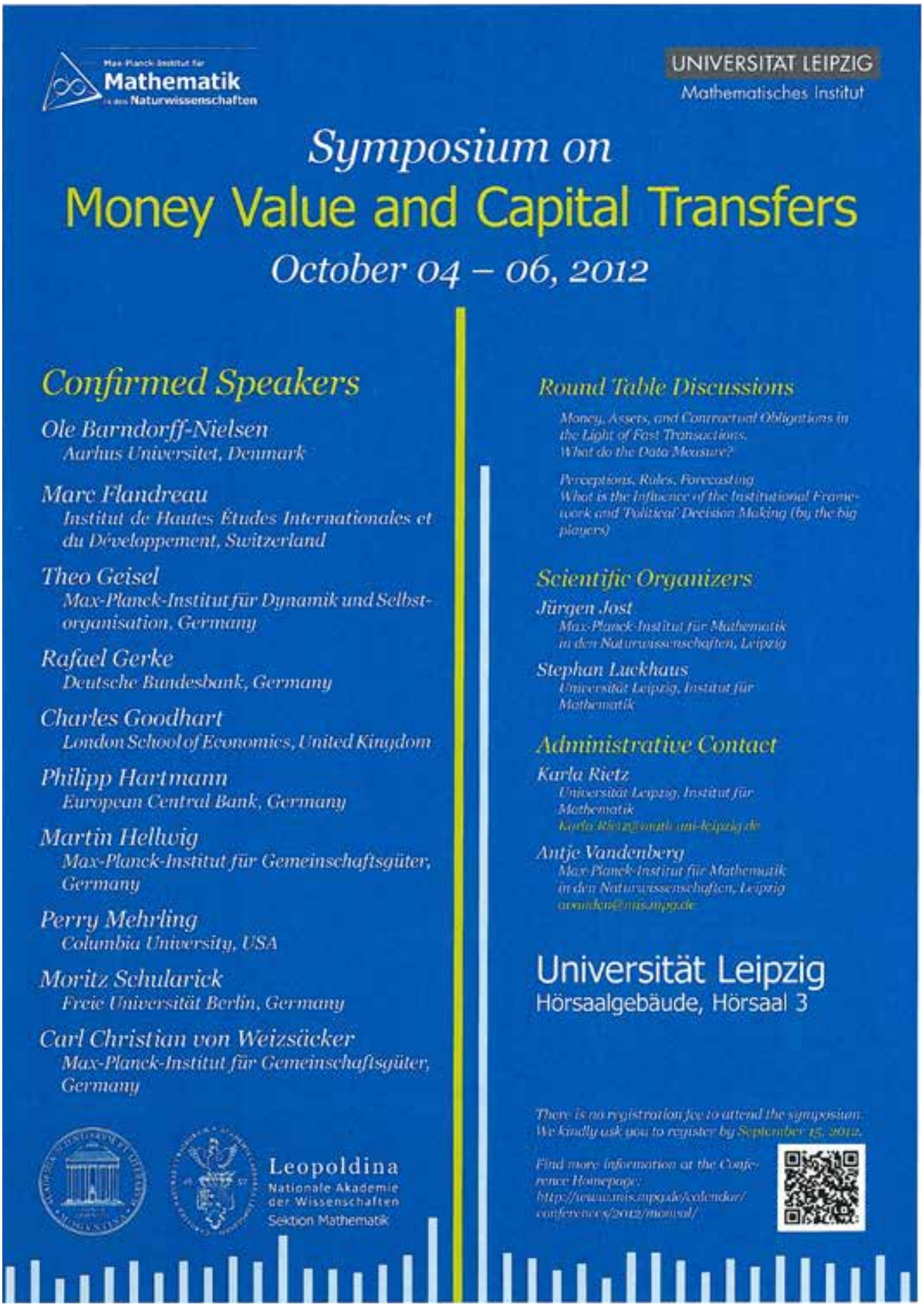
This three day event is not meant to be a workshop on mathematical modeling in economics in the strict sense. Rather we want to discuss what it is that should or could be modeled in the interaction between capital markets, souverain debt, central banks, and the economy as a whole. Current political issues, however, will not be our primary focus.

The plan is to devote one day to historical aspects of capital, money, and transactions; one day to modern statistics of capital flows and financial transactions; and one day to the influence of the political and instutional framework.

The questions behind our symposium include: do there exist 'universal' aspects and systematic relations in the aforementioned economic interactions, what are observable quantities, and what is the states' influence.

We hope for a lively discussion, so beside the lectures we will organize two round table discussions ("Money, Assets, and Contractual Obligations in the Light of Fast Transactions. What do the Data Measure?" and "Perceptions, Rules, Forecasting. What is the Influence of the Institutional Framework and 'Political' Decision Making (by the big players)").

The symposium is a joint symposium of the Institute of Mathematics at the University of Leipzig, the Max Planck Institute for Mathematics in the Sciences, the Academy of Science and Literature Mainz, and the National Academy of Sciences Leopoldina.

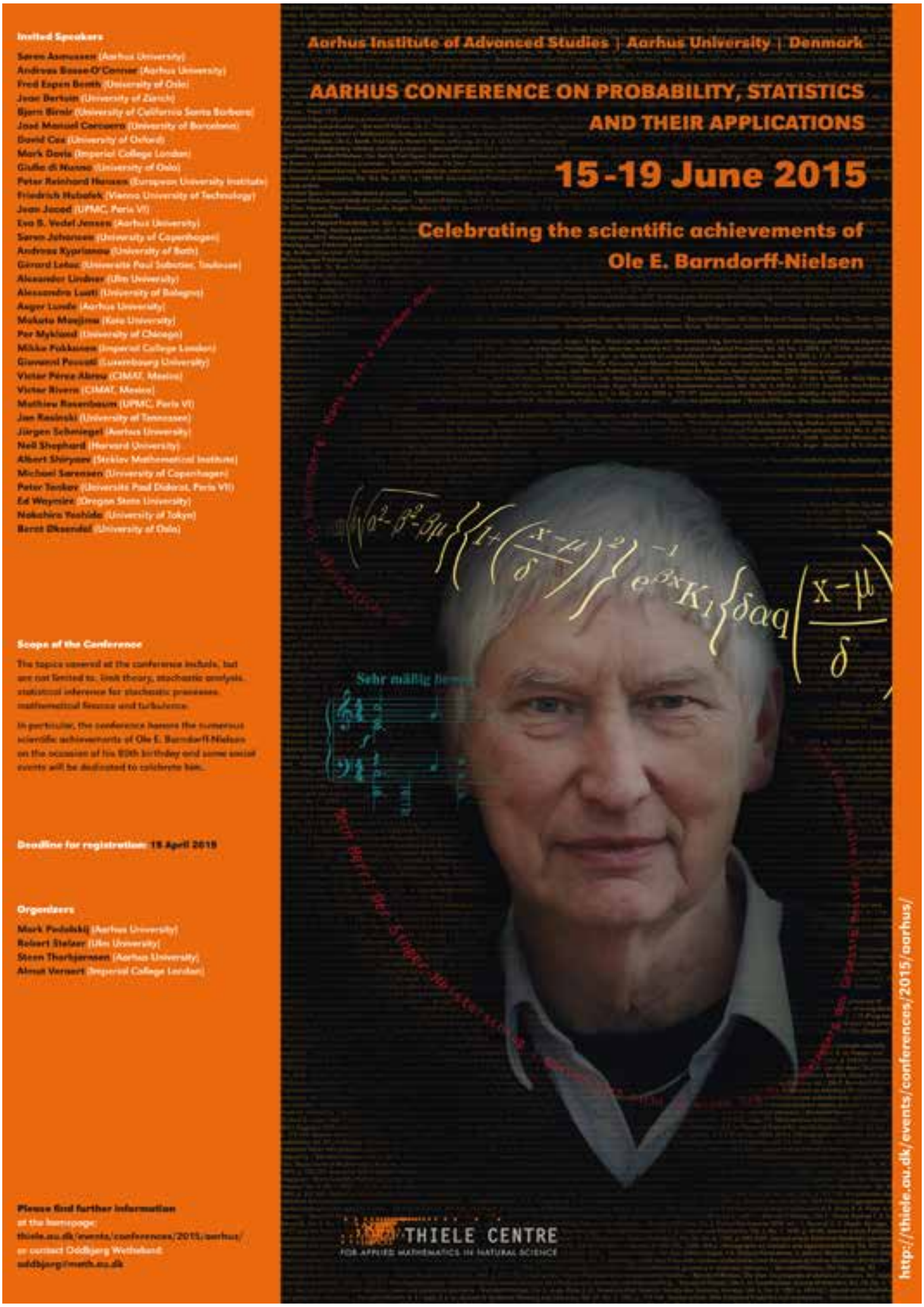


Max-Planck-Institut für Mathematik in den Naturwissenschaften
UNIVERSITÄT LEIPZIG
Mathematisches Institut
Symposium on
Money Value and Capital Transfers
October 04 – 06, 2012
Confirmed Speakers
Ole Barndorff-Nielsen
Aarhus Universitet, Denmark
Marc Flandreau
Institut de Hautes Études Internationales et du Développement, Switzerland
Theo Geisel
Max-Planck-Institut für Dynamik und Selbstorganisation, Germany
Rafael Gerke
Deutsche Bundesbank, Germany
Charles Goodhart
London School of Economics, United Kingdom
Philipp Hartmann
European Central Bank, Germany
Martin Hellwig
Max-Planck-Institut für Gemeinschaftsgüter, Germany
Perry Mehrling
Columbia University, USA
Moritz Schularick
Freie Universität Berlin, Germany
Carl Christian von Weizsäcker
Max-Planck-Institut für Gemeinschaftsgüter, Germany
Round Table Discussions
Money, Assets, and Contractual Obligations in the Light of Fast Transactions. What do the Data Measure?
Perceptions, Rules, Forecasting. What is the Influence of the Institutional Framework and 'Political' Decision Making (by the big players)
Scientific Organizers
Jürgen Jost
Max-Planck-Institut für Mathematik in den Naturwissenschaften, Leipzig
Stephan Luckhaus
Universität Leipzig, Institut für Mathematik
Administrative Contact
Karla Rietz
Universität Leipzig, Institut für Mathematik
Antje Vandenberg
Max-Planck-Institut für Mathematik in den Naturwissenschaften, Leipzig
Universität Leipzig
Hörsaalgebäude, Hörsaal 3
There is no registration fee to attend the symposium. We kindly ask you to register by September 15, 2012.
Find more information at the Conference Homepage:
Leopoldina
Nationale Akademie der Wissenschaften
Sektion Mathematik


**Mephisto and Faust in front of Auerbach's Keller in Leipzig.**
**The Keller is famous for its association to Goethe's Faust and to Offenbach's master piece 'Hoffmann's Erzählungen' (The Tales of Hoffmann) based on some of the stories of E.T.A. Hoffmann. Both Goethe's Faust and Offenbach's opera – one of my favourites – take partly place in Auerbach's Keller.**

The reason for my being in Leipzig was that I was giving a talk at a small Conference at the Max Planck Institute for Mathematics in the Sciences.

# AARHUS CONFERENCE 15.–19. JUNE 2015


Invited Speakers
Søren Asmussen (Aarhus University)
Andreas Basse-O'Connor (Aarhus University)
Fred Espen Benth (University of Oslo)
Jean Bertoin (University of Zürich)
Bjørn Birnir (University of California Santa Barbara)
José Manuel Corcuera (University of Barcelona)
David Cox (University of Oxford)
Mark Davis (Imperial College London)
Giulia di Nunno (University of Oslo)
Peter Reinhard Hansen (European University Institute)
Friedrich Hubalek (Vienna University of Technology)
Jean Jacod (UPMC, Paris VI)
Eva B. Vedel Jensen (Aarhus University)
Søren Johansen (University of Copenhagen)
Andreas Kyprianou (University of Bath)
Gérard Letac (Université Paul Sabatier, Toulouse)
Alexander Lindner (Ulm University)
Alessandra Luati (University of Bologna)
Asger Lunde (Aarhus University)
Makoto Maejima (Keio University)
Per Mykland (University of Chicago)
Mikko Pakkanen (Imperial College London)
Giovanni Peccati (Luxembourg University)
Victor Pérez Abreu (CIMAT, Mexico)
Victor Rivero (CIMAT, Mexico)
Mathieu Rosenbaum (UPMC, Paris VI)
Jan Rosinski (University of Tennessee)
Jürgen Schmiegel (Aarhus University)
Neil Shephard (Harvard University)
Albert Shiryaev (Steklov Mathematical Institute)
Michael Sørensen (University of Copenhagen)
Peter Tankov (Université Paris Diderot, Paris VII)
Ed Waymire (Oregon State University)
Nakahiro Yoshida (University of Tokyo)
Bernt Øksendal (University of Oslo)
Scope of the Conference
The topics covered at the conference include, but are not limited to, limit theory, stochastic analysis, statistical inference for stochastic processes, mathematical finance and turbulence.
In particular, the conference honors the numerous scientific achievements of Ole E. Barndorff-Nielsen on the occasion of his 80th birthday and some social events will be dedicated to celebrate him.
Deadline for registration: 15 April 2015
Organizers
Mark Podolskij (Aarhus University)
Robert Stelzer (Ulm University)
Steen Thorbjørnsen (Aarhus University)
Almut Veraart (Imperial College London)
Please find further information
at the homepage:
thiele.au.dk/events/conferences/2015/aarhus/
or contact Oddbjørg Wethelund:
oddbjorg@math.au.dk
Aarhus Institute of Advanced Studies | Aarhus University | Denmark
AARHUS CONFERENCE ON PROBABILITY, STATISTICS AND THEIR APPLICATIONS
15-19 June 2015
Celebrating the scientific achievements of Ole E. Barndorff-Nielsen
Sehr mäßig
THIELE CENTRE
FOR APPLIED MATHEMATICS IN NATURAL SCIENCE
http://thiele.au.dk/events/conferences/2015/aarhus/

NUFFIELD COLLEGE
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OX1 1NF

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23 May 2015

Dear Ole,

After a lot of hesitation I've reluctantly decided on health grounds that it is best that I not come to your celebration next month. I'm sure the occasion will be a great one and I am deeply disappointed to miss it.

More importantly, I hope you know how enormously I appreciate, and have always appreciated, the privilege of working with you over those years, and, more importantly still the personal contact which I recall with the greatest pleasure: In particular the Aarhus Rings was unforgettable!

Very best wishes to you + Bente

David

Registered Charity No. 1137506. Registered Office: New Road, Oxford OX1 1NF

AARHUS UNIVERSITY
DEPARTMENT OF MATHEMATICS

Dear David

Many thanks for your letter.

Of course your health comes first. But you will be greatly missed by all your friends here and at the conference.

You and Joyce know, I trust, that the heartwarming feelings you express are fully reciprocated by Bente and me.

Ole

**Department of Mathematics**

**Ole E. Barndorff-Nielsen**

Professor

Date: 04 June 2015

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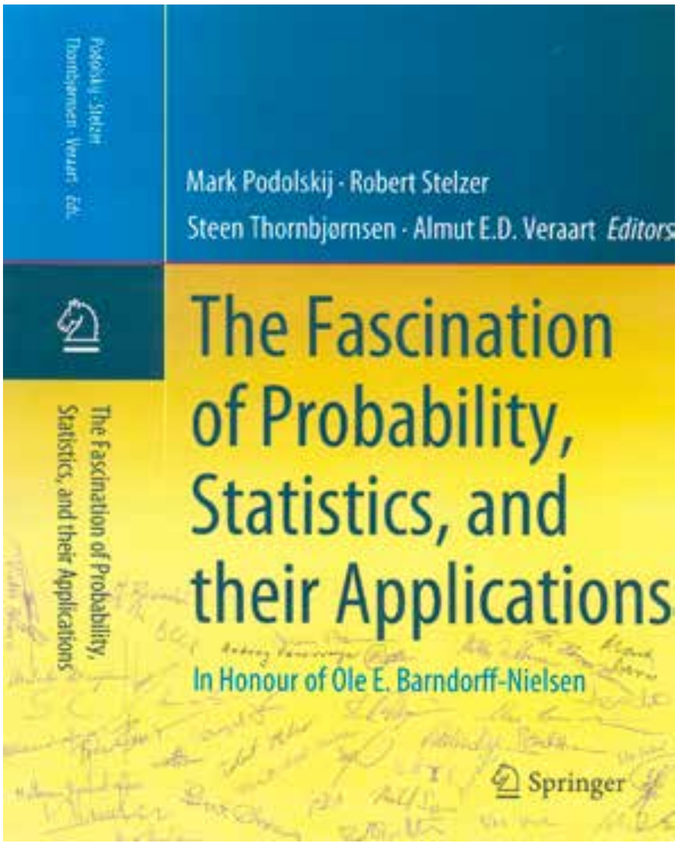


**A splendid gift**

Hvis denne skal være større, må vi scanne bogens forside

Alle de foreslåede fotos er for lav opløsning til at bruges.

ENDNOTES

# HANS SACHS

Throughout my life, from early teenage years til now, has Wagner's opera 'Die Meistersinger von Nürnberg' transported me to a euphoric state, at first solely through the music and then gradually from an increasing understanding of the depth and multifaceted nature of the libretto and its complete symbiosis with the music.

The portrait of the real historic figure Hans Sachs, 'Schuhmacher und Poet darzu', inherent in the opera's Sachs seems to represent him fairly as a person but, of course, Wagner's Sachs has its own stature, as a thoroughly human being, guided by noble thinking but also with a clear eye for human follies and how best to react to them.


**Hans Sachs, wood engraving by Michael Ostendorfer**

The real Hans Sachs (1494-1576) is a prominent figure in German cultural history, both as a poet and as a representative of life in Nürnberg around 1500. After some Wanderjahre he settled down in Nürnberg which at the time was the leading German city, being the hub of trading and cultural influences from East, West, North and South.

He earned his living as shoemaker but besides he was an incredibly productive poet and playwright and a central person in the famous Meistersinger guild, from the traditions of which Wagner build a background for his opera 'Die Meistersinger von Nürnberg'. Sachs wrote about 4000 Meisterlieder in addition to other poems and songs, a number of carnival plays, tragedies, comedies, prose dialogues, fables and religious tracts.

The name and work of Sachs fell largely into oblivion until Goethe became aware of his works and life's history, in connection to his own work on Faust.

When it comes to contemplating the idea of a universally human being the name of Hans Sachs always comes to my mind, both the historical figure and the splendid portrait of him given by Richard Wagner in 'Die Meistersinger von Nürnberg'.

**WEBSOURCE**

Hans Sachs im Holzschnitt von Michael Ostendorfer (1545) https://www.flickr.com/photos/fbever/17277510936

# EPILOGUE

Hans Sachs, aged 68, to his wife:

> *Wohlauf Herz, Sinn, Mut und Vernunft,*
> *Hilf mir auch itzt und in Zukunft*
> *Loben die Auserwählt und Zart,*
> *Ihr Gstalt, Sitten und gute Art,*
> *Auf dass ich mit Lob müg bekrönen*
> *Die Auserwählt Tugendreich Schönen,*
> *Dass von mir ausbreit mit Begierd*
> *Werd ihr weiblich Natur, geziert*
> *Vor allen Frauen und Jungfrauen*
> *So ich vor tät mit Augen schauen*
> *Hin und wieder in manchem Land,*
> *Dergleich mir keine war bekannt,*
> ...
> *Dass unser ehlich Lieb und Treu*
> *Sich täglich alle Tag verneu,*
> *Zunehm und fruchtbarlich aufwachs*
> *Bis an das End, das wünscht Hans Sachs.*
>
> (Anno salutis 1562, am 4. Tag Septembris)

**Bente at Hans Sachs Brunnen in Nürnberg, summer 2014**

## CV FOR OLE EILER BARNDORFF-NIELSEN

September 19, 2018 (?)

### Ab initio

Mag. scient. in mathematics and mathematical statistics from Aarhus University, June 1960

1954-1958 part-time employment at Department of Biostatistics, the Danish State Serum Institute

1958-1959 teaching assistant and 1959-1960 scientific assistant in mathematical statistics at University of Copenhagen

Since 1 July 1960 affiliated with Institute of Mathematics, now Department of Mathematical Sciences, Aarhus University. First as amanuensis and lektor (assistant professor); from 1 November 1965 as afdelingsleder and lektor (associate professor); and from 1 August 1973 as Professor.

1962-1963 and 1963-1964 stays at, respectively, University of Minnesota and Stanford University

### Various professional data

Sc.D. from University of Copenhagen, July 1973

August 1974 - February 1975 Overseas Fellow at Churchill College, Cambridge, and visitor at Statistical Laboratory, Cambridge University. April 1990 visiting Japan under Fellowship from the Japan Society for the Promotion of Science.

Member of the Royal Danish Academy of Sciences and Letters

Member of Academia Europaea (M.A.E.)

Corresponding Fellow of the Royal Society of Edinburgh (CorrFRSE)

Honorary Fellow of the Royal Statistical Society; Member of the Bernoulli Society for Probability and Mathematical Statistics; Fellow of the Institute of Mathematical Statistics; Honorary Member of the Danish Society for Theoretical Statistics; Honorary Member of the International Statistical Institute.

R.1 11 September 1998

Former Chairman of European Regional Committee of the Bernoulli Society. Former member of Council of the Bernoulli Society, of Council of the Danish Society for Theoretical Statistics, of Council of the Institute of Mathematical Statistics, and of the Council of the International Statistical Institute.

President-Elect, 1991-1993, President, 1993-1995, Bernoulli Society for Mathematical Statistics and Probability

Chairman of ERCOM (European Research Centres on Mathematics) under EMS (European Mathematical Society), 1997-2002

Editor of ***International Statistical*** *Review* 1980-1987

Member of the Editorial Board of **Scandinavian Journal of Statistics** 1974-1985

Associate Editor of ***Annals of Statistics*** 1992-1994

Member of the Editorial Board of **Annales de Toulouse, Mathématiques** 1992-2002

Editor-in-Chief of **Bernoulli** 1994-2000

Member of the Editorial Board of **JEMS** (Journal of the European Mathematical Society) 1998-2011

Member of the Editorial Board of **Lévy Matters** 2010-

Member of Scientific Advisory Board for Mathematisches Forschungsinstitut Oberwolfach 2006-2007

Organiser/Coorganiser of many international conferences in the mathematical sciences

Invited lecturer/visitor at universities in Europe, Australia, Brazil, Japan, Mexico, North America, Russia

Dr. Honoris Causa, Université Paul Sabatier, Toulouse, June 1993

Dr. Honoris Causa, Katholieke Universiteit Leuven, April 1999

Scienti c Director, **MCAA** (Mathematical Research Centre at Aarhus University), 1995-1997.

Scientific Director, **MaPhySto** (Centre for Mathematical Physics and Stochastics). Funded by The Danish National Research Foundation, 1998-2003

Humboldt-Forschungspreis 2001

Fellow, Institute of Advanced Studies, TUM. 2008-

Faculty Prize 2010 from Faculty of Science, Aarhus University

Holst Knudsen Prize 2014 from Aarhus University

International Expert, Russian Science Foundation. 2015-

### Scientific Works

6 Research Monographs; about 275 Research Papers; 9 Edited Volumes

### Film

***Blown Sand.*** Film on the physics of blown sand and the life of Brigadier R.A. Bagnold, F.R.S. Produced for the Faculty of Science, Aarhus University, with financial support from Svend Bundgaard's Fond and from Tuborgfondet. (1984). Web address: http://www.youtube.com/watch?v=u73mtDZXZV8